Math 662- Midterm Exam **Name:_____**

October 25, 2006 **Student #: _____**

Must show all work for full credit!!!

I pledge that I have not violated the NJIT code of honor_____

1. A person answers each of two multiple choice questions at random. If there are four possible choices on each question, what is the conditional probability that both answers are correct given that at least one is correct?

(10 pts)

2. A laboratory blood test is 95 percent effective in detecting a certain disease when it is, in fact, present. However, the test also yields a "false positive" result for 1 percent of the health persons tested. (That is, if a healthy person is tested, then, with probability 0.01, the test result will imply he or she has the disease.) If 0.5 percent of the population actually has the disease, what is the probability a person has the disease given that the test result is positive?

(12 pts)

3. Let X be a random variable whose probability mass function (*pmf*) is given by $p(x) = c\left(\frac{2}{3}\right)^x, \quad x = 1, 2, 3, \ldots$ zero elsewhere. Find the constant c > 0 and the cumulative distribution function (*cdf*) of X.

(12 pts)

4. Let X be a random variable whose *pmf* is given by $p(x) = \left(\frac{1}{2}\right)^x, \quad x = 1, 2, 3, \ldots$ zero elsewhere. Let $Z = (X-2)^2$. Find the *pmf* of Z.

(12 pts)

5. A random variable X has its *cdf* given by $F(x) = (1+e^{-x})^{-1}, -\infty < x < \infty$. Find the density function of X. Also find the 60th percentile for the given distribution.

(12 pts)

6. Let X have the density function given by $f(x) = \begin{cases} 4x^3, 0 < x < 1 \\ 0, \ elsewhere. \end{cases}$

Find the cdf and the density function of and $Y = -\ln(X^4)$.

(14 pts)

7. Let X and Y have the joint probability density function given by

$$f(x, y) = \begin{cases} 3x, 0 < y < x < 1, \\ 0, \quad elsewhere. \end{cases}$$

Are X and Y independent? Why or why not? Compute $E(X / y)$.

(16 pts)

8. Let X_1 and X_2 have joint probability density function given by

$$f(x_1, x_2) = \begin{cases} 2e^{-x_1-x_2}, 0 < x_1 < x_2 < \infty, \\ 0, \qquad\qquad elsewhere. \end{cases}$$

Find the joint density function of $Y_1 = 2X_1$ and $Y_2 = X_2 - X_1$.
(12 pts)