



Useful Thermodynamics Helpful Relations

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Helpful Relations

General Relations: $h = u + pv$, $g = h - Ts$, $du = Tds - Pdv$, $dh = Tds + vdp$

$$C_v = \left(\frac{\partial u}{\partial T} \right)_v = \left(\frac{du}{dT} \right)_{IG}, \quad C_p = \left(\frac{\partial h}{\partial T} \right)_p = \left(\frac{dh}{dT} \right)_{IG}, \quad \eta_c = \frac{w_{c,s}}{w_{c,a}}, \quad \eta_t = \frac{w_{t,a}}{w_{t,s}}, \quad \epsilon_{regen} = \frac{\Delta h_{comp\ gas,act}}{\Delta h_{comp\ gas,max}}$$

Closed System First Law: $Q_{net} - W_{net} = \Delta E$

Conservation of Mass for Steady-Flow: $\sum_{inlets} \dot{m}_i = \sum_{exits} \dot{m}_e$ $\dot{m} = \rho A \vec{V} = \frac{A \vec{V}}{v}$

First Law for Steady-Flow: $\dot{Q}_{net} + \sum_{inlets} \dot{m}_i \left(h + \frac{\vec{V}^2}{2} + gz \right)_i = \dot{W}_{net} + \sum_{exits} \dot{m}_e \left(h + \frac{\vec{V}^2}{2} + gz \right)_e$

Boundary Work: $W_b = \int_1^2 P dV$

Steady-flow rev Work: $w_{rev-sf} = \frac{\dot{W}_{rev,sf,by}}{\dot{m}} = - \int_1^2 v dP - \Delta ke - \Delta pe$

Ideal Gas Relations: $PV = mRT$, $\frac{P_2 V_2}{T_2} = \frac{P_1 V_1}{T_1}$, $C_p = \frac{k}{k-1} R$, $C_v = \frac{R}{k-1}$, $k = \frac{C_p}{C_v}$

$$\Delta s = \left[\int_{T_1}^{T_2} \frac{C_p(T) dT}{T} - R \ln \left(\frac{P_2}{P_1} \right) \right] = [s^0(T_2) - s^0(T_1) - R \ln \left(\frac{P_2}{P_1} \right)]$$

$$\Delta S = m \left[\int_{T_1}^{T_2} \frac{C_v(T) dT}{T} + R \ln \left(\frac{v_2}{v_1} \right) \right]$$

For ideal gases undergoing an isentropic process with constant specific heats, $n = k = C_p/C_v$. Cold analysis specific heats are constants and

$$\Delta h = c_{p,avg} \Delta T \quad \text{and} \quad \Delta u = c_{v,avg} \Delta T$$

$$c_{p,avg} = c_{v,avg} + R; \quad R = R_u / MW$$

hot analysis requires the use of ideal gas tables A-17,18 etc

Polytropic Process Relations:

$$\text{general: } P_2 V_2^n = P_1 V_1^n, \quad W_b = \frac{P_2 V_2 - P_1 V_1}{1-n}, \quad n \neq 1 \quad \text{and} \quad W_b = PV \ln \left(\frac{V_2}{V_1} \right), \quad n = 1$$

$$\text{ideal gas: } \frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} = \left(\frac{V_1}{V_2} \right)^{n-1}, \quad W_b = \frac{mR(T_2 - T_1)}{1-n}, \quad n \neq 1 \quad \text{and} \quad W_b = mRT \ln \left(\frac{V_2}{V_1} \right), \quad n = 1$$

For ideal gases undergoing an isentropic process with constant specific heats, $n = k = C_p/C_v$.

Ideal Gas Mixture Relations:

$$y_i = \frac{mf_i / M_i}{\sum (mf_i / M_i)} \quad mf_i = \frac{y_i M_i}{\sum (y_i M_i)}$$

$$h = \sum h_i mf_i; \quad s = \sum s_i mf_i; \quad u = \sum u_i mf_i; \quad c_p = \sum c_{p,i} mf_i$$

$$k = \frac{\sum c_{p,i} mf_i}{\sum c_{v,i} mf_i}; \quad R = \sum R_i mf_i; \quad MW = \sum y_i MW_i$$

For an ideal gas mixture

$$\bar{s}_i = \bar{s}_i^0(T) - R_U \ln(y_i P_{i-am} / 1)$$

Psychrometric Relations:

$$\phi = \frac{P_v}{P_g} = \frac{m_v}{m_g(T)}; \quad T_{dp} = T_{sat}(P_v); \quad \omega = \frac{m_v}{m_a} = 0.622 \frac{P_v}{P - P_v}$$

$$h_a = C_{p,a} T \quad h_v = 2501.3 + 1.82T \quad (\text{kJ / kg}) \quad T \text{ in } C^\circ$$

$$h = h_a + \omega h_v$$

Compressibility factor Law (of corresponding states):

$$\mathbf{Z(P_r, T_r, Z_{critical}) =}$$

$$Z = P\hat{v} / RT = P\bar{v} / R_u T = v / v_{ideal\text{gas}}$$

$$P_r = P / P_{cr}; \quad T_r = T / T_{cr}; \quad v_r' = v / [RT_{cr} / P_{cr}]$$

Carnot efficiency: thermal efficiency = Wnet/ Qact to per cycle

$$\eta_{che} = (1 - T_L / T_H) ;$$

$$\text{Entropy ; for a reversible process ; } \Delta S = \int_1^2 dQ / T_{bdy}$$

$$S_{gen} = \Delta S - \sum Q / T_{bdy} \geq 0$$

$$\text{Isolated system; } S_{gen} = \sum \Delta S$$

$$S_{gen, heattransferspace} = Q[1/T_{low} - 1/T_{hi}]$$

open system

$$\dot{S}_{gen} = \frac{DS}{Dt} + \sum \dot{m}_o s_o - \sum \dot{m}_{in} s_{in} - \sum Q / T_{bdy} \geq 0$$

Carnot engine: $Q_H/Q_{rej} = T_H/T_L$; $\eta = 1 - T_L/T_H$; $COP_{hp} = Q_H/W_{in}$; $COP_{REF} = Q_{rej}/W_{in}$

Entropy

Ideal gas isentropic process variable specific heats ; $P_{r2}/P_{r1} = P_2/P_1$

$$\Delta s = 0 = \Delta s^0 - R \ln \frac{P_2}{P_1}$$

Entropy change for a Reservoir and for a system

Entropy change of an incompressible substance

$$\Delta S = mc \ln(T_{final} / T_{initial})$$

Isothermal Re *servoir*

$$\Delta S = Q_{toreservoir} / T_{res}$$

General - System

$$\Delta S = \int dQ_{to} / T \}_{rev}$$

Efficiency: Component -Adiabatic efficiency= for steady flow, c = compressor, t= turbine

$$\eta_c = \frac{w_{c,s}}{w_{c,a}}, \quad \eta_t = \frac{w_{t,a}}{w_{t,s}}, \quad s \text{ is for isentropic, } a \text{ is for actual}$$

Exergy

Effectiveness = [output or outcome] / [input or required]

Fixed mass extensive exergy

$$\Phi = U - U_0 + P_0 [Vol - Vol_0] - T_0 [S - S_0] + mV^2 / 2J + mgZ / J$$

$$\dot{\Phi}_Q = \int d\dot{Q}_{to} (1 - T_0 / T_{BDY}) \approx \Sigma \dot{Q}_{to} (1 - T_0 / T_{BDY})$$

OPEN System

Energy equation

$$\dot{Q}_{net} + \sum_{inlets} \dot{m}_i \left(h + \frac{\vec{V}^2}{2} + gz \right)_i = \dot{W}_{net} + \sum_{exits} \dot{m}_e \left(h + \frac{\vec{V}^2}{2} + gz \right)_e$$

Exergy:

$$\dot{m}\psi_{in} + \int d\dot{Q}_{to} (1 - T_0 / T_{BDY}) = D\Phi / Dt]_{cv} + \dot{W}_{useful-out} + \dot{m}\psi_{out} + [T_0 \dot{S}_{gen} = \dot{I} = \dot{X}_{destroy}]$$

$$\dot{W}_{useful-out} = \dot{m}(\psi_{in} - \psi_{out}) - T_0 \dot{S}_{gen} + \int d\dot{Q}_{to} (1 - T_0 / T_{BDY}) \quad ; \text{SS+ fixed}$$

$$\text{where } \psi = h - h_0 + (V^2 - V_{0-atm}^2) / 2J + g(z - z_0) / J - T_0(s - s_0)$$

$$\dot{W}_{useful-out} = \dot{m}(\psi_{in} - \psi_{out}) + \int d\dot{Q}_{to} (1 - T_0 / T_{BDY}) - T_0 \dot{S}_{gen}; \text{neglecting } ke \text{ and potential differences}$$

$$\dot{W}_{useful-out} = \dot{m}[h_{in} - h_{out} - T_0(s_{in} - s_{out})] + \int d\dot{Q}_{to} (1 - T_0 / T_{BDY}) - T_0 \dot{S}_{gen}$$

If the entropy equation is used the Irreversibility is $= T_0 \dot{S}_{gen} =$

$$\dot{I} = T_0 \dot{m}(s_{out} - s_{in}) - \int d\dot{Q}_{to} (T_0 / T_{BDY})$$

2ndlaw efficiency $= \varepsilon = \text{output} / \text{necessary} - \text{input of exergy}$

Chemical Reactions.

$$h = h_f^0 + (h(T, P) - h(T_0, P_0)); \quad h_f^0(l) = h_f^0(g) - h_{fg}; \quad \text{on mol base } L\bar{H}V = H\bar{H}V - N_{H2O} \bar{h}_{fg}$$

sssf $Q - W = H_P - H_R$; For complete combustion with stoichiometric air

$$\text{enthalpy of reaction} = \sum N h_p - \sum N h_R \quad \text{for combustion} = h_c; \quad -h_c = \text{heating value}$$

For compound the enthalpy of reaction in forming compound from stable elements

all at same T and P = enthalpy of formation.

For an ideal gas mixture

$$\bar{s}_i = \bar{s}_i^0(T) - R_U \ln(y_i P_{in-atm} / 1)$$