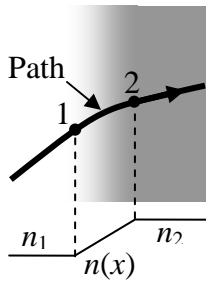


Section 6.3

Problem 6.A: Fermat's Principle states that light travels along the path for which its transit time is a minimum. We have a region of refractive index n_1 , and a region of refractive index n_2 , with a linear transition region between them where the index goes smoothly from n_1 to n_2 as $n(x) = ax$. (Note: Refractive index $n = c/v$, where c is the speed of light in a vacuum, and v is the speed of light in the medium.) **(a)** Given the transit time along a path $\tau = \int_1^2 \frac{ds}{v}$, write down the equation for the path integral over the transition



region (from point 1 to point 2) in terms of $n(x)$ integrated over x , i.e. express ds in terms of $y(x)$ and dx . **(b)** Identify the function f , and evaluate the Euler-Lagrange equation $\frac{\partial f}{\partial y} = \frac{d}{dx} \frac{\partial f}{\partial y'}$. Solve for y' . **(c)** Integrate to find the path, in the form $x(y)$. [Hint:

$$\int \frac{du}{\sqrt{u^2 - 1}} = \operatorname{arccosh} u .]$$