Throughput Analysis of Type-I HARQ Strategies in Two-Way Relay Channels

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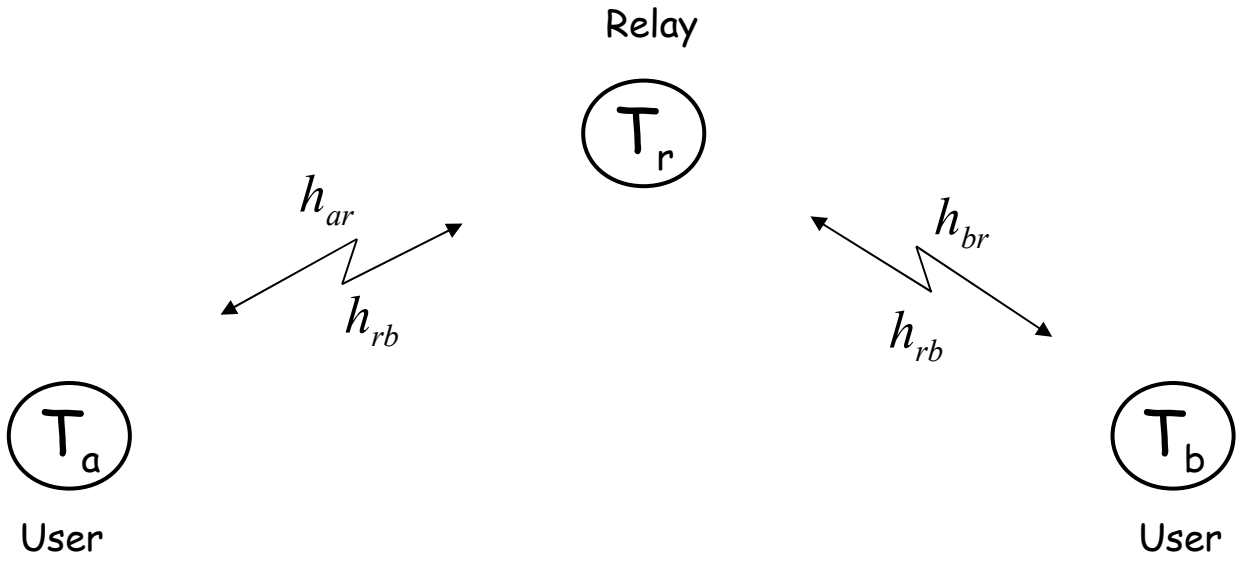
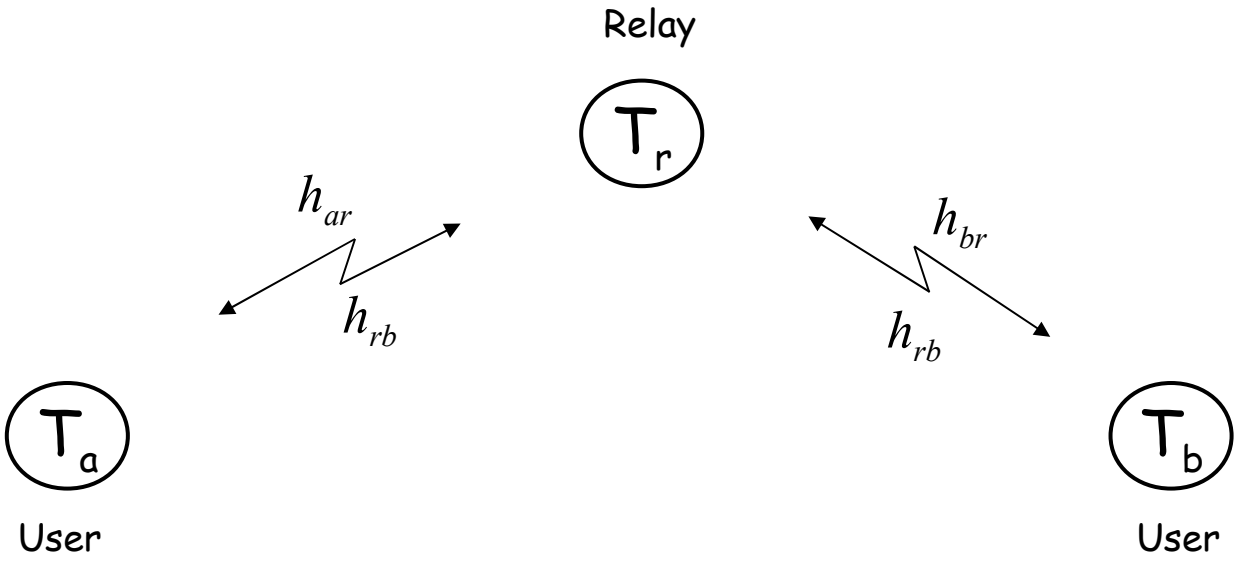
Two-way Relay Channel (TWRC)



- Terminals T_a and T_b want to establish a bidirectional communication
- The direct channels between terminal T_a and T_b are unavailable

The bidirectional communication fully relies on the help of the relay terminal T_r

Two-way Relay Channel (TWRC)



Wireless channel:

- Superposition (Uplink)
- Broadcast (Downlink)

Design of relaying techniques exploiting these properties

Basic relaying strategies

- Decode-and-Forward (DF): the relay decodes users messages.
  - Pro: Denoising by decoding messages.
  - Con: Possible bottleneck at the relay.

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  - Pro: All the processing left to the receivers, no relay bottleneck.
  - Con: Noise amplification.
- Compress-and-Forward (CF): the relay compresses and retransmits the noisy observations.
  - Pro: No relay bottleneck.
  - Con: No denoising at the relay.

Basic relaying strategies

- DF: Decoding of the users' messages at the relay may create bottleneck, but is useful to denoise.
- AF, CF: No decoding no bottleneck, but no denoising.

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- AF, CF: No decoding no bottleneck, but no denoising.

Can a strategy denoise while not creating a bottleneck by fully decoding?

Structured (Lattice) codes

[Narayanan '07, Nam '08, Nazer '08]

Lattice codes principle

Superposition property of the wireless channel

The two users' codewords get summed to the relay

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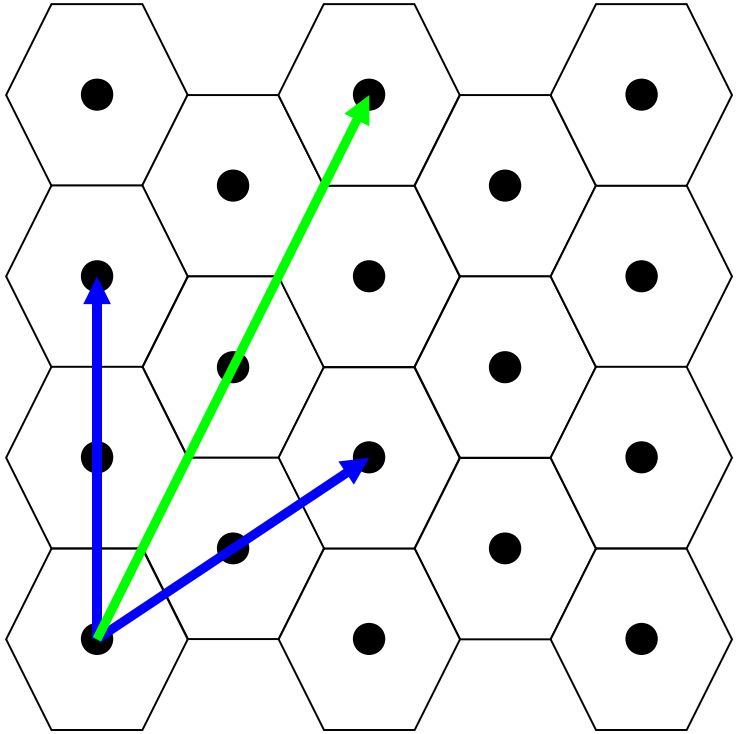
The sum of two codewords is still a codeword of the lattice (group property)

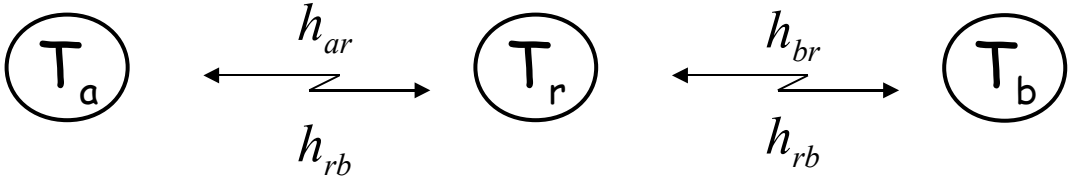


The relay can decode the combined message instead single messages → Denoising without full decoding

Previous work

- Most previous work on the TWRC is based on AWGN channels
  - CF → ½bit to the capacity of the symmetric AWGN TWRC [Gunduz '08]
  - CF → 3/2 bits for general AWGN TWRC [Avestimehr '08]
  - Lattice codes → ½bit to the capacity of the general AWGN TWRC (Narayanan '07, Nam '08)
- Block-fading channels with type-I ARQ
  - single user detection-DF and AF with uncoded transmissions [Popovski '06]

Setting and requirements



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- Non symmetric TWRC
- Reliable communication without concerns about delays

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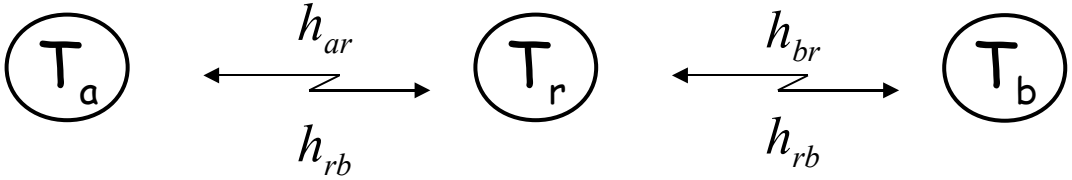

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Type-I HARQ

- Copacity-achieving codes
- Retransmission of copies of the codewords
- "Memoryless" decoding

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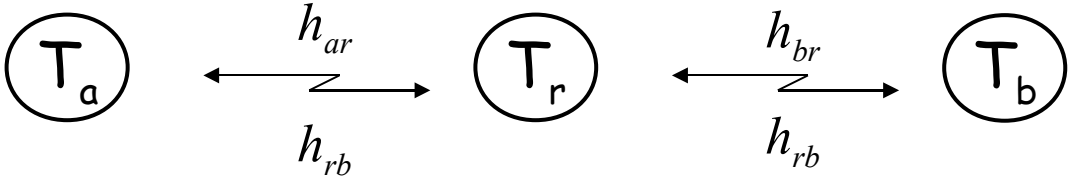

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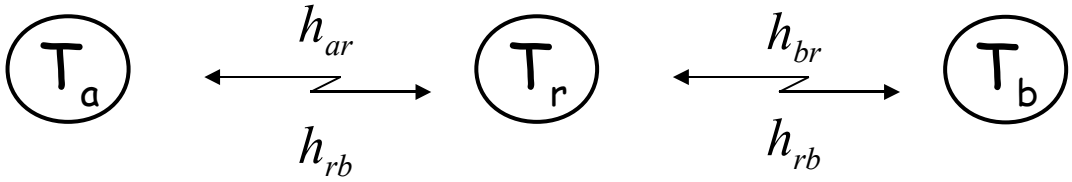
Questions: what is the maximum throughput? How can it be approached?

Contributions of the paper

- Find an Upper Bound (UB) on the sum-throughput
- Achievable sum-throughputs with
  - AF
  - DF (single and multiple user decoding)
  - Lattice
- Numerical results

Problem statement



- Block Rayleigh fading.
- Time is slotted (with fixed duration of the time slot).
- Average power constraint on the transmitted signal.
- Users with infinite backlogs of data intended for the other user.
- The relay has no message to transmit of its own.
- Decoding at the nodes based only on the current (re)transmission.
- No combination of different packets of the same user is allowed.

Throughput definition

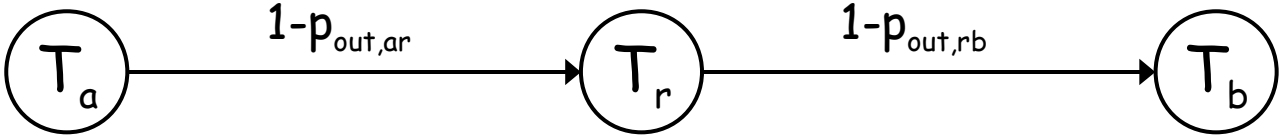
- Long term sum-throughput

$$\eta = \lim_{M \to \infty} \frac{1}{M} \sum_{m=1}^{M} R_a I_a[m] + R_b I_b[m]$$

- R_i : transmission rate of terminal T_i (fixed)
- $I_j[m]$: the destination T_i decodes a packet from T_j during time slot $m, i, j \in \{a, b\}$, with $i \neq j$

Upper Bound for the sum-throughput

- Upper bound for type-I HARQ
  - Enhanced system: each user operates without interference
  - In the enhanced system, decode-and-forward (multihop) with Gaussian codebooks is optimal.



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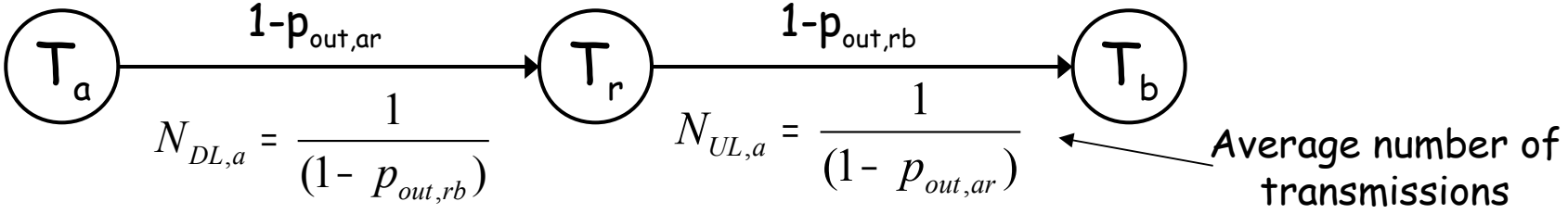

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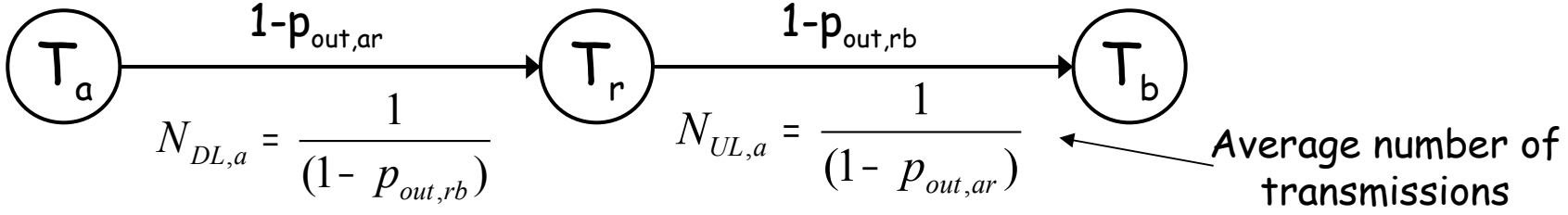

$$\eta_a = R_a\left(N_{UL,a} + N_{DL,a}\right)^{-1}$$

Similarly for $T_b \rightarrow T_a$

$$\eta \leq \eta_{UB} = \eta_a + \eta_b$$

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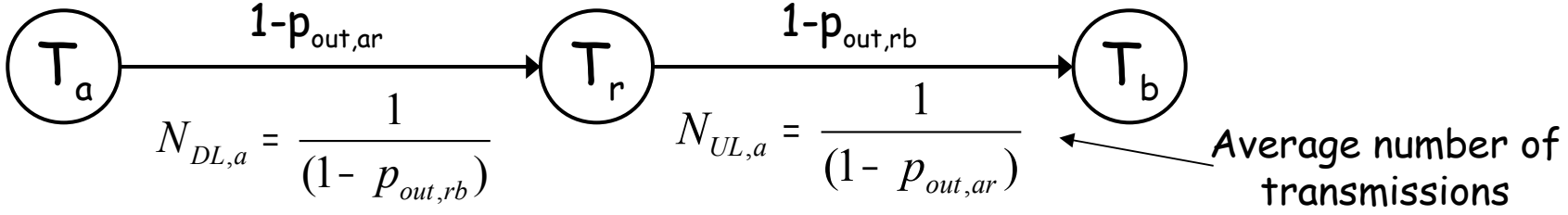

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- Outage probabilities:

$$p_{out,ij} = \Pr\left[R_i > \log_2\left(1+ P_i\left|h_{ij}\right|^2\right)\right] = 1- \exp\left(\frac{2^{R_i} - 1}{P_i}\right) \qquad i, j \in \{a,b,r\},\ i \neq j,\ P_i: \text{SNR}$$

Protocols

- We combine the basic relaying techniques (DF, AF and lattice) with type-I HARQ protocols.
- All the proposed protocols are based on the state of the relay's buffer [Popovski '06].

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- Analysis of the throughput using Markov chains.

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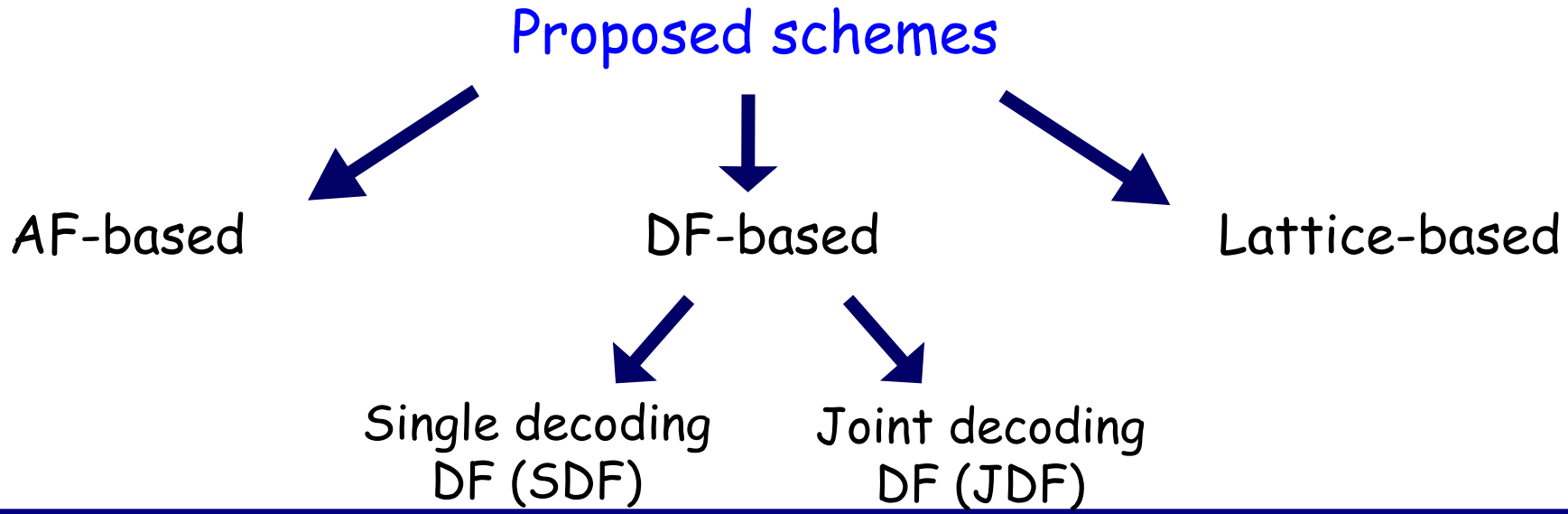
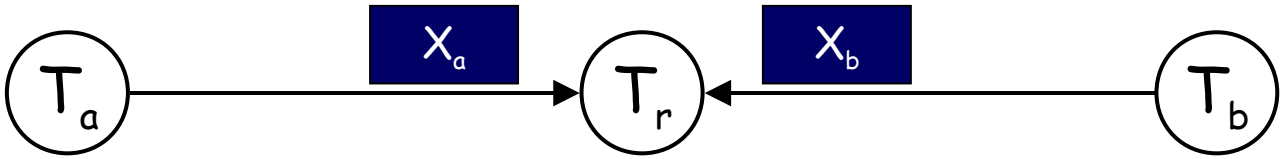


AF-based scheme: protocol

- Even time slots:
  - Both users transmit (or eventually retransmit) simultaneously a message for the other user



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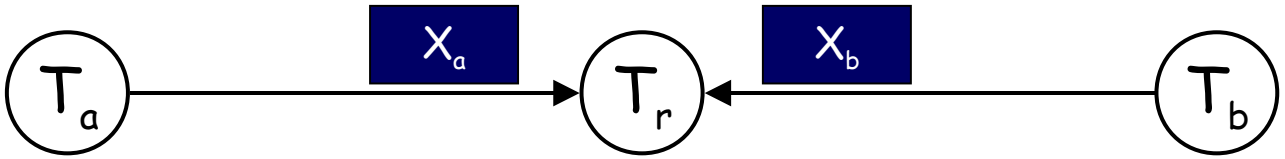
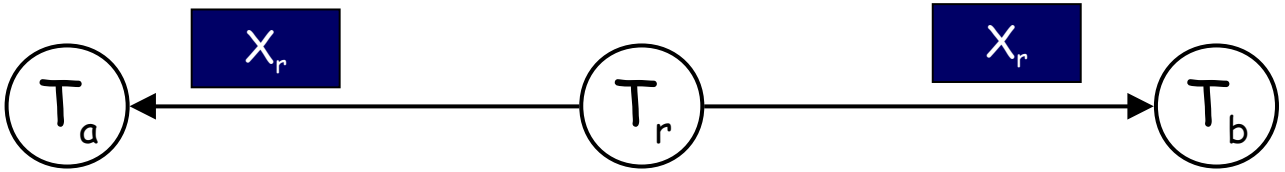


- Odd time slots:
  - The relay transmits the amplified version of the signal received during the previous even time slot



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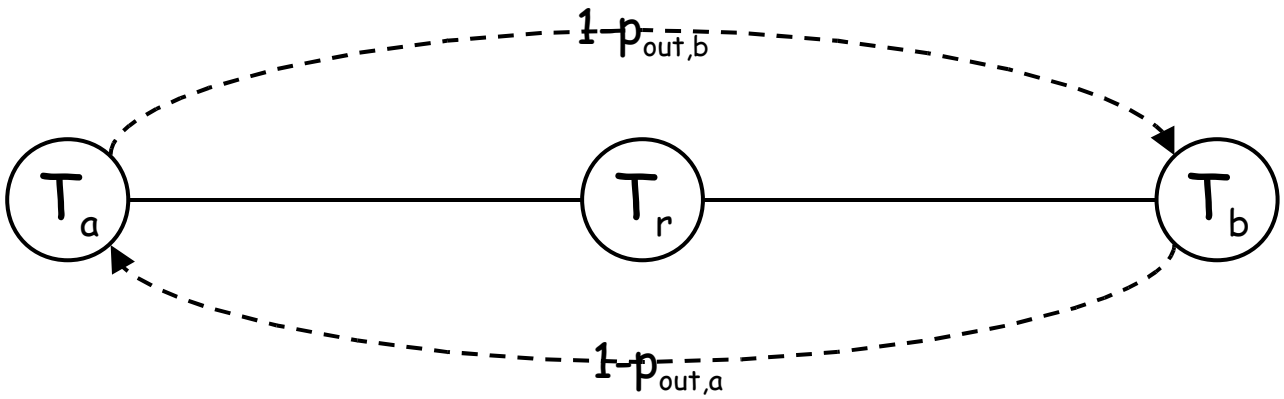

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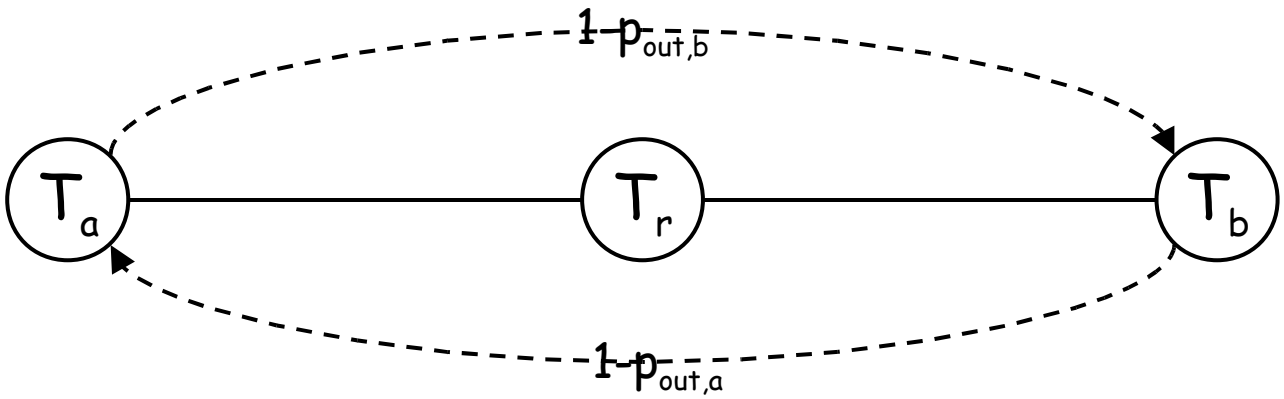

Indicator variables:

$$\bar{I}_a = (1 - p_{out,b})/2$$
$$\bar{I}_b = (1 - p_{out,a})/2$$

$$\Rightarrow \eta^{AF} = \frac{1}{2}\left[R_a(1 - p_{out,b}) + R_b(1 - p_{out,a})\right]$$

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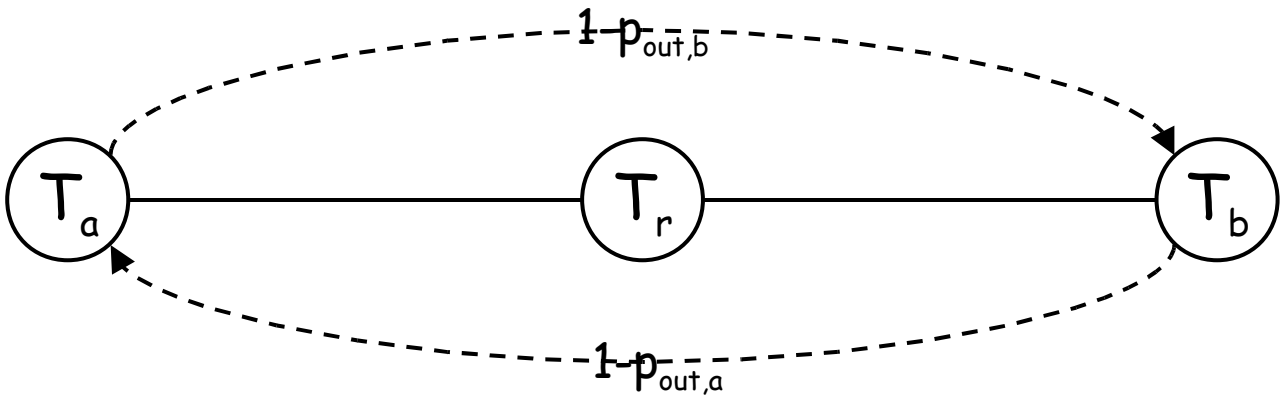

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Outage probabilities [Laneman '04]:

$$p_{out,i} = \Pr\left[R_i > \log_2\left(1 + \frac{GP_j|h_{ir}|^2|h_{rj}|^2}{1 + G|h_{rj}|^2}\right)\right], \; i \neq j \quad i, j \in \{a, b\}$$

DF-based schemes

DF-based schemes:

Single user decoding Decode-and-Forward (SDF)

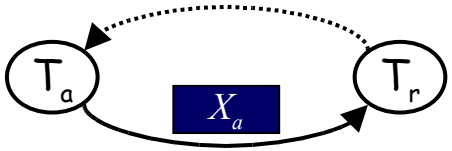
- Users cannot transmit simultaneously
- The relay performs only single user detection

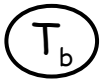
Joint user decoding Decode-and-Forward (JDF)

- Users can transmit simultaneously
- The can relay perform joint detection

SDF-based scheme: protocol

State | Action | Buffer





$B_r = \{\}$

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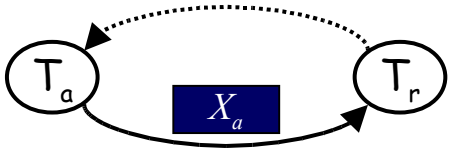
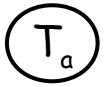
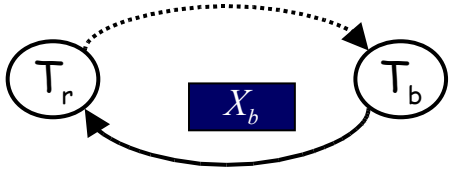
		 $B_r = \{\}$
	 	 $B_r = \{W_a\}$

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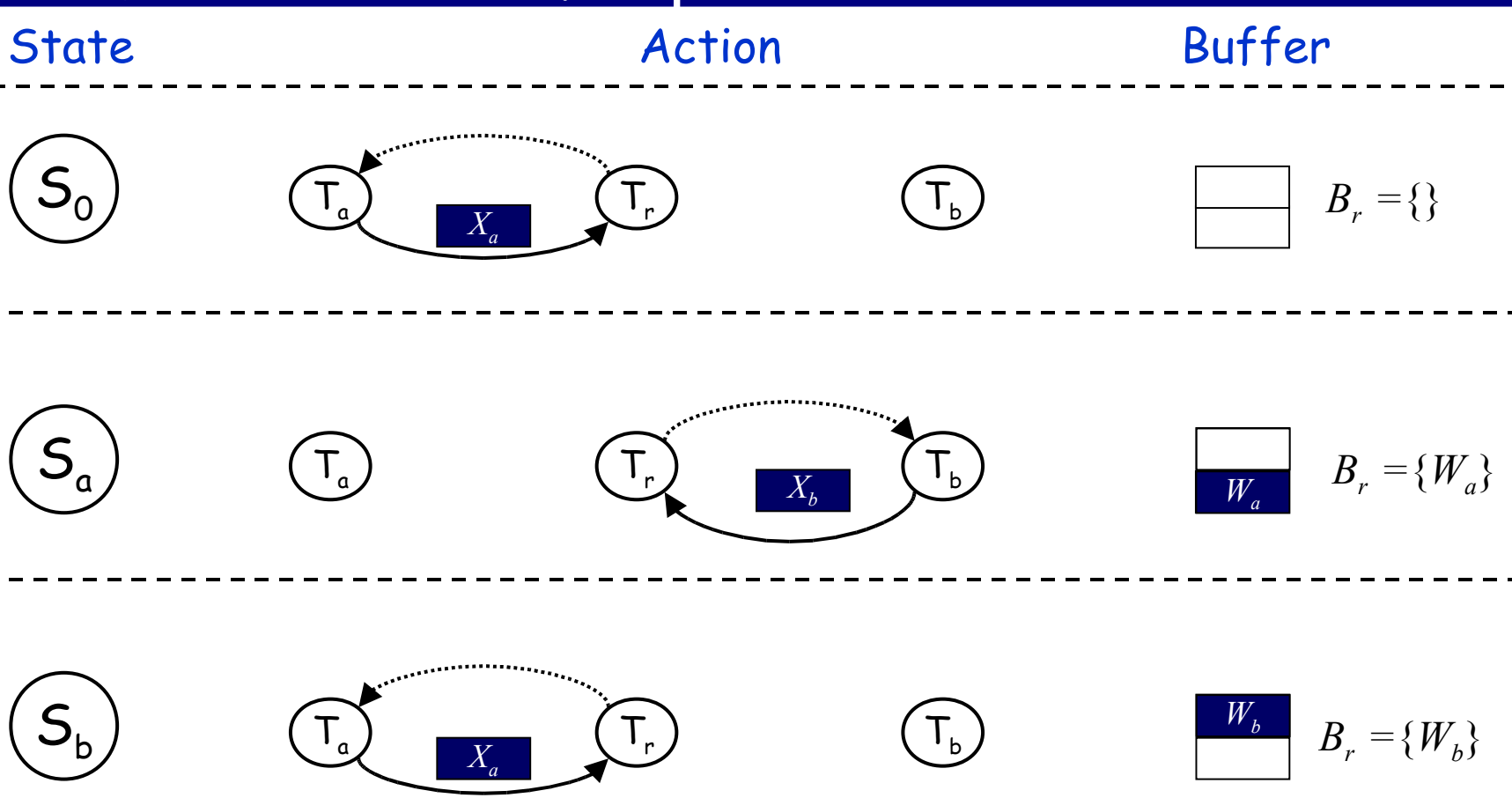

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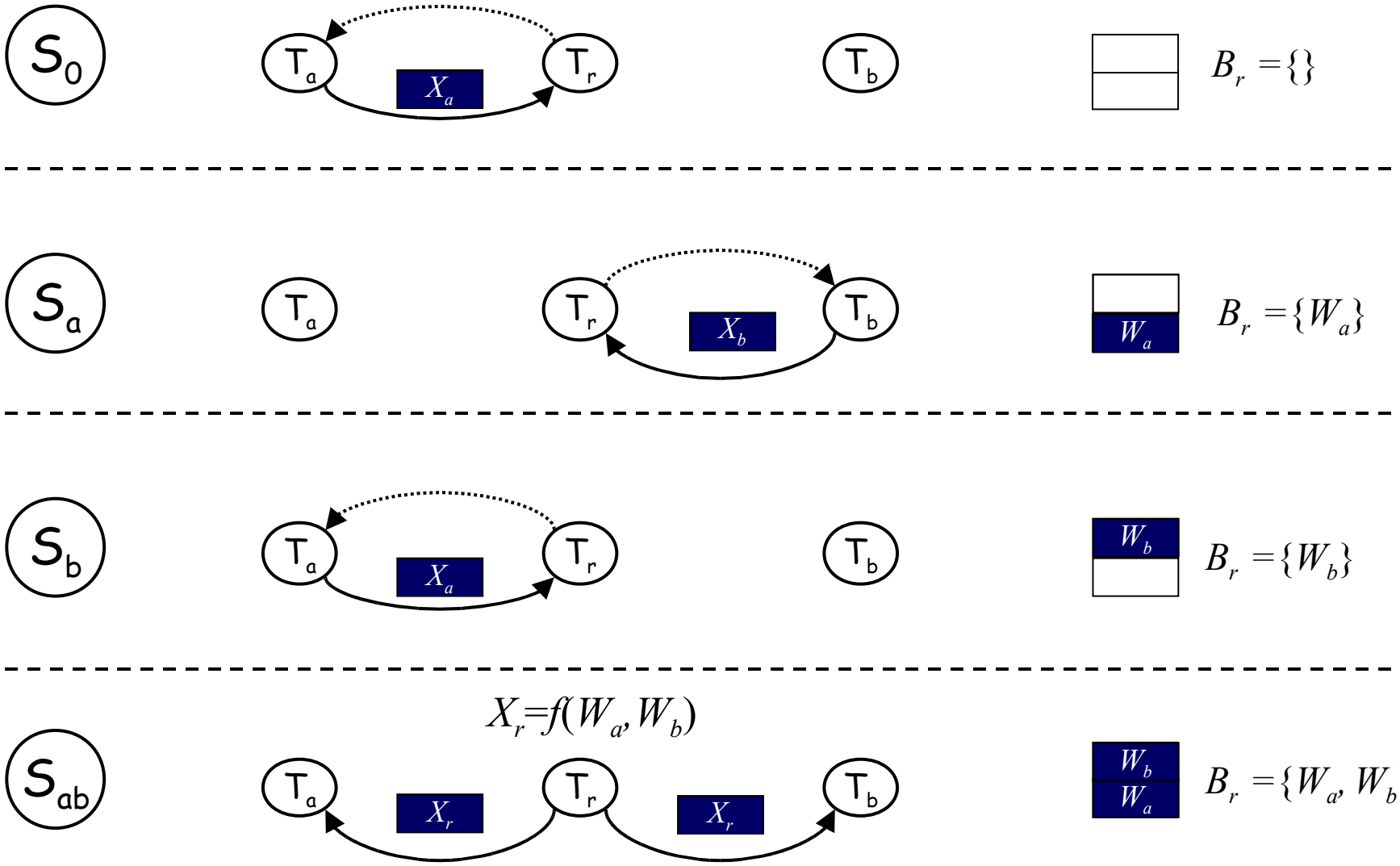
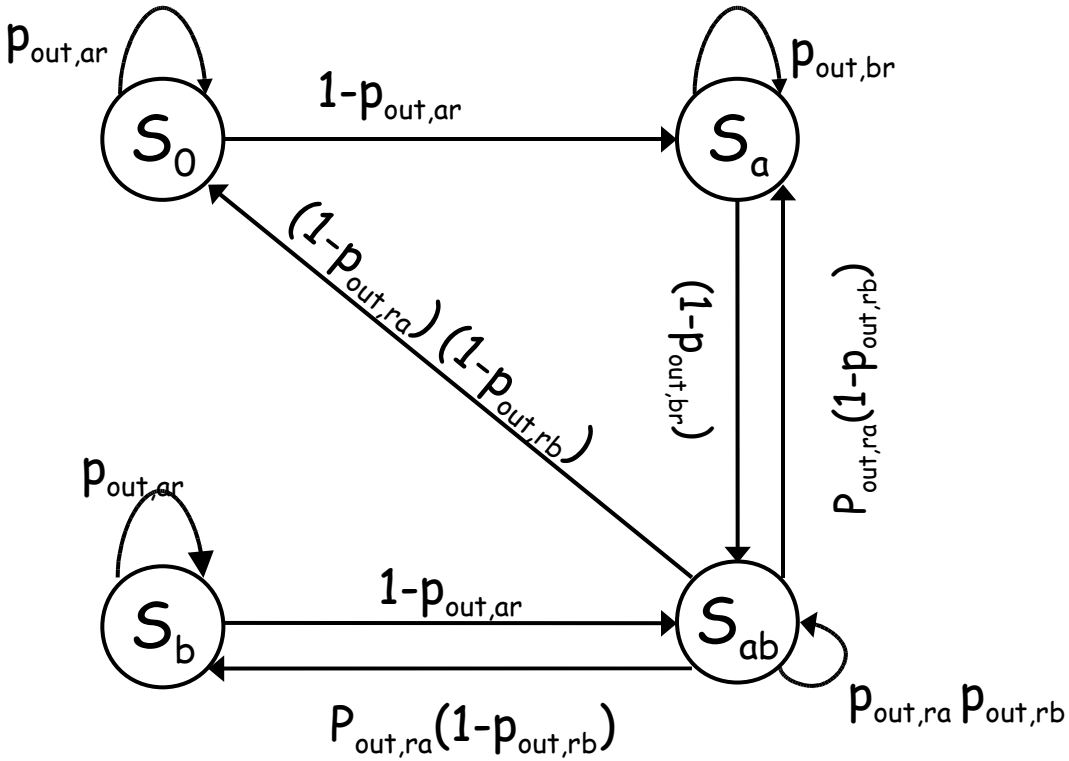


SDF-based scheme: throughput analysis



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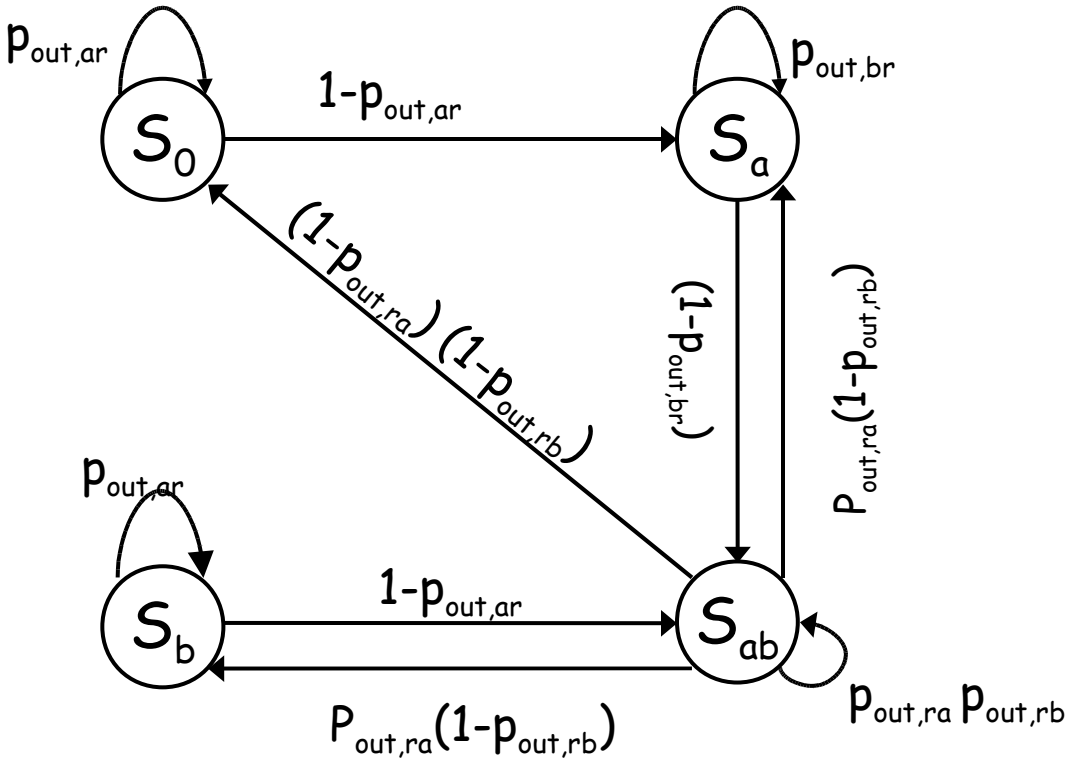

Steady state distribution:

$$\boldsymbol{\pi} = \left[\pi_0, \pi_a, \pi_b, \pi_{ab}\right]^T$$

Succesfull decoding variables:

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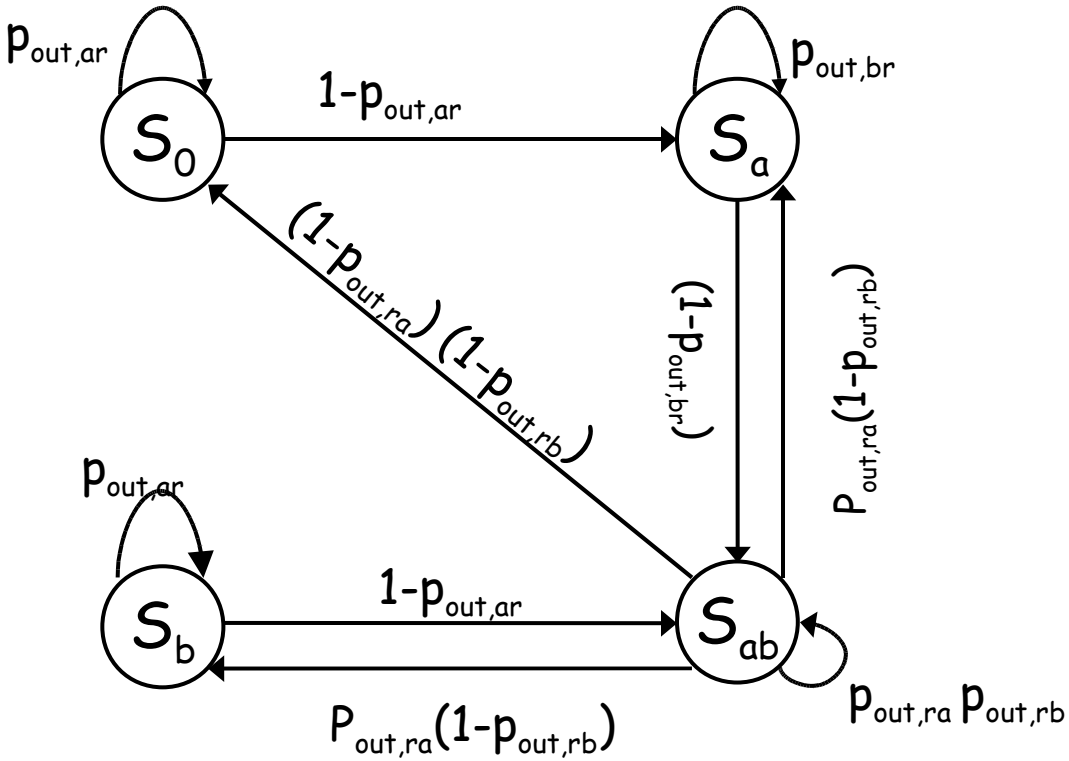

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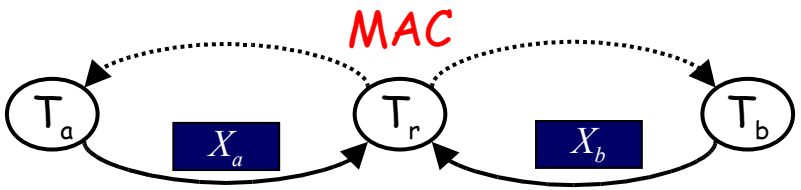
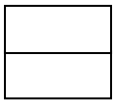
Throughput:

$$\eta^{SDF} = R_a \bar{I}_a^{SDF} + R_b \bar{I}_b^{SDF}$$

JDF-based scheme: protocol

State | Action | Buffer





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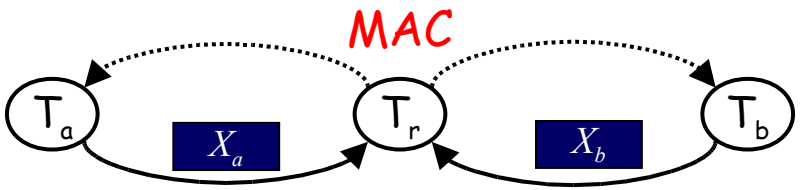

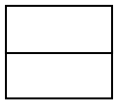

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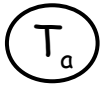

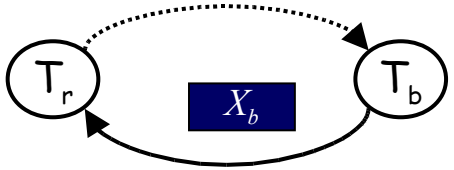

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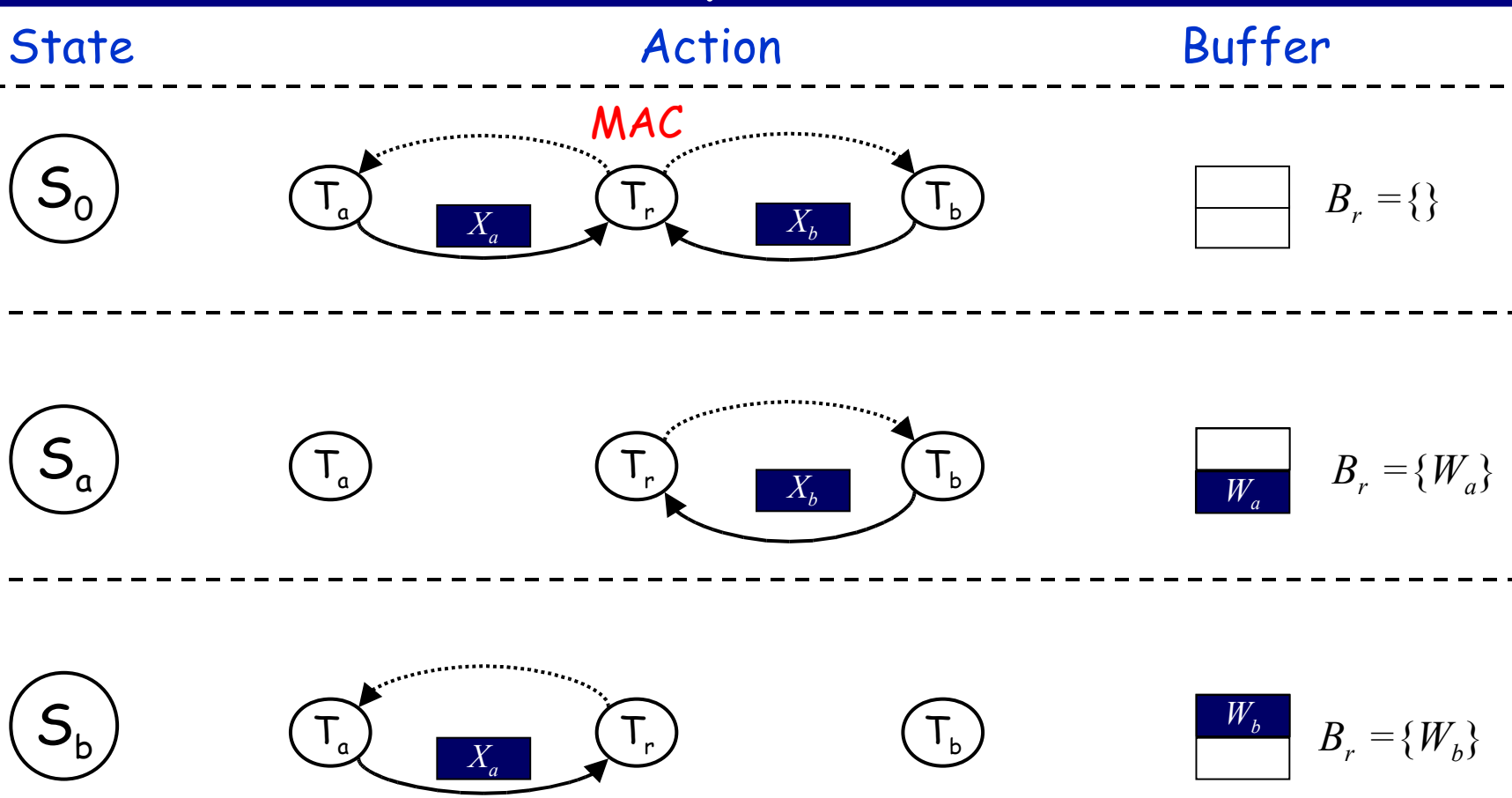
JDF-based scheme: protocol



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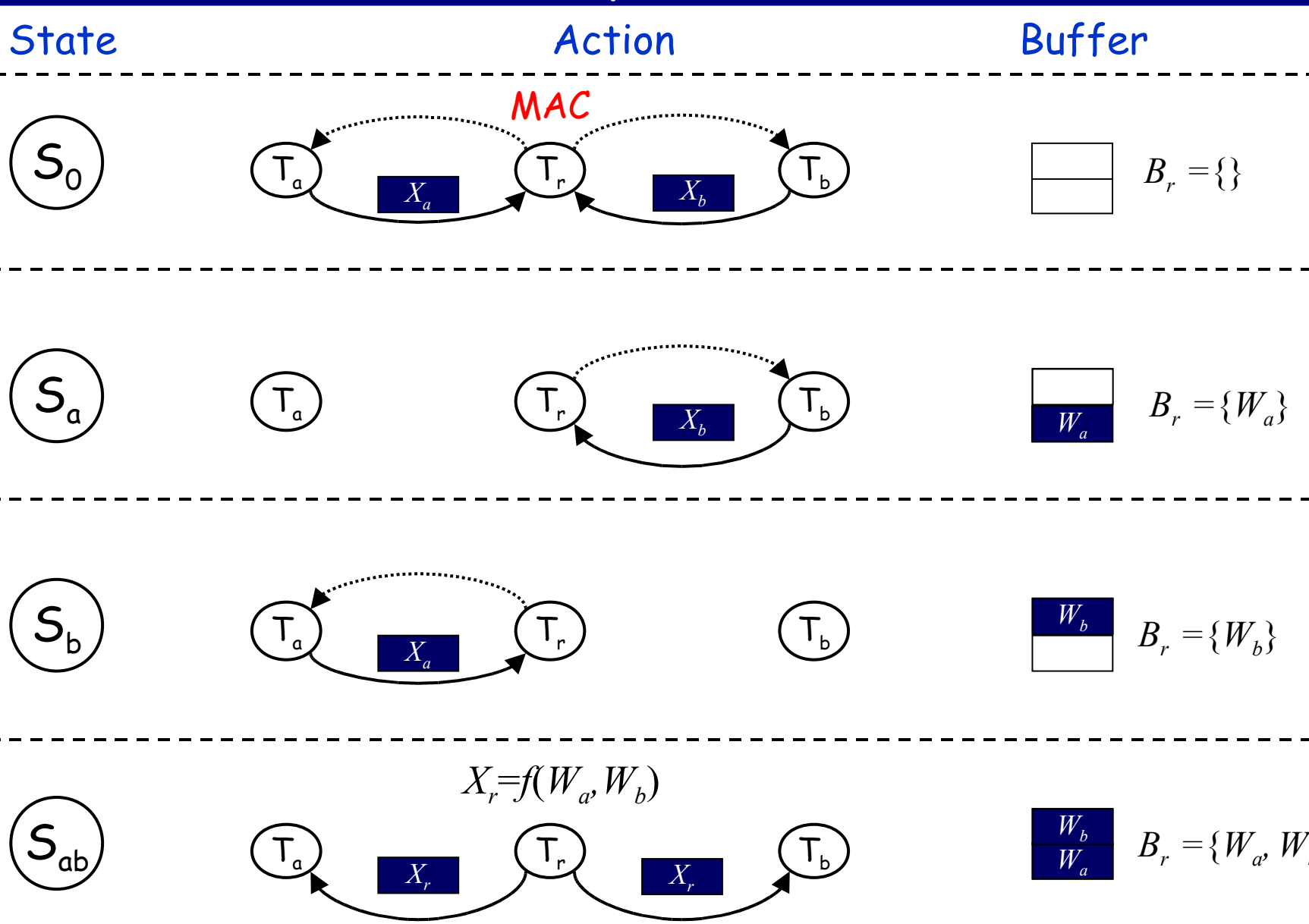
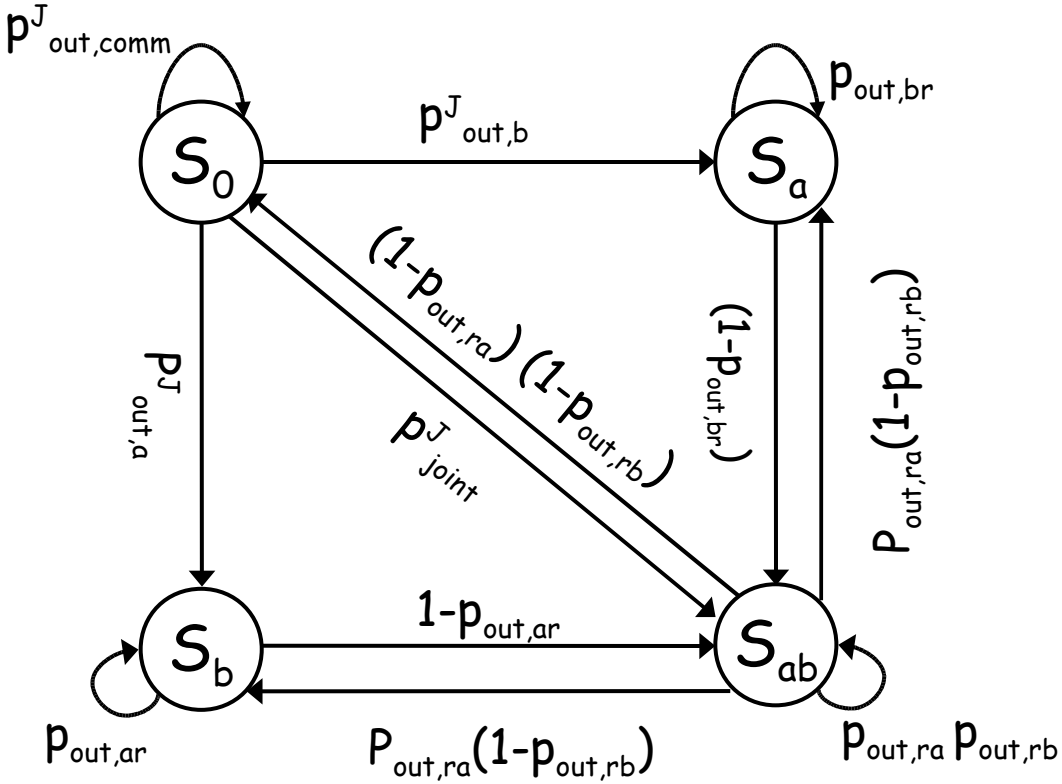


JDF-based scheme: throughput analysis



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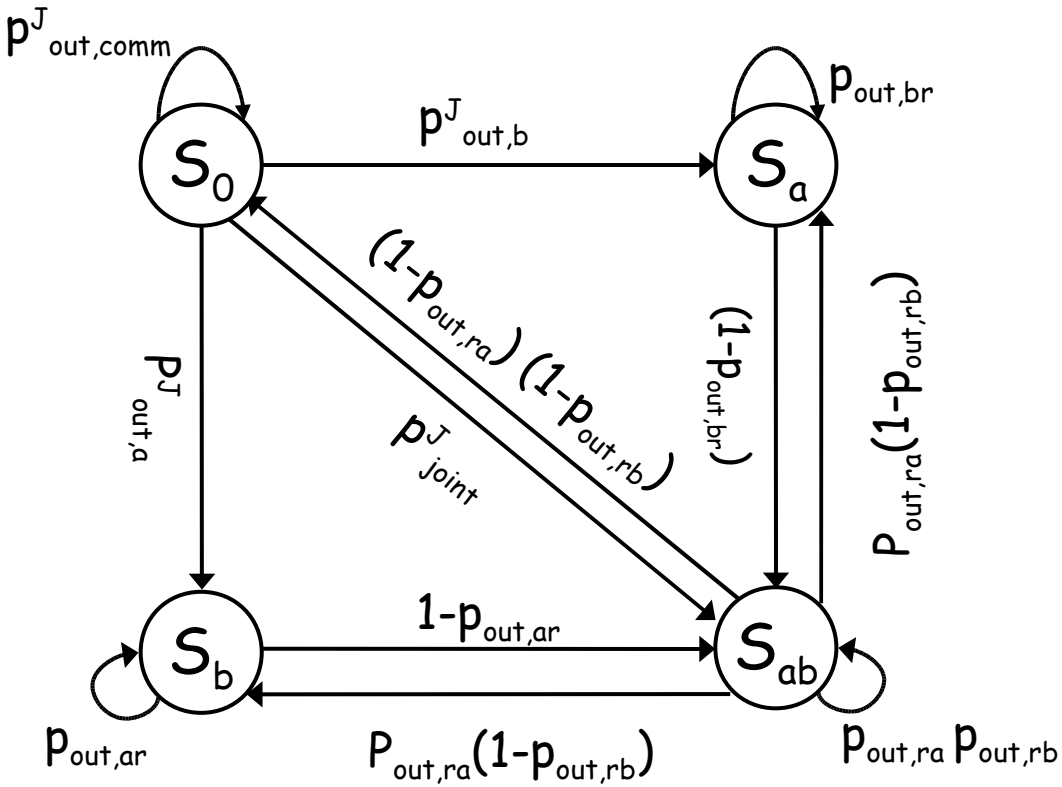
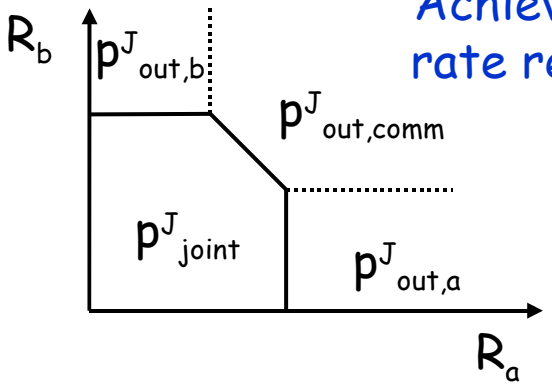
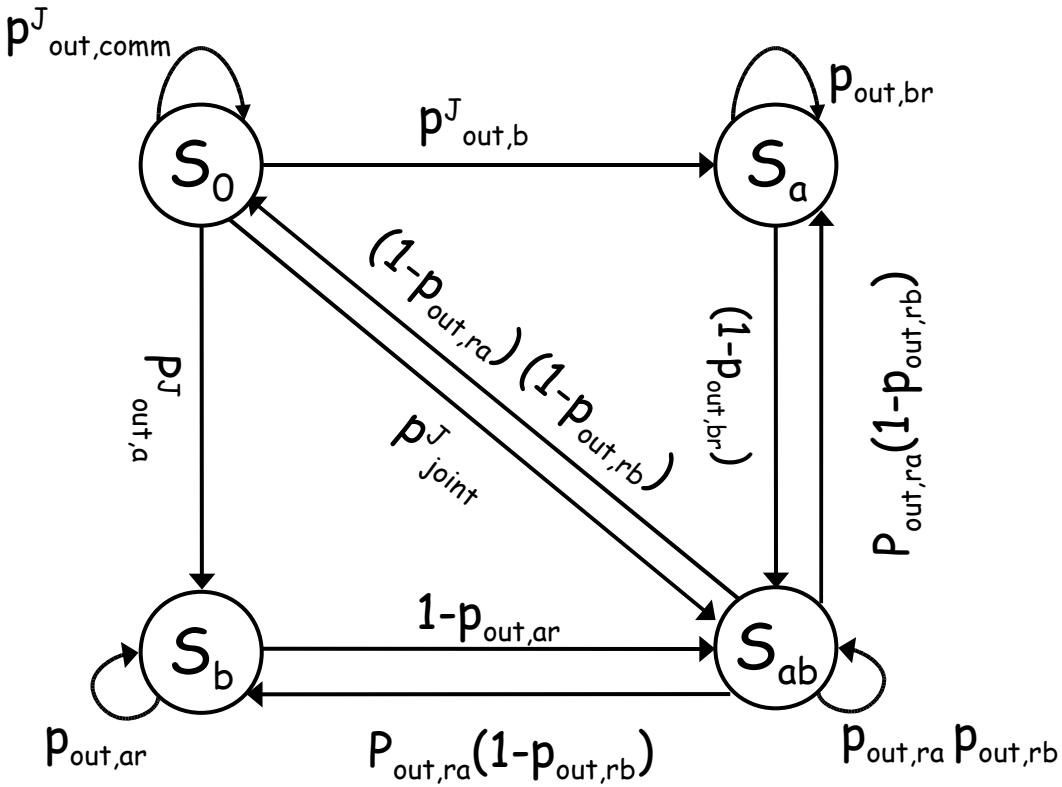
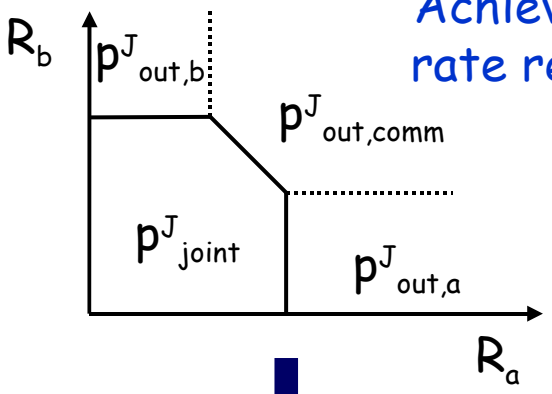




Achievable rate region

JDF-based scheme: throughput analysis





Achievable rate region

Indicator variables:

$$\bar{I}_a^{JDF} = \pi_{ab}^{JDF}(1 - p_{out,rb})$$

$$\bar{I}_b^{JDF} = \pi_{ab}^{JDF}(1 - p_{out,ra})$$

Throughput:

$$\eta^{JDF} = R_a \bar{I}_a^{JDF} + R_b \bar{I}_b^{JDF}$$

JDF: outage probabilities [Narasimhan '07]

- JDF MAC – common outage:

$$p_{out,comm}^{J} = \Pr\left[\begin{array}{l} R_a > \log_2\left(1+\dfrac{P_a|h_{ar}|^2}{1+P_b|h_{br}|^2}\right), R_b > \log_2\left(1+\dfrac{P_b|h_{br}|^2}{1+P_a|h_{ar}|^2}\right) \\ R_a + R_b > \log_2\left(1+P_a|h_{ar}|^2 + P_b|h_{br}|^2\right) \end{array}\right]$$

- JDF MAC – single user outage:

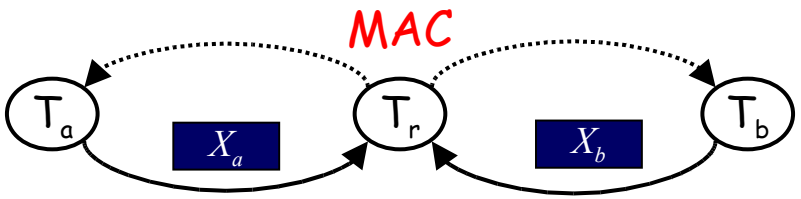
$$p_{out,i}^{J} = \Pr\left[R_i > \log_2\left(1+P_i|h_{ir}|^2\right), R_j < \log_2\left(1+\frac{P_j|h_{jr}|^2}{1+P_i|h_{ir}|^2}\right)\right], \begin{array}{l} i \neq j \\ i,j \in \{a,b\} \end{array}$$

- JDF MAC – succesfull joint decoding:

$$p_{joint}^{J} = 1 - p_{out,a}^{J} - p_{out,b}^{J} - p_{out,comm}^{J}$$

Lattice-based scheme: protocol

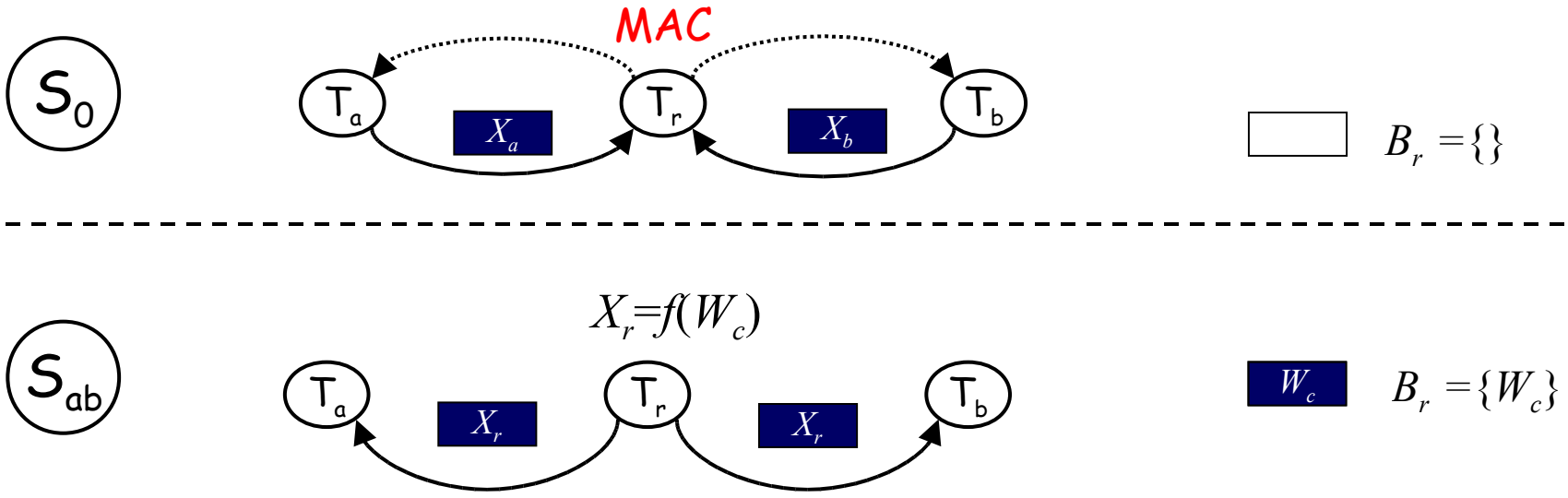
Whenever users transmit they do that simultaneously and pre-compensate for different fading phases (transmit CSI) to operate as in [Nam '08].

State	Action	Buffer
S_0		$B_r = \{\}$

Lattice-based scheme: protocol

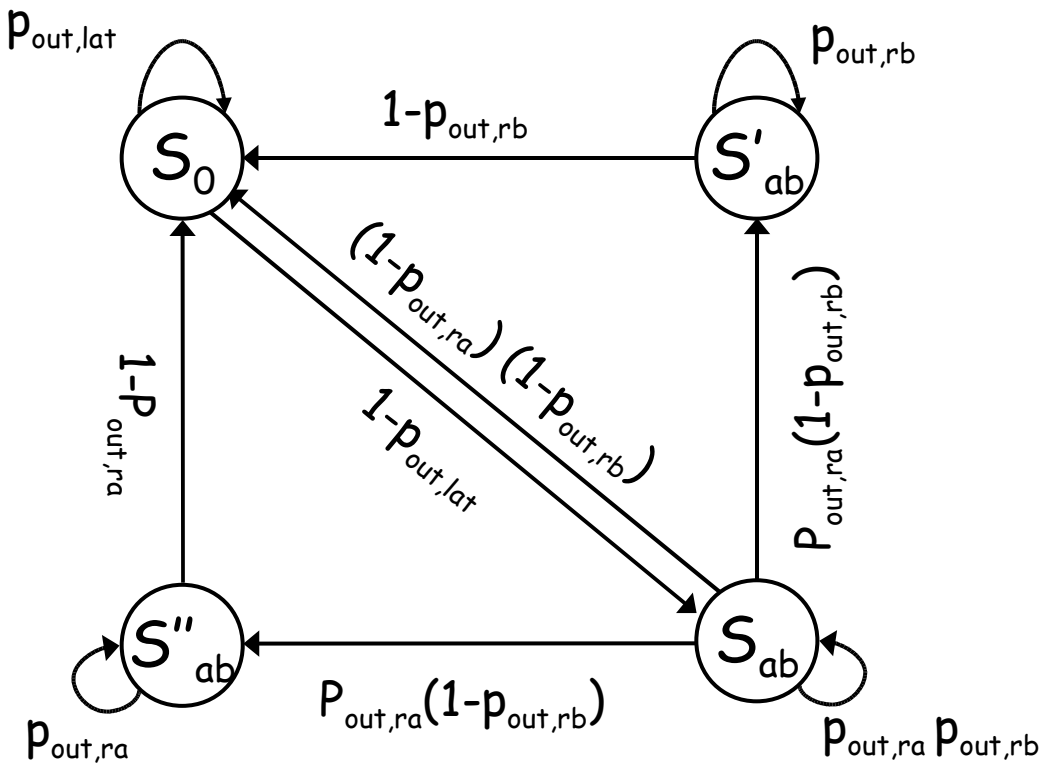
Whenever users transmit they do that simultaneously and pre-compensate for different fading phases (transmit CSI) to operate as in [Nam '08].

State | Action | Buffer

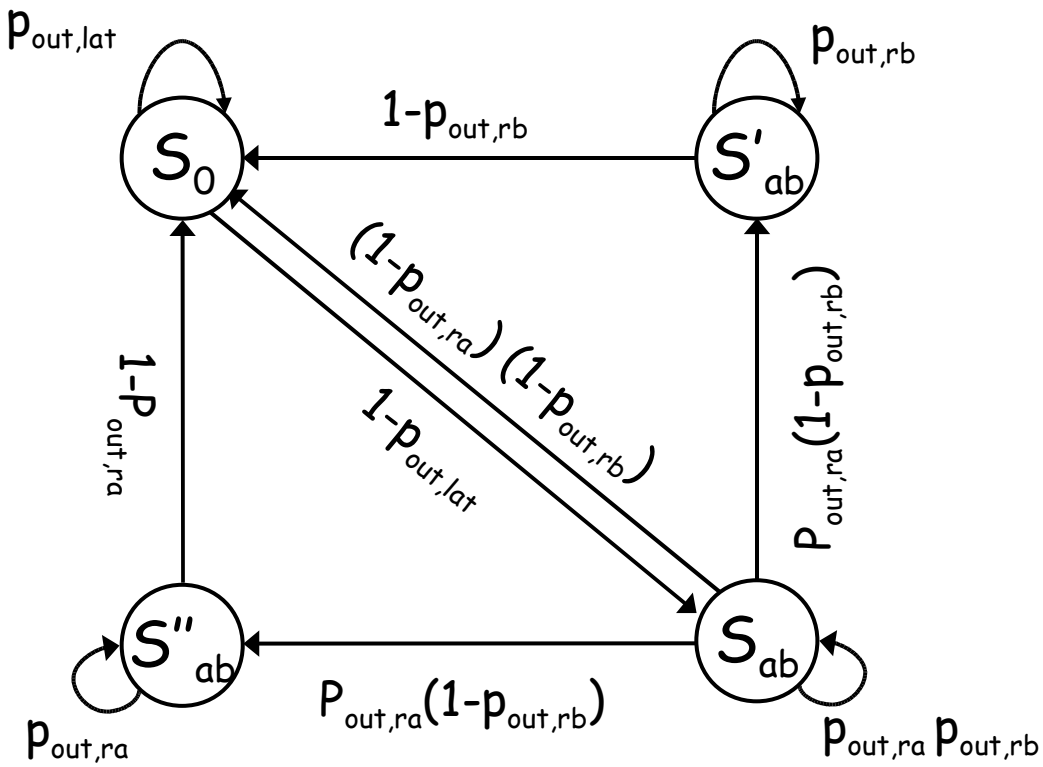


The relay keeps transmitting the same codeword till both users correctly decode. The destination can decode the other user's message thanks to the side information.

Lattice-based throughput analysis



Lattice-based throughput analysis



Indicator variables:

$$\bar{I}_a = \left(\pi_{ab}^{LAT} + \pi''^{LAT}_{ab}\right)(1 - p_{out,rb})$$

$$\bar{I}_b = \left(\pi_{ab}^{LAT} + \pi''^{LAT}_{ab}\right)(1 - p_{out,ra})$$

$$\eta^{LAT} = R_a \bar{I}_a^{LAT} + R_b \bar{I}_b^{LAT}$$

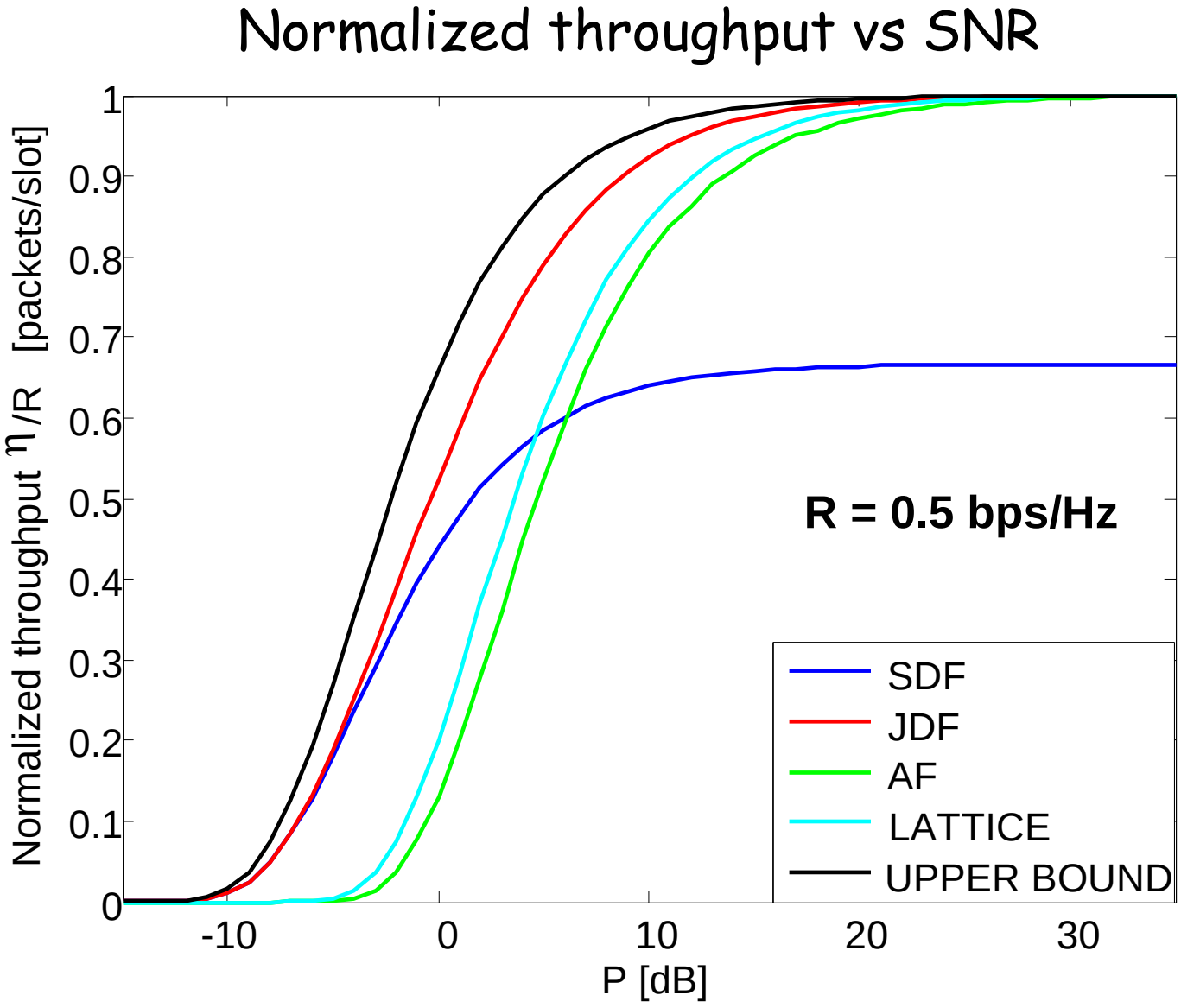
Lattice-based outage probability

Probability that the compound codeword is not decoded [Nam '08]:

$$p_{out}^{lat} = 1 - \Pr\left[\begin{array}{l} R_a \le \left[\log_2\left(\dfrac{P_a|h_{ar}|^2}{P_a|h_{ar}|^2 + P_b|h_{br}|^2} + P_a|h_{ar}|^2\right)\right]^+ \\ R_b \le \left[\log_2\left(\dfrac{P_b|h_{br}|^2}{P_a|h_{ar}|^2 + P_b|h_{br}|^2} + P_b|h_{br}|^2\right)\right]^+ \end{array}\right]$$

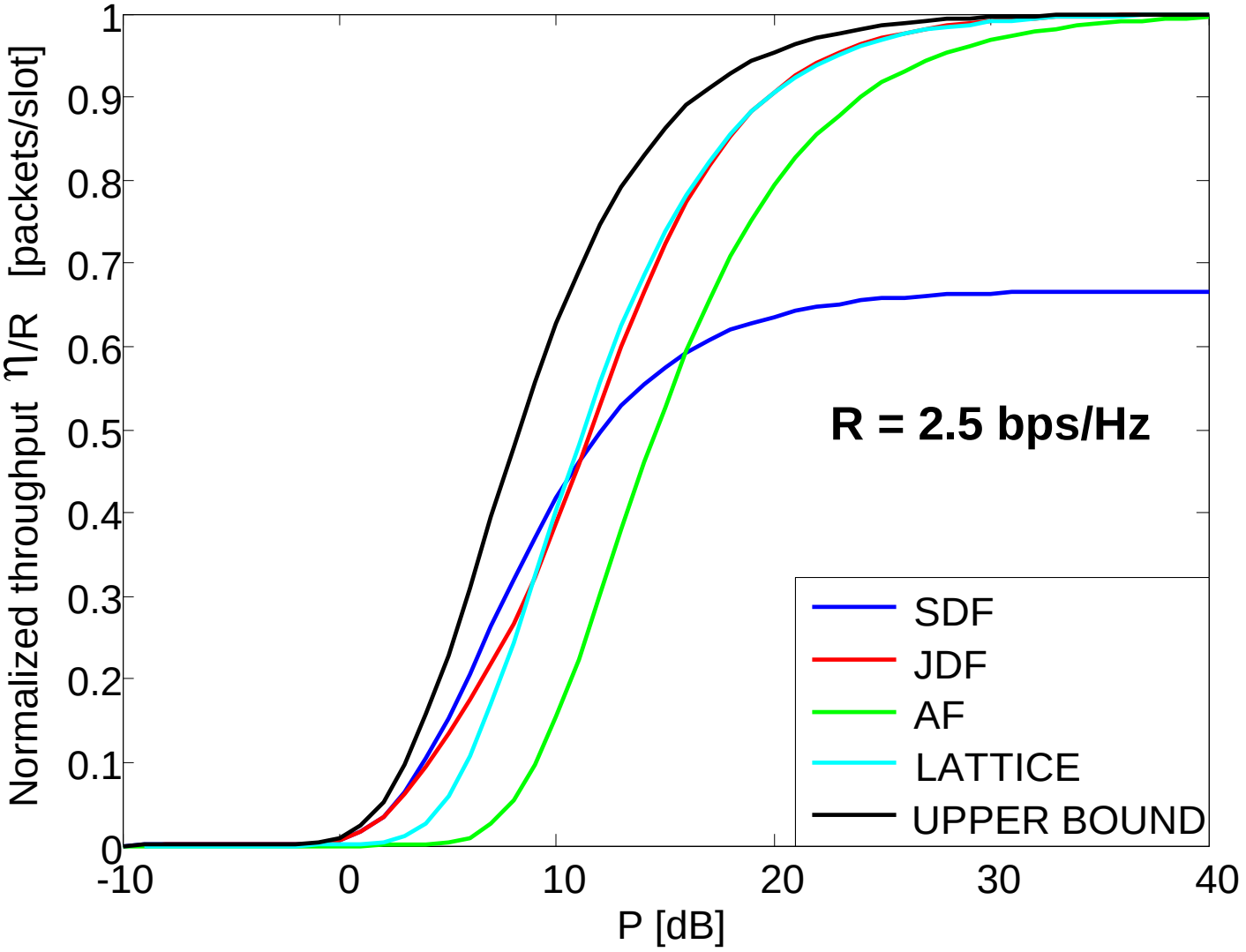
Where $[x]^+ = \max\{0, x\}$

Numerical results for the symmetric TWRC

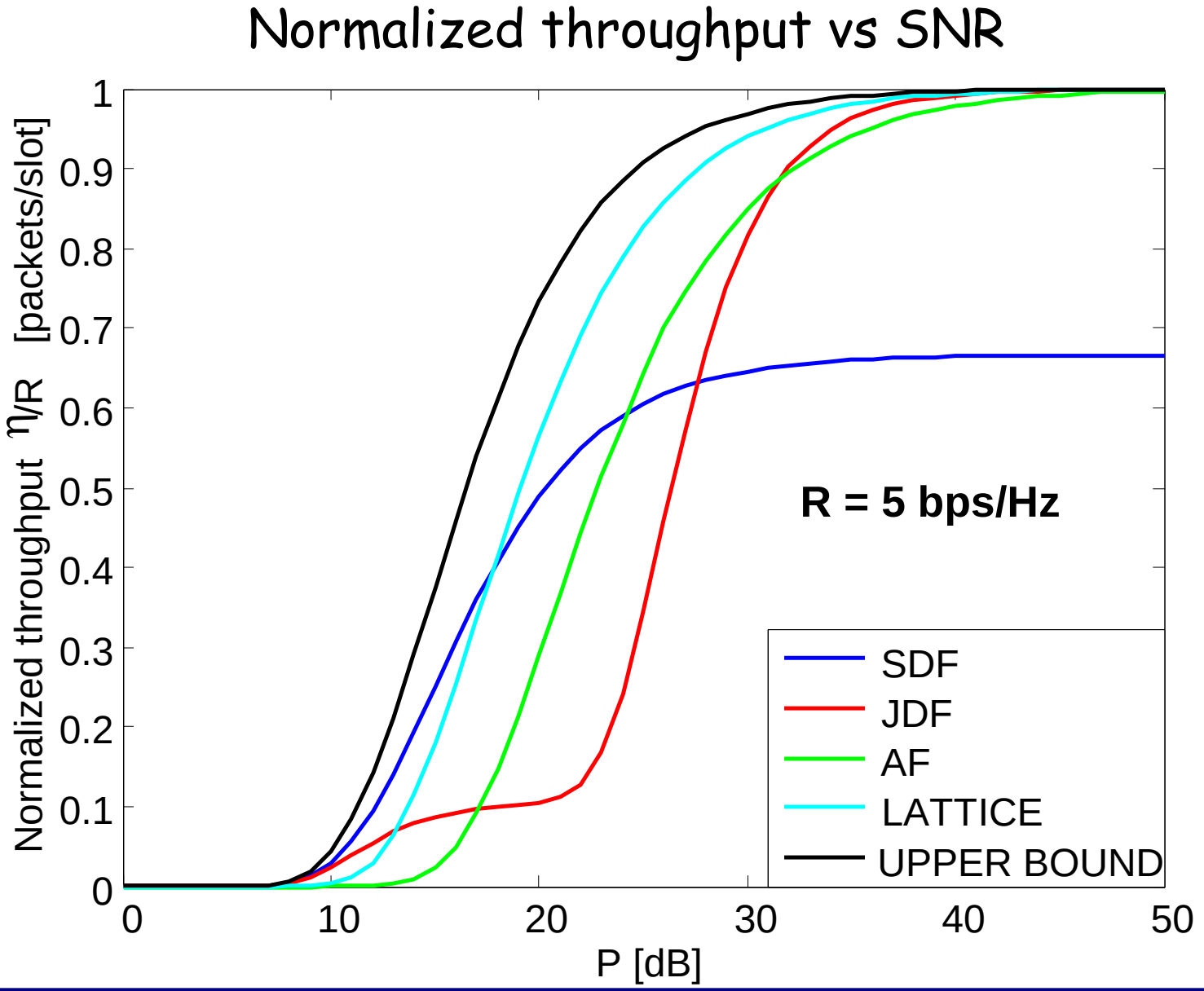


Numerical results for the symmetric TWRC

Normalized throughput vs SNR

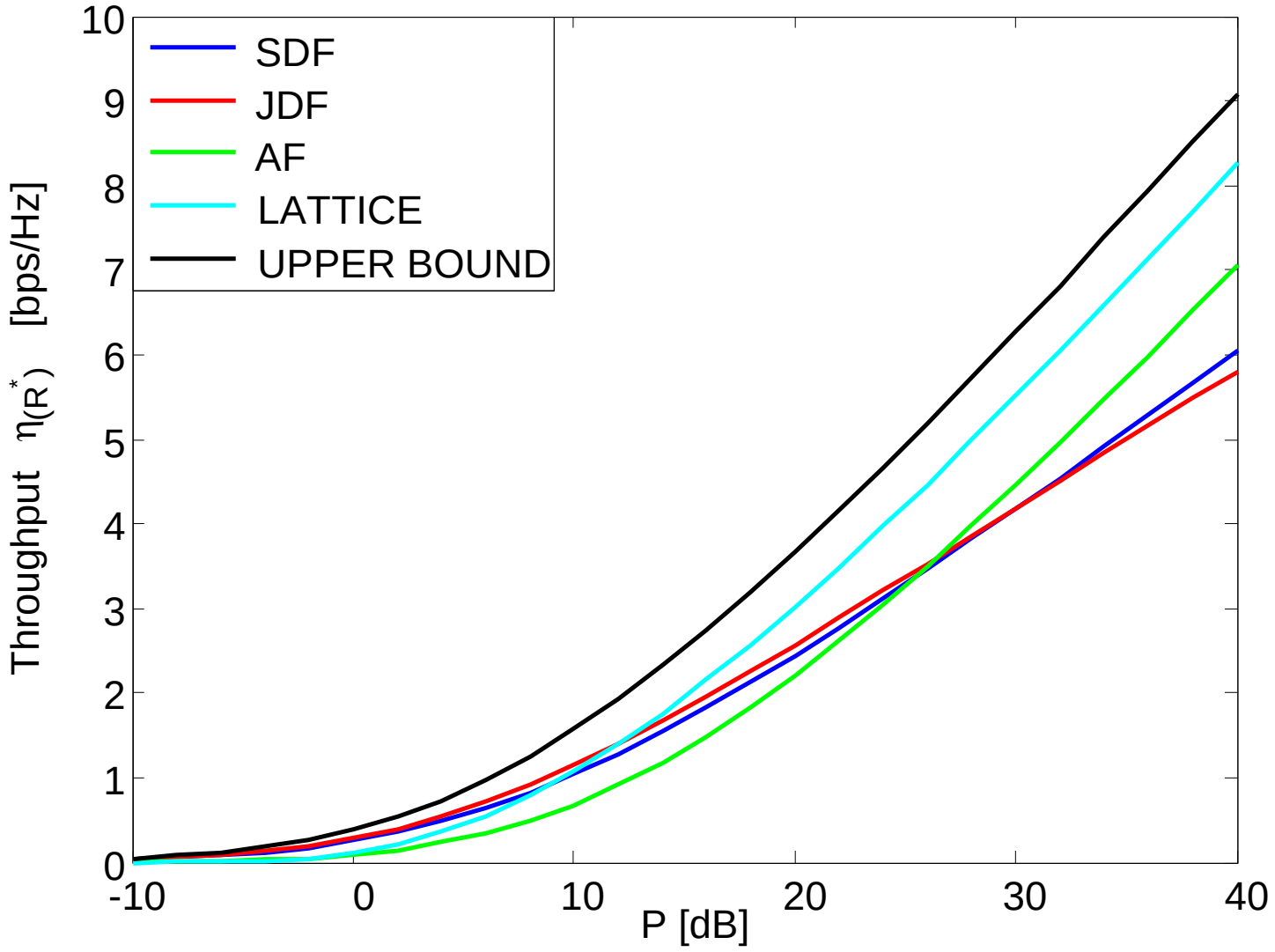


Numerical results for the symmetric TWRC



Numerical results for the symmetric TWRC

Throughput optimizing the transmission rate vs SNR



Conclusion

- Analysis of Type-I HARQ strategies combined with different relaying techniques (AF, DF, Lattice)
- There is no a scheme performing better than the others in all the regimes
- Lattice-based scheme outperforms the others scheme in some regimes

Thank you

CSI requirements

- AF:
  - Relay: no CSI
  - Users: Receive CSI (product channel)
- DF:
  - Relay and Users: Receive CSI
- Lattice-based:
  - Relay: receive CSI
  - Users: transmit and receive CSI