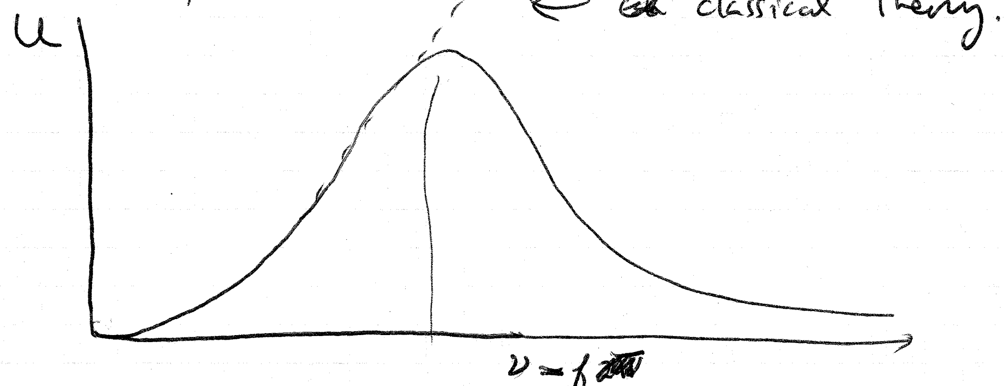
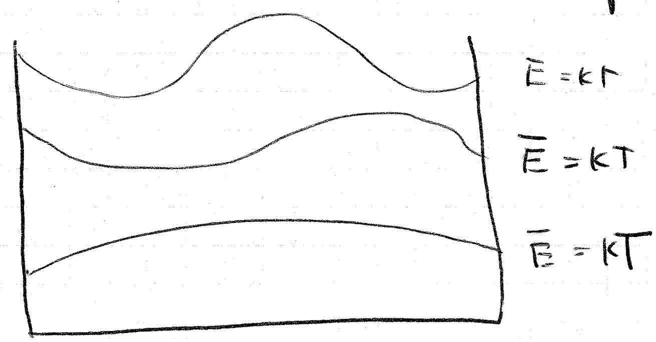
7.4 Blackbody Radiation



↑ ultraviolet frequencies — "ultraviolet catastrophe"

Classically

$U = 2 \cdot \frac{1}{2} kT \cdot N$



total energy <u>infinite</u>.

since # of oscillators infinite

⇒ wavelength can always be smaller and smaller.

Plank to the rescue ⟹ Energies of harmonic oscillators quantized.

$$E_n = 0,\ hf,\ 2hf,\ 3h\nu \ldots$$

$$Z = 1 + e^{-\beta hf} + e^{-2\beta hf} + \ldots$$

$$= \frac{1}{1 - e^{-\beta hf}}$$

$$\bar{E} = -\frac{1}{Z}\frac{\partial Z}{\partial \beta} = \frac{hf}{e^{hf/kT} - 1}$$

$$\bar{n}_{PI} = \frac{1}{e^{h\nu/kT}-1}$$ $\Leftarrow$ # of units of energy in the oscillator

plank distribution

<u>note</u> $\bar{n}_{PI} \ll 1$ for $h\nu/kT \gg 1$

$\Rightarrow$ high frequency photons "frozen out"

Compare with ~~BE~~ Bose - Einstein

$$\bar{n}_{BE} = \frac{1}{e^{(\varepsilon-\mu)/kT}-1}$$ $\Rightarrow$ $\mu = 0$ for photons.

why? $\left(\frac{\partial F}{\partial N}\right)_{T,V} = \mu$ (at equilibrium)

$\Rightarrow$ but # of photons is not constrained.

$\Rightarrow$ they can be created or destroyed (unlike electrons)

so $\frac{\partial F}{\partial N} = 0 \Rightarrow \mu = 0$

Let's calculate total energy $\lambda = \frac{2L}{n}$ $p = \frac{h}{\lambda}$

$\Rightarrow$ sum over modes $\varepsilon = pc = \left(\frac{hn}{2L}\right)c$

L is size of box containing photons.

$$\varepsilon = c\sqrt{p_x^2+p_y^2+p_z^2} = \frac{hc}{2L}\sqrt{n_x^2+n_y^2+n_z^2}$$

$$U = 2 \sum_{n_x} \sum_{n_y} \sum_{n_z} \varepsilon \bar{n}_{PL} = 2 \sum\sum\sum \left(\frac{hn}{2L}\right) c \frac{1}{e^{\frac{hcn}{2L}/kT} - 1}$$

Polarizations

convert to integral in spherical coordinates

$$U = \int_0^\infty dn \int_0^{\pi/2} d\theta \int_0^{\pi/2} d\phi \, n^2 \sin\theta \; \frac{hcn}{L} \frac{1}{e^{hcn/2LkT} - 1}$$

Change variables $\quad \varepsilon = \dfrac{hcn}{2L} \qquad d\varepsilon = dn \left(\dfrac{hc}{2L}\right)$

$$U = \left(\frac{2L}{hc}\right)^3 \int_0^\infty d\varepsilon \left(\frac{\pi}{2}\right) \frac{2\varepsilon^3}{e^{\varepsilon/kT} - 1}$$

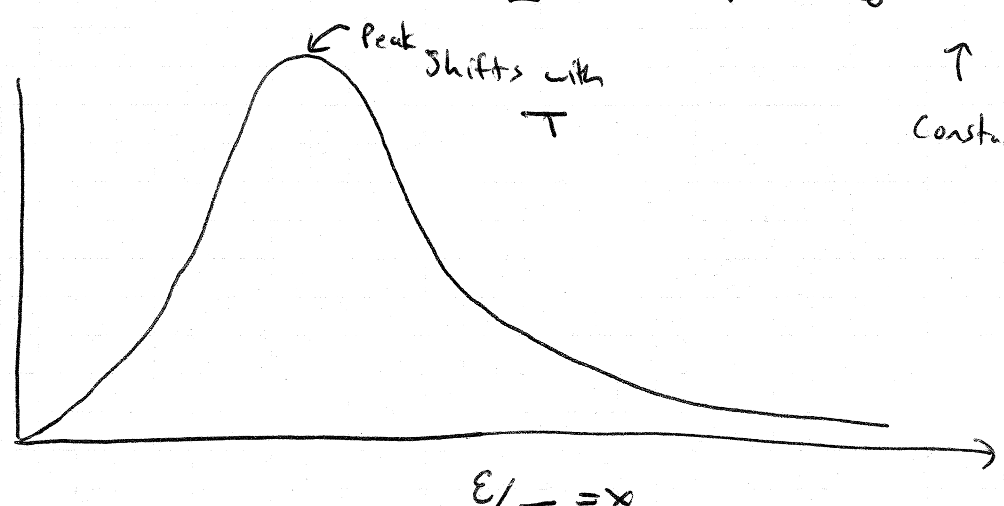
energy density $\Rightarrow$

$$\frac{U}{L^3} = \frac{8\pi}{(hc)^3} \int_0^\infty \frac{d\varepsilon \, \varepsilon^3}{e^{\varepsilon/kT} - 1}$$

let $x = \dfrac{\varepsilon}{kT} \Rightarrow \dfrac{U}{L^3} = \dfrac{8\pi}{(hc)^3} (kT)^4 \left[\displaystyle\int_0^\infty \frac{dx \, x^3}{e^x - 1}\right]$

Constant.

$$\frac{x^3}{e^x - 1} = \frac{(\varepsilon/kT)^3}{e^{\varepsilon/kT} - 1}$$



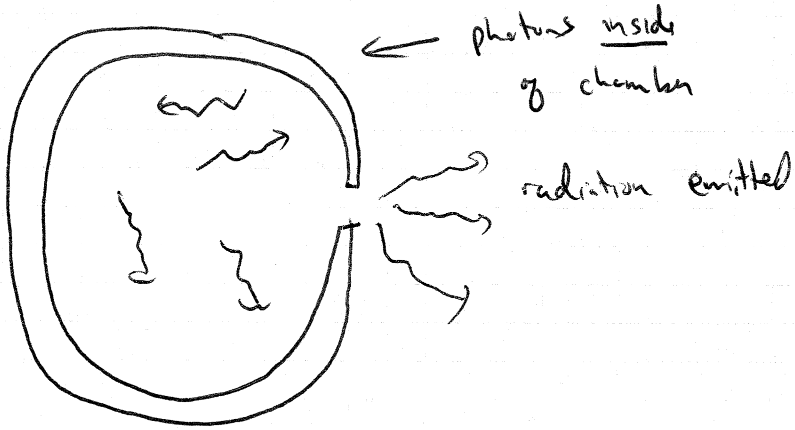
Note: $\frac{U}{V} \sim T^4$

Note: curve peaks at $x = 2.82$ or $\varepsilon = 2.82\,kT$

Wien's displacement Law $\Rightarrow$ as T increases, peak of Blackbody emission shifts to higher photon energy.

Blackbody radiation from Cosmos (~ 2.7 K) $\Longrightarrow$ evidence of "Big bang" remnant.

How does one actually **measure** the Black body radiation?

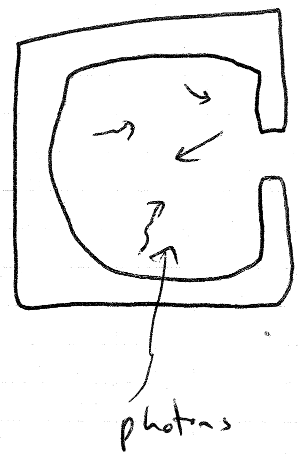


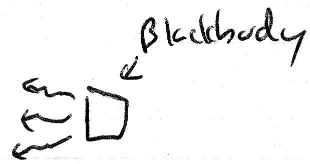
measuring emitted spectra is same as spectra inside assuming that hole is ~~make~~ small.

$$\frac{\text{Power}}{\text{Area}} \cancel{\neq} \sim \frac{cU}{V} = \sigma T^4 \leftarrow \text{power per area emitted}$$

σ ← stefan-Boltzmann constant.

What if we don't have a cavity, but just a "black" body?





T of both the same.

Consider size of hole and body to ~~be~~ be same.

hole/body absorb radiation from each other.

Spectrum and power must be same for both....

Why?

If not, more energy flows to black body (for example) and its T should rise. ⇒ violates 2nd law of thermo

⇒ so black body spectrum (derived for a cavity) must still hold for arbitrarily shaped "black" object.

$$\text{Power} = \sigma e A T^4$$

A ← objects surface area

emissity → e

emissity → measure of how "black" an object is

1 ⇒ ~~per~~ perfect blackbody   0 ⇒ perfect reflector.

7.5 Debye Theory of Solids

In Problem 3.25, you solved for heat capacity of an "Einstein" solid. $\varepsilon_n = 0, \varepsilon, 2\varepsilon, 3\varepsilon, \ldots$ etc.

"Einstein model"
$$C_v = \frac{3Nk\,(\varepsilon/kT)^2\, e^{\varepsilon/kT}}{\left(e^{\varepsilon/kT}-1\right)^2}$$

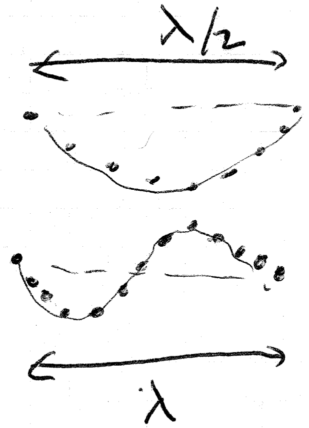
as $T \to 0$ $\quad C_v \sim e^{\varepsilon/kT}$ $\Leftarrow$ exponential decay.

<u>Measured</u> behavior is $C_v \sim T^3$

Model must not be correct....

Debye model $\Rightarrow$ vibrations in solid are collective phenomena.

line of atoms $\Rightarrow$



Similar to EM waves of black body <u>except</u>

- motion of atoms about equilibrium corresponds to <u>sound</u> propagation of <u>mechanical</u> wave

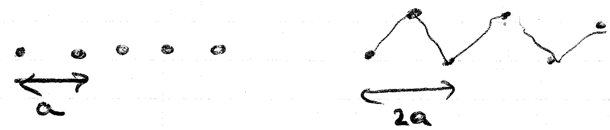
$$c \to c_s$$

$\nwarrow$ speed of sound

- mechanical wave $\Rightarrow$ 2 transverse modes
  1 compressional mode

  $\Rightarrow$ degeneracy of 3

- $\lambda_{max} = 2a$



whereas for E+M waves, $\lambda \to 0$.

so...

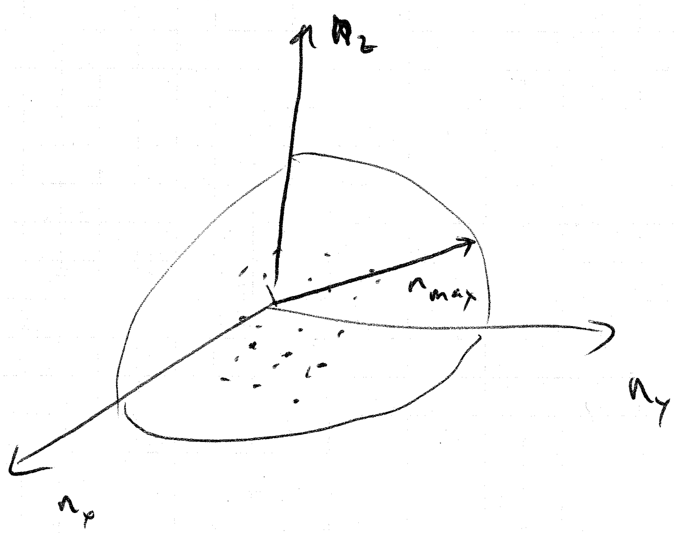
$$\varepsilon = hf = \frac{hc_s}{\lambda} = \frac{hc_s n}{2L} \qquad n = \sqrt{n_x^2 + n_y^2 + n_z^2}$$

$$\bar{n}_{PI} = \frac{1}{e^{\varepsilon/kT} - 1} \qquad \mu = 0 \Rightarrow \text{"phonons"}$$

$$U = 3 \sum_{n_x} \sum_{n_y} \sum_{n_z} \varepsilon \, \bar{n}_{PI}(\varepsilon)$$

we would like to convert to integral in spherical coordinates since $n = \sqrt{n_x^2 + n_y^2 + n_z^2}$

as with Fermi gas and Black body radiation



$$N = \frac{1}{8}(\text{volume sphere})$$

$$N = \frac{1}{8}\left(\frac{4}{3}\pi n_{max}^3\right)$$

$$n_{max} = \left(\frac{6N}{\pi}\right)^{1/3}$$

$$U = 3\int_0^{n_{max}} dn \int_0^{\pi/2} d\theta \int_0^{\pi/2} d\phi \; n^2 \sin\theta \; \frac{\varepsilon}{e^{\varepsilon/kT}-1}$$

$$U = \frac{3\pi}{2}\int_0^{n_{max}} \frac{n^2 \varepsilon \, dn}{e^{\varepsilon/kT}-1}$$

let $\varepsilon = \frac{hc_s n}{2L}$

and $x = \frac{\varepsilon}{kT}$

$$U = \frac{3\pi}{2}\left[\int_0^{x_{max}} \frac{x^3}{e^x-1}\, dx\right] kT \left(\frac{kT}{hc_s}\right)^3 (2L)^3$$

$$x_{max} \equiv \overbrace{\frac{hc_s n_{max}}{2Lk}}^{T_D}\frac{1}{T} \equiv \frac{T_D}{T}$$

← debye T

$$T_D = \frac{hc_s n_{max}}{2Lk} = \frac{hc_s}{2Lk}\left(\frac{6N}{\pi}\right)^{1/2} = \frac{hc_s}{2k}\left(\frac{6N}{\pi V}\right)^{1/3}$$

depends on # of atoms/volume.

$$\cancel{U} = U = N\frac{3\pi}{2} kT^4 \frac{k^3 8V}{(hc_s)^3 N} \int_0^{T_D/T} \frac{x^3}{e^x-1}\, dx$$

← substitute for T_D

$$U = \frac{3\cancel{\pi}}{\cancel{2}} NkT^4 \frac{\cancel{k^3 8}}{\cancel{(hc_s)^3}} \frac{\cancel{(hc_s)^3}}{T_D^3 \cancel{8k^3}} \frac{\overset{3}{\cancel{6}}}{\cancel{\pi}} \int_0^{T_D/T} \frac{x^3}{e^x-1}$$

$$U = \frac{9NkT^4}{T_D^3}\int_0^{T_D/T} \frac{x^3}{e^x-1}\, dx$$

Look at limits $T >> T_D$ (High T limit)

$\Rightarrow$ in ~~the~~ integral $e^x \sim 1 + x$

so
$$\int_0^{T_D/T} \frac{x^3}{e^x - 1}\,dx \simeq \int_0^{T_D/T} x^2\,dx = \frac{1}{3}\left(\frac{T_D}{T}\right)^3$$

so
$$U = 3NkT \qquad (T >> T_D)$$

expected from equipartion theorom.

For $T << T_D$

so
$$\int_0^{\infty} \frac{x^3}{e^x - 1}\,dx = \frac{\pi^4}{15}$$

so
$$U = \frac{3}{5}\pi^4 \frac{NkT^4}{T_D^3} \qquad (T << T_D)$$

$$C_v = \frac{\partial U}{\partial T} = \frac{12\pi^4}{5}\left(\frac{T}{T_D}\right)^3 Nk$$

$C_v \to 0$ as $T \to 0$

Heat capacity for metals at low T

$$C = \left(\frac{\pi^2 N k^2}{2\varepsilon_F}\right)T + \frac{12\pi^4}{5}\left(\frac{T}{T_D}\right)^3 Nk$$

electronic

photons