Section 6.3

Problem 6.A: Fermat's Principle states that light travels along the path for which its transit time is a minimum. We have a region of refractive index n_1, and a region of refractive index n_2, with a linear transition region between them where the index goes smoothly from n_1 to n_2 as $n(x) = ax$. (Note: Refractive index $n = c/v$, where c is the speed of light in a vacuum, and v is the speed of light in the medium.) **(a)** Given the transit time along a path $\tau = \int_1^2 \frac{ds}{v}$, write down the equation for the path integral over the transition region (from point 1 to point 2) in terms of $n(x)$ integrated over x, i.e. express ds in terms of $y'(x)$ and dx. **(b)** Identify the function f, and evaluate the Euler-Lagrange equation $\frac{\partial f}{\partial y} = \frac{d}{dx}\frac{\partial f}{\partial y'}$. Solve for y'. **(c)** Integrate to find the path, in the form $x(y)$. [*Hint:* $\int \frac{du}{\sqrt{u^2 - 1}} = \text{arccosh}\, u$.]