On the Throughput Region of Single and Two-way Multi-hop Fading Networks with Relay Piggybacking

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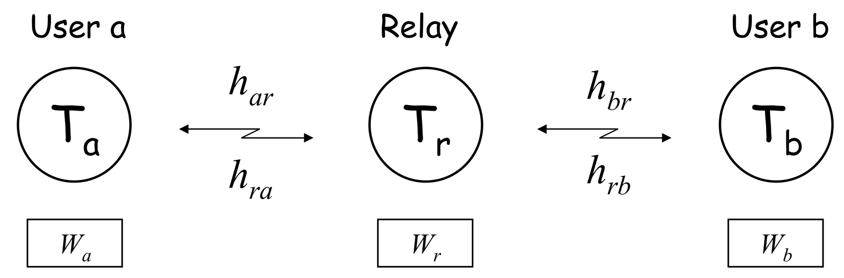
Two-hop Networks



System requirement and settings:

- ✓ Communication between T_a and T_b only through the help of the relay T_r (one or two-way).
- ✓ Terminal T_a (T_b) wants to transmit messages W_a's (W_b's) of nR_a (nR_b) bits each to T_b (T_a) (in the one way scenario T_b has no messages to transmit).
- ✓ The relay T_r needs to transmit common messages W_r's of nR_r bits each to both users [Tannious '07 and Oechtering '08].
- ✓ Messages are taken from an infinite backlog of data.
- ✓ Reliable communication for applications with no delay constraints.

System model:

- ✓ Links subject to i.i.d. Block-Rayleigh fading (symmetric channels).
- ✓ No concerns about decoding and processing delays.
- ✓ Half-duplex nodes.

Proposed solution

Type-I HARQ

Capacity-achieving codes — Retransmission of the codewords — Memoryless decoding

Goal: Determine the throughput regions for different relaying strategies for both one and two-way scenarios.

Related work on Type –I HARQ without relay's messages can be found in [Popovski '06 and Iannello '09]

System Constraints

We consider the following operational constraints:

- ✓ Memoryless operations at all the nodes.
- ✓ Decoding at the nodes is based only on the current packet (no combination of different packet "generations" is allowed).
- ✓ A new message is taken from the backlog only when the last packet is correctly decoded at the intended destination.
- ✓ The relay can only piggyback its own messages W_r to user messages (users' throughput has higher priority than relay's throughput). This avoids the relay to use all the downlink slots for itself.
- ✓ The relay cannot transmit a new message W_r until the current user messages are not correctly delivered.

Throughput Definition

Given non-negative transmission rates (R_a, R_b, R_r) [*bit/s/Hz*], where I_i is a successful decoding event indicator variable for packet W_i:

✓ The users' sum-throughput is (one-way, R_b=0 and $\eta_u=\eta_a$):

$$\eta_u = \lim_{M\to\infty}\frac{1}{M}\sum_{m=1}^{M} R_a I_a[m] + R_b I_b[m]$$

✓ The relay's throughput is:

$$\eta_r = \lim_{M\to\infty}\frac{1}{M}\sum_{m=1}^{M} R_r I_r[m]$$

Throughput Region: Outer Bounds

Proposition: The throughput region is included in the union of all pairs (η_a,η_r) (one-way) or (η_u,η_r) (two-way) that satisfy:

One-way	Two-way
• $2\eta_a+\eta_r \le R^* \exp\left(-\frac{2^{R^*}-1}{P}\right)$	• $\eta_u+\eta_r \le R^* \exp\left(-\frac{2^{R^*}-1}{P}\right)$
• $\eta_r \le \max_{R_r} \frac{R_r}{1+\exp\left(\frac{2^{R_r}-1}{P}\right)}$	• $\eta_r \le \max_{R_r} \frac{R_r}{1+\exp\left(\frac{2^{R_r}-1}{P}\right)}$

Proof outline: cut-set principle and half-duplex constraint.

With $R^*=W_0(P)$, where $W_0(\bullet)$ is the Lambert W function main branch and P is the SNR (unitary noise variance).

One-way: Achievable Throughput Regions

Uplink ($T_a \to T_r$)

T_a transmits its message W_a of nR_a bits through a codeword $x_a(W_a)$. T_a keeps transmitting x_a until W_a is correctly decoded by T_r.

Downlink ($T_r \to T_b$)

Joint encoding (JE)

- ✓ T_r encodes W_a and W_r in a single compound codeword $x_r(W_a,W_r)$.
- ✓ T_r retransmits x_r until W_a and W_b are simultaneously decoded by T_b (no single codeword can be retrieved).

Superposition encoding (SUP)

- ✓ The relay superimposes two codewords to obtain:

$$\mathbf{x}_r = \sqrt{\beta}\mathbf{x}'_r(W_a) + \sqrt{1-\beta}\mathbf{x}''_r(W_r)$$

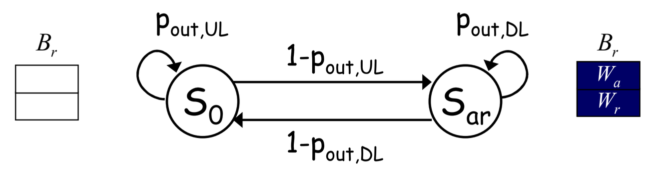
- ✓ If T_b decodes neither W_a nor W_b, T_r retransmits x_r.
- ✓ If T_b decodes W_a (W_r) but not W_r (W_a), T_r transmits x_r setting β=0 (β=1).

The codewords are picked up from a Gaussian codebook of size $2^{n(R_i)}$, where $R_i=R_a$ for $x_a(W_a)$ and $x'_r(W_a)$, $R_i=R_a+R_r$ for $x_r(W_a,W_r)$ and $R_i=R_r$ for $x''_r(W_r)$. All the codewords have an average power constraint: $\|\mathbf{x}_i\|^2 \le nP$

One-way: Throughput Analysis

Throughput analysis using Markov chains. States are based on the content of the relay's buffer B_r.

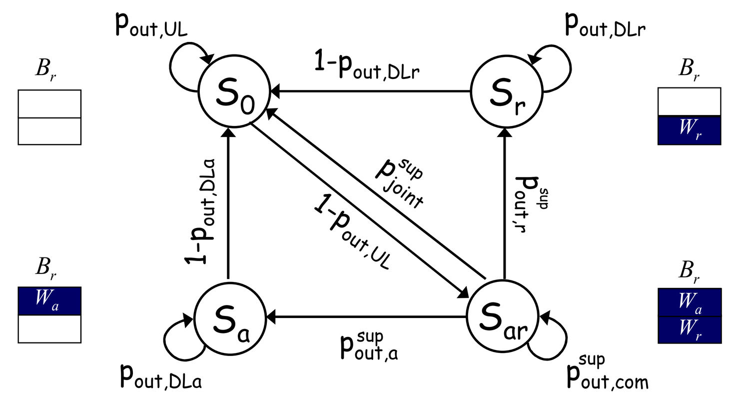
Joint encoding



$$\eta_i^{JE} = R_i \bar{I}_i^{JE}$$

$$\bar{I}_i^{JE} = \pi_{ar}^{JE}(1-p_{out,DL})$$

Superposition encoding



$$\eta_i^{SUP} = R_i \bar{I}_i^{SUP}$$

$$\bar{I}_i^{SUP} = \pi_{ar}^{SUP}\left(p_{joint}^{SUP}+p_{out,j}^{SUP}\right)+\pi_i^{SUP}\left(1-p_{out,DL_i}^{SUP}\right)$$

Where: $\bar{I}_i^{SUP/JE} = E\left[I_i^{SUP/JE}\right]$; $i,j \in \{a,r\}, i \ne j$

The expressions of the transition probabilities can be found in [Narasimhan '07]. π_i indicates the steady state probability of state S_i.

Two-way: Achievable Throughput Regions

Uplink ($T_a \to T_r$, $T_b \to T_r$)
Decode-and-Forward (DF)

No multiple access → **Single-DF (S-DF)**

- ✓ T_a and T_b are not allowed to transmit simultaneously.
- ✓ T_a (T_b) keeps transmitting until W_a (W_b) is correctly decoded by T_r.

Multiple access → **Joint-DF (J-DF)**

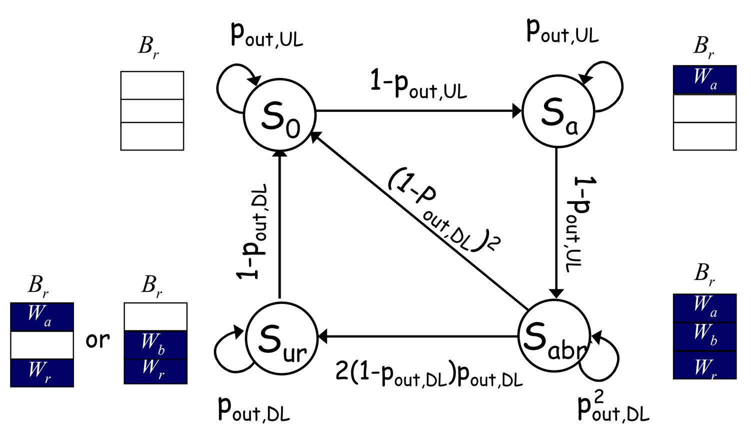
- ✓ T_a and T_b always transmit simultaneously.
- ✓ T_a and T_b keep transmitting until both W_a and W_b are simultaneously decoded by T_r.

Downlink ($T_r \to T_a$, $T_r \to T_b$)

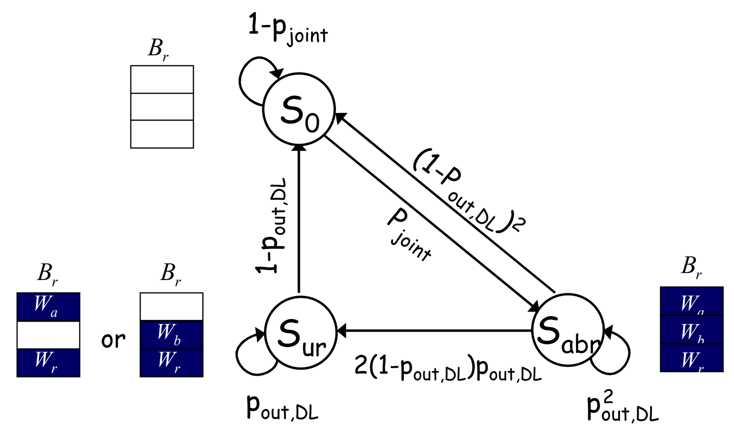
- ✓ T_r encodes messages W_a and W_b both of nR_u bits (symmetric case), and W_r of nR_r bits, in a single codeword $x_r(W_a, W_b, W_r)$ picked up from a Gaussian codebook of size $2^{n(2R_u+R_r)}$.
- ✓ Decoding at the destination nodes T_a and T_b is performed spanning a codebook of size $2^{n(R_u+R_r)}$, thanks to the fact that end users already know their own messages (side information).
- ✓ T_r retransmits x_r until the last of the two users correctly decodes its intended messages, namely W_b (W_a) and W_r for T_a (T_b).

Two-way: Throughput Analysis

S-DF



J-DF

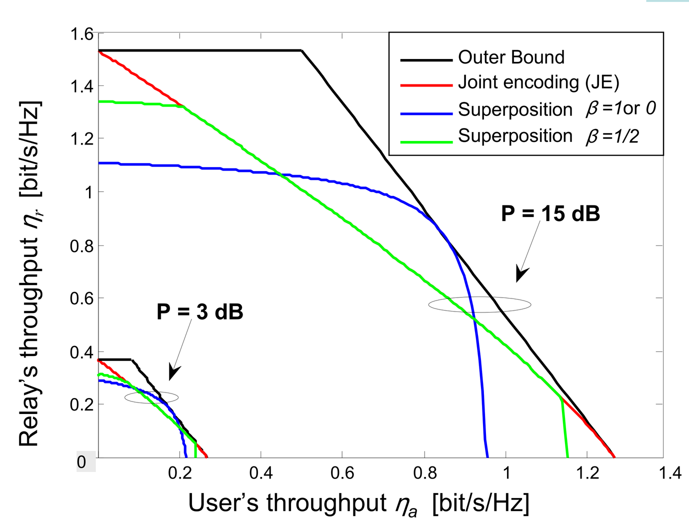


$$\eta_u^{(S/J)-DF} = R_u(1-p_{out,DL})\left(2\pi_{abr}^{(S/J)-DF}+\pi_{ur}^{(S/J)-DF}\right)$$

$$\eta_r^{(S/J)-DF} = R_r(1-p_{out,DL})\left(\pi_{abr}^{(S/J)-DF}(1-p_{out,DL})+\pi_{ur}^{(S/J)-DF}\right)$$

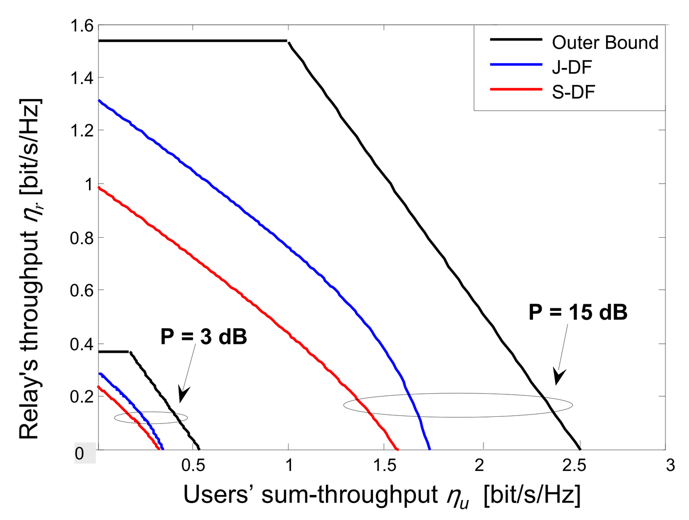
Numerical Results

One-way



- ✓ $\beta=1$ ($\beta=0$) corresponds to a time division (TD) scheme where W_a (W_r) is transmitted first.
- ✓ By time-sharing between TD and JE the outer bound can be approached for all the values such that $\eta_a \ge \eta_r$.
- ✓ The superposition scheme with TD is optimum for $\eta_a=\eta_r$, while joint encoding is optimum for the extreme points, that is, $\eta_a=0$ and $\eta_r=0$.

Two-way



- ✓ The performance of both S-DF and J-DF schemes are strongly affected by the uplink access. In the S-DF scheme the limiting factor is the time division access, while for the J-DF the bottleneck is due to the ability (or inability) of the relay to perform multiple users decoding.