Math 663-101, Spring 2011 **Name:**____________

Mid-Term Exam **Student ID:**____________

March, 28. *Please show the complete solution (with all steps) to each problem to receive perfect score!*

I pledge I have not violated the NJIT Honor Code______________________

1. The following data reflect the time in minutes between prescription drop-off and pick-up at a local pharmacy:

   8 15 7 6 4 20 3 6 4 3

   Compute the:

   a. Mean time between drop-off and pick-up. $\doteq 7.6$ min. (2 points)

   b. The standard deviation in time between drop-off and pick-up. $\doteq 5.6015587077$ (2 points)

   c. Using 1 a. and 1 b. above find outliers in the data. Give reason for your answer. (8 points)

   d. Compute the first and third quartiles of this data (which is used to create box-plot). (4 points)

c. Smallest observation $= 3$

$$z\text{-score} = \frac{3 - 7.6}{5.6017587077} \doteq -0.8212$$

Since $0.8212 < 3$, 3 is not an outlier

Hence the are no outliers on the low end of the data.

Highest observation is 20

$$\frac{20 - 7.6}{5.6017587077} = 2.21 < 3$$

Hence 20, is not an outlier.

Therefore there are no outliers.

d. $n = 10$ the position of 25^{th} percentile $=$

1^{st} Quartile $= \left\lceil \frac{1+5}{2} \right\rceil = \lceil 3 \rceil = 3$

or $\left\lceil \frac{1+\frac{n}{2}}{2} \right\rceil = \left\lceil \frac{n+2}{4} \right\rceil = 3$. Ordered data $=$ 3, 3, $\boxed{4}$, 4, 6, 6, 7, $\boxed{8}$, 15, 20

Hence $Q_1 = 4$ and $Q_3 = 8$.

Answer $(21.05'', 28.95'')$ is normal

2. The heights of children are approximately normally distributed at each age and gender. If the mean height for two year old females is 25 inches, with a standard deviation of 2.4 inches, find the following:
   a. The probability that a two year old female is between 24 and 26 inches.(8 points)
   b. Suppose that the smallest 5% and largest 5% of heights are considered "abnormal." What range of heights is "normal"? Give answers to two decimal places accurate. (10 points)

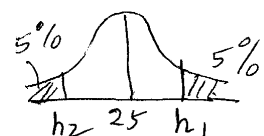
X: height of 2-year old girls.

(a) $P(24 < X < 26) = P\left(\frac{24-25}{2.4} < Z < \frac{26-25}{2.4}\right)$

$= P(-.42 < Z < .42) = 1 - 2(1 - .6628)$

$= 1 - 2(.3372) = 1 - .6744$

$= 0.3256$

(b) 

$P\left(Z > \frac{h_1 - 25}{2.4}\right) = .05$

$\frac{h_1 - 25}{2.4} = 1.645$ $\quad h_1 = 28.948''$ $\quad P\left(Z < \frac{h_2 - 25}{2.4}\right) = .05 \Rightarrow h_2 - 25/2.4 = -1.645 \Rightarrow h_2 = 21.05''$

3. An HMO is considering a marketing plan to increase the numbers of patients who get flu shots, particularly patients with chronic diseases. Before implementing such a plan, some preliminary analyses are conducted. Available data indicates that 40% of patients with chronic diseases get flu shots.
   a. If 12 patients with chronic diseases are sampled at random, what is the probability that at least two patients get flu shots regularly? (8 points)
   b. If a particular clinic which is part of the HMO serves 5220 patients with chronic disease, how many would be expected to get flu shots? Also, what would be the standard deviation of the number of patients out of 5220 who would get flu shots? (4 points)
   c. If nobody out of 5220 patients showed up to get flu shots, would you consider it to be a rare event? Why? Or why not? (4 points)

b $\mu = (5220)(.4) = 2088$

$\sigma = \sqrt{npq} = \sqrt{(5220)(.4)(.6)}$

$= \sqrt{(2088)(.6)} = 35.395$

(c) $P(\text{nobody show up for flu shot})$

$= \binom{5220}{0}(.4)^0(.6)^{5220}$

$= 0$ is impossible.

Hence, Yes it is a rare event.

a. X: # that get flu shots out of $n = 12$

$P(X \geq 2) = 1 - P(X < 2)$

$= 1 - \{P(X=0) + P(X=1)\}$

$= 1 - \left[\binom{12}{0}(.4)^0(.6)^{12} + \binom{12}{1}(.4)^1(.6)^{11}\right]$

$= 1 - [.0021768 + .017414259]$

$= .980408959.$

4. Scores on a standardized exam are assumed to follow a normal distribution, with mean of 100 and standard deviation of 36. If a simple random sample of 5 exams is selected:
   a. What is the standard error of the exam scores? (5 points)
   b. What is the probability that the sample mean score will exceed 125? (8 points)
   c. Give justification for the method used in 4.b. above. (3 points)

$\mu = 100 \quad \sigma = 36 \quad n = 5$

(a) $\sigma_{\bar{X}} = \dfrac{36}{\sqrt{5}} = 16.0997$

(b) $P(\bar{X} > 125) = P\left(Z > \dfrac{125-100}{16.0997}\right)$
$= P(Z > 1.55) = 1 - 0.9394 = .0606$

(c) 'Z' is used because a random sample from normal gives $\bar{X}$ to be normal because $\sigma = 36$ is given.

5. A randomized trial is run to compare two competing medications for peripheral vascular disease. Fifty patients with the disease are randomized in to one of the two medication treatment groups. One of the outcomes is self-reported physical functioning. After taking the assigned medication for 6 weeks, patients provide data on their abilities to perform various physical activities, and a score is computed for each individual. The physical functioning score range from 0 to 100, with higher scores indicative of better functioning. The data are:

Medication	Number of patients	Mean score	Standard deviation of score
1	25	70.6	24.4
2	25	77.23	19.8

Based on the data, test whether Medication 2 is significantly better than Medication 1.
   a. Justify the test used. Take alpha = 0.05 for all tests used in this problem. 3 (10 points)
   b. Find the p-value of the test. (7 points)

$F = \dfrac{S_1^2}{S_2^2} = \dfrac{(24.4)^2}{(19.8)^2} = 1.5186206$

$H_0: \sigma_1^2 = \sigma_2^2$ is not rejected in the region $\dfrac{1}{F_{24,24,.975}}$ to $F_{24,24,.975}$

$\doteq \dfrac{1}{F_{20,24,.975}}$ to $F_{20,24,.975}$

$.429 = \dfrac{1}{2.33}$ to 2.33

Hence do not reject H_0 $\sigma_1^2 = \sigma_2^2$

$S_p^2 = \dfrac{(24.4)^2 + (19.8)^2}{2} = 493.7$

$\Rightarrow S_p = 22.21936$

$H_0 \quad \mu_1 = \mu_2$
$H_1 \quad \mu_1 < \mu_2$

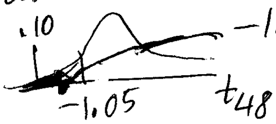
P.T.O.

$$t = \frac{\overset{-6.63}{70.6 - 77.23} - 0}{22.21936\sqrt{\frac{1}{25} + \frac{1}{25}}} = -1.05$$

$df = 24 + 24 = 48$

Since $-1.677 < -1.05$
do not reject H_0. Medication 2 may not be better that Medication 1 (not significant) at $\alpha = .05$.

$\boxed{\text{p-value} > .10}$ =



6. An investigator wished to design a study to estimate the proportion of patients in a particular hospital whose primary insurance coverage is Medicaid.
   a. How many subjects would be required to estimate the true proportion within 3% with 98% confidence? (7 points)
   b. Suppose a similar study was conducted in 2003 and produce the following 98% confidence interval for the proportion of patients in the same hospital whose primary insurance coverage was Medicaid: 27% ± 6%. If the point estimate from the 2003 study is used, how does that affect the answer given in 6.a.? (7 points)
   c. Which answer a. or b. will cost less to implement? Explain. (3 points)

$$Z_{1-\alpha/2}\frac{\sqrt{p(1-p)}}{\sqrt{n}} \le .03$$

(a)

$$\frac{Z_{(1-.01)}\sqrt{(.5)(.5)}}{.03} \le \sqrt{n} \qquad Z_{.99} = 2.33$$

$$\frac{Z_{.99}(.5)}{.03} \le \sqrt{n} \Rightarrow n \ge \left[\frac{(2.33)(.5)}{.03}\right]^2 = 1508.03$$

$\boxed{n = 1509.}$

b

$$\frac{2.33\sqrt{(.27)(.73)}}{(.03)} \le \sqrt{n}$$

$$n \ge \left(\frac{2.33\sqrt{.27(.73)}}{.03}\right)^2 = 1188.9291$$

$n = 1189$

c. b. will cost less because smaller sample size.