Math 663-102, Spring 2012 **Name:**____________
Mid-Term Exam **Student ID:**_____________

March, 26. *Please show the complete solution (with all steps) to each problem to receive perfect score!*
I pledge I have not violated the NJIT Honor Code______________________

1. a. ~ d. A sample was taken of 20 salaries of employees in a large health insurance company. The following are the annual salaries (in thousands of dollars). For convenience, the data have been ordered.

28	31	34	35	37	41	42	42	42	47
49	51	52	52	60	61	67	72	75	77

a. What is the median salary of the 20 employees? in \$ (5 points)

Median = average of 10th & 11th observation

$\frac{47+49}{2} = 48$ // i.e \$48,000.00

b. What is the first quartile of the 20 salaries? in \$ (5 points)

Q_1 is the $\left[\frac{1+10}{2}\right]^{th}$ observation $\Rightarrow$

$= [5.5] = 5^{th}$ observation $= 37$ //

i.e., \$37,000.00

c. What is the interquartile range of the 20 salaries? in \$ (5 points)

$Q_3 = 5^{th}$ observation from the right (high salaries)

$= 61$, i.e., \$61,000.00

$IQR = 61 - 37 = 24$, ie, \$24,000.00

d. Suppose each employee in the company receives a \$3000 raise for next year (each employee's salary is increased by \$3000). For each of the following summary measures, indicate how it would change after the raise. (5 points)

A) The median salary. in \$ median will increase by \$3,000,
B) The interquartile range of the salaries. in \$ no change,
C) The standard deviation of the salaries. in \$ no change.

2. a. – c. The following data were collected from the 2003 administrative records at a local community center:

	Primary Diagnosis				
Age	Diabetes	Asthma	Arthritis	Cardiac	Total
30 - 39	27	56	20	30	133
40 - 49	32	32	25	24	113
50 – 59	30	14	43	43	130
60+	29	7	65	41	142
Total	118	109	153	138	518

a. Are the variables Age and Primary Diagnosis independent of each other? (8 points)

$$P(Age_{30-39}) = \frac{133}{518} = .2568 \qquad P(Diabetes) = \frac{118}{518} = .2278$$

Not independent because $P(Age_{30-39}) \cdot P(Diabetes) = .0585 \neq P(Age_{30-39} \cap Diabetes) = \frac{27}{518} = .0521$

b. What proportion of 60+ years of age has either Diabetes or Cardiac as their primary diagnosis? (7 points)

$$P(Diabetes \cup Cardiac / 60+) = \frac{29+41}{142} = \frac{70}{142} = .493$$

c. What proportion of patients is of age 30 - 39 or has Asthma as their primary diagnosis? (Please see problem 6, page 138 and 7, page 139) (5 points)

$$P(Age\ 30\text{-}39 \cup Asthma) = \frac{\overbrace{(27+56+20+30)}^{133} + 32+14+7}{518}$$

$$= \frac{133+53}{518} = \frac{186}{518} = \frac{93}{259} \approx 0.3591$$

3. A longitudinal study is conducted requiring patients to follow up with research associates every month for assessments. The probability a patient fails to follow up in a given month is 12%. A pilot study is conducted to assess feasibility involving 15 patients. What is the probability at least two patients will fail to follow up in the first month? What model assumptions are necessary to validate this probability computation? (Please see problem 16, page 142) (15 points)

X: # who fail to follow up out of $n=15$

$$P(X \geq 2) = 1 - P(X<2)$$

$$= 1 - [P(X=0) + P(X=1)]$$

$$= 1 - \left[\binom{15}{0}(.12)^0(.88)^{15} + \binom{15}{1}(.12)^1(.88)^{14}\right]$$

$$= 1 - .447602191 = .552397809$$

0. Each patient either follows up (p) or does not follow up (1-p).
1. The 15 patients follow up independently of each other.
2. The probability each patient follows up is .12 & does not is .88, for n=15.

4. a. –b. A dispensing machine is set to produce 1-pound lots of a particular compound. The machine is fairly accurate, producing mean weight of lots equal to 1.0 pounds with a standard deviation of 0.12 pounds. Thirty two lots are randomly selected.

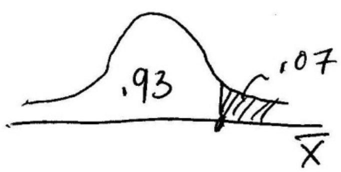
a. Find the probability that the mean weight is less than 0.96 pound.

$$P(\bar{X} < .96) = P\left(Z < \frac{.96 - 1}{\frac{.12}{\sqrt{32}}}\right)$$

$$= P(Z < -1.89)$$

$$= 0.0294$$

b. Find the mean weight in pound whose value is exceeded by only 7% of the mean weights. (Please see problem 3, page 168) (15 points)



$$\frac{\bar{X} - 1}{\frac{.12}{\sqrt{32}}} = 1.48 \Rightarrow$$

$$\bar{X} = 1 + (1.48)\left(\frac{.12}{\sqrt{32}}\right)$$

$$= 1.0313955 \text{ pound}$$

5. We wish to run the following test H_0: $\mu = 100$, H_1: $\mu > 100$ at $\alpha = 0.05$. If $\sigma = 10$, how large a sample would be required so that the power = 0.80 when $\mu = 105$? (10 points)

$$n = \left[\frac{Z_{1-\alpha} + Z_{1-\beta}}{ES}\right]^2 \qquad ES = \frac{|\mu_0 - \mu_1|}{\sigma}$$

$$= \left[\frac{1.645 + 0.84}{.5}\right]^2 \qquad = \frac{|100 - 105|}{10}$$

$$= [4.97]^2 \qquad = .5$$

$$= 24.7009$$

Ans = 25

#6 Treatment A $\bar{x}_A = 69.125$

$s_A = 8.773946823$

$\bar{x}_B = 82.875$

$s_B = 5.383506558$

$$F = \frac{s_A^2}{s_B^2} = 2.656$$

$F_{7,7,.025} = 4.99$ $\quad \frac{1}{4.99} < 2.656 < 4.99$

4.99

hence do not reject H_0 $\sigma_A^2 = \sigma_B^2$.

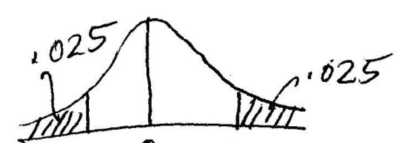
$$s_p^2 = \frac{76.98214285 + 28.98214286}{2} = 52.98214286$$

H_0 $\mu_A = \mu_B$

H_1 $\mu_A \neq \mu_B$

$$t = \frac{69.125 - 82.875 - 0}{\sqrt{52.9824429}\sqrt{\frac{1}{8} + \frac{1}{8}}} = \frac{-13.75}{3.639441676}$$

$= -3.778052027$



$t_{14,.025} = 2.145$

Since $-3.778052027 < -2.145$

Reject H_0, treatment A is better. Assumption: need both data sets to be normally distributed independent random samples.