Information Rates of Time Varying Rayleigh Fading Channels

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***Abstract*— In this paper, information rates of mobile radio channels without channel state information (CSI) at either the transmitter or the receiver are investigated. The channel is modeled as a time varying Rayleigh fading process whose dynamics are characterized by a Doppler spectrum with specified normalized fading rate. Results are presented in terms of the block length, the normalized fading rate, and the signal-to-noise ratio (SNR), for the Clarke's Doppler spectrum and uniform spectrum.**

I. Introduction

A growing body of literature is available that explores the ultimate limits of achievable rates of wireless communications systems over fading channels [3]. Results have been published for a large variety of cases, depending on channel models and on the availability of channel state information (CSI) at the transmitter and receiver.

It is well known that when the CSI is perfectly known to the receiver, the channel capacity is achieved by independent and identically distributed (i.i.d.) Gaussian inputs. If, in addition, the transmitter has the CSI, then Gaussian signaling is still optimal, and the capacity can be achieved by "waterfilling" the transmit power. However, in scenarios where neither the transmitter nor the receiver have CSI, the channel is not as well understood. The capacity problem without CSI has been studied for some simplified channel models, e.g., memoryless Rayleigh-fading channels [1], [18], block-fading Rayleigh channels [13], [19], block MPSK channels [17], and finite-state Markov channels [6], [10]. It has been shown in [1], [13], [18] that when CSI is not available, the capacity achieving distribution is discrete. This is in contrast to channels with perfect CSI for which continuous Gaussian signaling is optimal.

Traditionally, CSI is obtained by training with known pilot symbols inserted in the transmitted sequence. Due to the presence of noise, the receiver is provided with imperfect CSI and therefore the performance depends on the quality of the CSI [11], [14]. Capacities of pilot aided systems have been studied in [2],[5],[9].

In this paper, we investigate the time correlated fading channel, a channel model that received less attention in the literature. In particular, we study information rates of time varying, Rayleigh fading channels with no CSI available at either the receiver or the transmitter. The dynamics of the fading channel are characterized by its Doppler spectrum. We do not attempt to find the exact channel capacity and the optimal input distribution due to the difficulty of the problem. Rather, we derive bounds on information rates for i.i.d. input symbol sequences, for constant power input symbols and Gaussian inputs. Constant power signaling deserves attention because it is appealing from a practical point of view, whereas Gaussian signaling is considered because of its optimality in the presence of perfect CSI.

The remainder of this paper is organized as follows. The system and channel models are described in Section II. Section III contains the evaluation of the information rates. Numerical results are provided in section IV and conclusions are given in Section V.

In the sequel, lower- and upper-case boldface letters are used to denote column vectors and matrices, respectively. We use $\mathbf{0}$ to denote the all-zero vector. Matrix $\mathbf{I}$ stands for the identity matrix. Superscripts ^{T} and ^{H} denote the transpose operation and Hermitian operation, respectively. The function $\log$ is the natural logarithm so that information rates and entropies are expressed in nats/symbol.

II. System Model

The channel between the transmitter and receiver is modeled as a flat fading process. The discrete time received signal at time k, r_k, is given by

$$r_k = h_k s_k + n_k,$$

where s_k is the transmitted symbol, h_k and n_k are the samples of the fading and noise processes, respectively. Rayleigh fading is assumed and the fading process is normalized so that h_k is complex Gaussian with zero mean and unit variance. The additive noise sample is complex Gaussian with zero mean and variance $N_0/2$ per dimension. The average transmitted signal power is $\mathcal{E}_s = E[|s_k|^2]$. Hence, the signal-to-noise ratio (SNR) per symbol at the receiver is equal to $\mathcal{E}_s/N_0$.

We consider the transmission of a block (a *frame*) of L symbols, $s_1, s_2, \cdots, s_L$. The received sequence, denoted by an $L \times 1$ vector $\mathbf{r} = [r_1\ r_2\ \cdots\ r_L]^T$, is of the form

$$\mathbf{r} = \mathbf{S}\mathbf{h} + \mathbf{n},$$

where $\mathbf{S} = \mathrm{diag}(s_1, s_2, \cdots, s_L)$, $\mathbf{h} = [h_1\ h_2\ \cdots\ h_L]^T$, and $\mathbf{n} = [n_1\ n_2\ \cdots\ n_L]^T$. The noise process is assumed to be white, hence the noise sample vector $\mathbf{n}$ is complex Gaussian, distributed in L dimensions with zero mean vector $\mathbf{0}$, and covariance matrix $N_0\mathbf{I}$. Assuming the fading process to be stationary, $\mathbf{h}$ is complex Gaussian with zero mean and Toeplitz positive semidefinite covariance matrix $\mathbf{R_h}$. For two-dimensional isotropic scattering (Clarke's model), the (i,j) entry of $\mathbf{R_h}$ can be expressed as

$$[\mathbf{R_h}]_{ij} = E[h_i h_j^*] = J_0(2\pi f_D(i-j)), \tag{1}$$

where J_0 is the zeroth order Bessel function of the first kind, and f_D is the normalized fading rate in the channel (the Doppler spread normalized by the symbol rate).

With these assumptions, one can readily show that given $\mathbf{S}$, $\mathbf{r}$ has an L-dimensional complex Gaussian distribution with mean vector $\mathbf{0}$ and covariance matrix $\mathbf{S}\mathbf{R_h}\mathbf{S}^H + N_0\mathbf{I}$ (see, e.g., [15]).

III. Achievable Information Rates

Let $\mathbf{s} = [s_1 \ s_2 \ \cdots \ s_L]^T$ be the sequence of transmitted symbols. The mutual information between $\mathbf{r}$ and $\{\mathbf{s}, \mathbf{h}\}$ can be expanded by using the chain rule as follows (see [4]):

$$\begin{aligned} \mathcal{I}(\mathbf{r};\mathbf{s},\mathbf{h}) &= \mathcal{I}(\mathbf{r};\mathbf{s}) + \mathcal{I}(\mathbf{r};\mathbf{h}|\mathbf{s}) \\ &= \mathcal{I}(\mathbf{r};\mathbf{h}) + \mathcal{I}(\mathbf{r};\mathbf{s}|\mathbf{h}). \end{aligned}$$

Thus, $\mathcal{I}(\mathbf{r};\mathbf{s})$, the mutual information between channel input $\mathbf{s}$ and channel output $\mathbf{r}$, without CSI, can be expressed

$$\begin{aligned} \mathcal{I}(\mathbf{r};\mathbf{s}) &= \mathcal{I}(\mathbf{r};\mathbf{s},\mathbf{h}) - \mathcal{I}(\mathbf{r};\mathbf{h}|\mathbf{s}) && \text{(2a)} \\ &= \mathcal{I}(\mathbf{r};\mathbf{h}) + \mathcal{I}(\mathbf{r};\mathbf{s}|\mathbf{h}) - \mathcal{I}(\mathbf{r};\mathbf{h}|\mathbf{s}). && \text{(2b)} \end{aligned}$$

Note that $\mathcal{I}(\mathbf{r};\mathbf{s}|\mathbf{h})$ in (2b) is the mutual information with perfect CSI. Therefore, $(\mathcal{I}(\mathbf{r};\mathbf{h}|\mathbf{s}) - \mathcal{I}(\mathbf{r};\mathbf{h}))$ is the penalty in information rate due to unknown CSI. Since $\mathcal{I}(\mathbf{r};\mathbf{h})$ is nonnegative, $\mathcal{I}(\mathbf{r};\mathbf{h}|\mathbf{s})$ can be interpreted as the upper bound on the penalty due to unknown CSI. It is not hard to see that the terms in (2) generally satisfy $0 \le \mathcal{I}(\mathbf{r};\mathbf{h}) \le \mathcal{I}(\mathbf{r};\mathbf{h}|\mathbf{s})$, implying that there is always a cost due to unknown CSI. The cost is annulled only if $\mathcal{I}(\mathbf{r};\mathbf{h}|\mathbf{s}) = \mathcal{I}(\mathbf{r};\mathbf{h}) = 0$, i.e., CSI is known.

In (2), the information rate without CSI $\mathcal{I}(\mathbf{r};\mathbf{s})$, is dependent on the distribution of the input $\mathbf{s}$. In subsequent subsections, we first evaluate $\mathcal{I}(\mathbf{r};\mathbf{h}|\mathbf{s})$. The evaluation of upper bounds on $\mathcal{I}(\mathbf{r};\mathbf{s})$ is accomplished by upper bounding $\mathcal{I}(\mathbf{r};\mathbf{s},\mathbf{h})$ and using (2a). A lower bound on $\mathcal{I}(\mathbf{r};\mathbf{s})$ for Gaussian signaling is obtained by starting from (2b), neglecting the nonnegative term $\mathcal{I}(\mathbf{r};\mathbf{h})$, and evaluating $\mathcal{I}(\mathbf{r};\mathbf{s}|\mathbf{h})$. We will also provide some asymptotic results for long block lengths and high SNRs.

A. Evaluation of $\mathcal{I}(\mathbf{r};\mathbf{h}|\mathbf{s})$

It was previously shown that the term $\mathcal{I}(\mathbf{r};\mathbf{h}|\mathbf{s})$ serves as an upper bound on the penalty of not knowing the CSI. Expressing the mutual information $\mathcal{I}(\mathbf{r};\mathbf{h}|\mathbf{s})$ as a difference of conditional differential entropies of the L dimensional vector $\mathbf{r}$,

$$\begin{aligned} \mathcal{I}(\mathbf{r};\mathbf{h}|\mathbf{s}) &= \mathcal{H}(\mathbf{r}|\mathbf{s}) - \mathcal{H}(\mathbf{r}|\mathbf{h},\mathbf{s}) \\ &= \mathcal{H}(\mathbf{r}|\mathbf{s}) - \mathcal{H}(\mathbf{n}) \end{aligned} \tag{3}$$

where

$$\mathcal{H}(\mathbf{n}) = L\log(\pi e N_0) \tag{4}$$

and

$$\begin{aligned} \mathcal{H}(\mathbf{r}|\mathbf{s}) &= E_{\mathbf{s}}\left[\log\det(\pi e(\mathbf{S}\mathbf{R_h}\mathbf{S}^H + N_0\mathbf{I}))\right] \\ &= L\log(\pi e N_0) + \\ &\quad E_{\mathbf{s}}\left[\log\det(\mathbf{I} + \frac{1}{N_0}\mathbf{R_h}\mathbf{S}^H\mathbf{S})\right]. \end{aligned} \tag{5}$$

For general input distributions, the difficulty of evaluating $\mathcal{H}(\mathbf{r}|\mathbf{s})$ (and hence $\mathcal{I}(\mathbf{r};\mathbf{h}|\mathbf{s})$) is obvious. Instead, we proceed to develop bounds on $\mathcal{I}(\mathbf{r};\mathbf{h}|\mathbf{s})$.

1) Upper bound on $\mathcal{I}(\mathbf{r};\mathbf{h}|\mathbf{s})$*:* Applying Jensen's inequality to (5), we have

$$\begin{aligned} \mathcal{H}(\mathbf{r}|\mathbf{s}) &\le L\log(\pi e N_0) + \\ &\quad \log\det\left(\mathbf{I} + \frac{1}{N_0}\mathbf{R_h}E_{\mathbf{s}}\left[\mathbf{S}^H\mathbf{S}\right]\right), \end{aligned} \tag{6}$$

where equality is obtained if the input symbols have constant power, i.e., $||s_l||^2 = \mathcal{E}_s$, $l = 1, 2, ..., L$, as in the case of M-ary phase shift keying (M-PSK). Continuing the upper bounding of $\mathcal{H}(\mathbf{r}|\mathbf{s})$, we have

$$\begin{aligned} \mathcal{H}(\mathbf{r}|\mathbf{s}) &\le L\log(\pi e N_0) + \log\det\left(\mathbf{I} + \frac{\mathcal{E}_s}{N_0}\mathbf{R_h}\right) \\ &= L\log(\pi e N_0) + \sum_{l=1}^{L}\log\left(1 + \frac{\mathcal{E}_s}{N_0}\lambda_l\right), \end{aligned} \tag{7}$$

where λ_l, $l = 1, 2, ..., L$, are the eigenvalues of $\mathbf{R_h}$. Substituting (4) and (7) into (3) yields

$$\mathcal{I}(\mathbf{r};\mathbf{h}|\mathbf{s}) \le \sum_{l=1}^{L}\log\left(1 + \frac{\mathcal{E}_s}{N_0}\lambda_l\right). \tag{8}$$

As mentioned earlier $\mathcal{I}(\mathbf{r};\mathbf{h}|\mathbf{s})$ is interpreted as the upper bound on the penalty paid for not having CSI. The penalty attains its maximum value given by the right-hand side (RHS) of (8) for constant power signals. For future use, we denote this value of the penalty normalized to the number of symbols

$$P_\Delta = \frac{1}{L}\sum_{l=1}^{L}\log\left(1 + \frac{\mathcal{E}_s}{N_0}\lambda_l\right). \tag{9}$$

Notice that P_Δ is a function of the SNR per symbol, the frame length, and, through the assumed known channel covariance matrix $\mathbf{R_h}$, of the channel normalized fading rate f_D. Moreover, through the eigenvalues of channel covariance matrix, the penalty function is also dependent on the spectrum of the channel fading process.

Before proceeding we note that for fixed f_D and $\mathcal{E}_S/N_0$, P_Δ is a nonincreasing sequence in L. This is a consequence of the fact that the matrix $\mathbf{I}+(E_S/N_0)\mathbf{R_h}$ is Toeplitz positive definite and Theorem 16.8.6 in [4]. This observation confirms the intuition that the channel dynamics can be learned better with an increasing sequence of observations leading to a reduced loss in information rate.

2) Lower Bound on $\mathcal{I}(\mathbf{r};\mathbf{h}|\mathbf{s})$: From (5), we have

$$\begin{aligned}\mathcal{H}(\mathbf{r}|\mathbf{s}) &\geq L\log(\pi e N_0)+\\ &E_{\mathbf{s}}\left[\log\left(1+\frac{1}{N_0}\sum_{l=1}^{L}||s_l||^2\right)\right]. \quad (10)\end{aligned}$$

For Gaussian signaling, the second term on the RHS of (10) can be expressed in closed form [8]. From (3), (4), and (10), we obtain a lower bound on the penalty on information rate per symbol for Gaussian signaling

$$\begin{aligned}&P_{LB,Gaussian}\\ =\;&\frac{1}{L}\left[-T_L\left(-\left(\frac{\mathcal{E}_s}{N_0}\right)^{-1}\right)\mathrm{Ei}\left(-\left(\frac{\mathcal{E}_s}{N_0}\right)^{-1}\right)+\right.\\ &\left.\sum_{l=1}^{L-1}\frac{1}{l}T_l\left(\left(\frac{\mathcal{E}_s}{N_0}\right)^{-1}\right)T_{L-l}\left(-\left(\frac{\mathcal{E}_s}{N_0}\right)^{-1}\right)\right], \quad (11)\end{aligned}$$

where Ei is the exponential-integral function, and $T_N(t)=\sum_{n=0}^{N-1}t^n e^{-t}/n!$. Notice this bound approaches 0 as L goes to infinity and therefore is useful only for small block lengths.

B. Constant Power Signaling

We have shown that in (8), equality holds for constant power signaling. An upper bound on $\mathcal{I}(\mathbf{r};\mathbf{s})$ can be obtained by upper bounding $\mathcal{I}(\mathbf{r};\mathbf{s},\mathbf{h})$ and using (2a). Expressing $\mathcal{I}(\mathbf{r};\mathbf{s},\mathbf{h})$ as a difference of differential entropies

$$\begin{aligned}\mathcal{I}(\mathbf{r};\mathbf{s},\mathbf{h}) &= \mathcal{H}(\mathbf{r})-\mathcal{H}(\mathbf{r}|\mathbf{s},\mathbf{h})\\ &= \mathcal{H}(\mathbf{r})-\mathcal{H}(\mathbf{n}), \quad (12)\end{aligned}$$

where $\mathcal{H}(\mathbf{n})$ is given in (4). Since $\mathbf{s}$, $\mathbf{h}$, and $\mathbf{n}$ are independent, the covariance matrix of $\mathbf{r}$ can be written

$$\mathbf{R_r}=E\left[\mathbf{S}\mathbf{h}\mathbf{h}^H\mathbf{S}^H+N_0\mathbf{I}\right],$$

where the (i,j) entry is

$$[\mathbf{R_r}]_{ij}=E\left[r_i r_j^*\right]=E\left[s_i s_j^*\right]E\left[h_i h_j^*\right]+N_0\delta_{ij},$$

where $\delta_{ij}=1$ if $i=j$ and $\delta_{ij}=0$ otherwise. Therefore,

$$\begin{aligned}\mathcal{H}(\mathbf{r}) &\leq \log\det\left(\pi e\mathbf{R_r}\right) && (13a)\\ &\leq \log\prod_{l=1}^{L}(\pi e[\mathbf{R_r}]_{ll}) && (13b)\\ &= L\log\pi e(\mathcal{E}_s+N_0), && (13c)\end{aligned}$$

where inequality (13a) holds because the entropy of a random vector is upper bounded by the entropy of a Gaussian random vector with the same covariance matrix; (13b) follows from Hadamard's inequality, with equality if and only if $[\mathbf{R_r}]_{ij}=0$, $i\neq j$, i.e., if the input symbols are uncorrelated.

From (4), (12), and (13c), we have

$$\begin{aligned}\mathcal{I}(\mathbf{r};\mathbf{s},\mathbf{h}) &\leq L\log\pi e(\mathcal{E}_s+N_0)-L\log\pi e N_0\\ &= LC_{\text{AWGN}}\left(\frac{\mathcal{E}_s}{N_0}\right), \quad (14)\end{aligned}$$

where

$$C_{\text{AWGN}}(x)\triangleq\log(1+x)$$

is the capacity per symbol of an additive white Gaussian noise (AWGN) channel with a per symbol SNR of x.

Let $\mathcal{I}_C(\mathbf{r};\mathbf{s})$ denote the achievable information rate per symbol with unknown CSI for constant power signaling. Then the following upper bound on $\mathcal{I}_C(\mathbf{r};\mathbf{s})$ follows immediately from (2a), (8) and (14):

$$\mathcal{I}_C(\mathbf{r};\mathbf{s})\leq C_{\text{AWGN}}\left(\frac{\mathcal{E}_s}{N_0}\right)-\frac{1}{L}\sum_{l=1}^{L}\log\left(1+\frac{\mathcal{E}_s}{N_0}\lambda_l\right). \quad (15)$$

C. Gaussian Signaling

Since the term $\mathcal{I}(\mathbf{r};\mathbf{h})$ in (2b) is nonnegative, we have

$$\mathcal{I}(\mathbf{r};\mathbf{s})\geqslant\mathcal{I}(\mathbf{r};\mathbf{s}|\mathbf{h})-\mathcal{I}(\mathbf{r};\mathbf{h}|\mathbf{s}). \quad (16)$$

The second term on the RHS of (16) is upper bounded in (8). The first term on the RHS of (16) is maximized if the input symbols are i.i.d. Gaussian, and is equal to

$$\begin{aligned}\mathcal{I}(\mathbf{r};\mathbf{s}|\mathbf{h}) &= \sum_{l=1}^{L}\mathcal{I}(r_l;s_l|h_l)\\ &= LC_{\text{Rayleigh}}\left(\frac{\mathcal{E}_s}{N_0}\right), \quad (17)\end{aligned}$$

where

$$\begin{aligned}C_{\text{Rayleigh}}(x) &\triangleq E_{\mathbf{h}}\log\left(1+x\left|h_l\right|^2\right)\\ &= -\exp\left(x^{-1}\right)\mathrm{Ei}\left(-x^{-1}\right) \quad (18)\end{aligned}$$

is the ergodic capacity per symbol of a Rayleigh fading channel in the presence of perfect CSI with average SNR x [12], [16].

For Gaussian signaling, the penalty $\mathcal{I}(\mathbf{r};\mathbf{h}|\mathbf{s})\leq P_\Delta$, where P_Δ is the penalty for constant power signaling in (9). Let $\mathcal{I}_G(\mathbf{r};\mathbf{s})$ denote the achievable information rate per symbol without CSI for Gaussian signaling. From (8), (16) and (17), we obtain

$$\mathcal{I}_G(\mathbf{r};\mathbf{s})\geq C_{\text{Rayleigh}}\left(\frac{\mathcal{E}_s}{N_0}\right)-\frac{1}{L}\sum_{l=1}^{L}\log\left(1+\frac{\mathcal{E}_s}{N_0}\lambda_l\right). \quad (19)$$

From (2a), (11) and (14), we have the following upper bound on the allowed information rates:

$$\begin{aligned}&\mathcal{I}_G(\mathbf{r};\mathbf{s})\\ \leq\;& C_{\text{AWGN}}\left(\frac{\mathcal{E}_s}{N_0}\right)-\\ &\frac{1}{L}\left[-T_L\left(-\left(\frac{\mathcal{E}_s}{N_0}\right)^{-1}\right)\mathrm{Ei}\left(-\left(\frac{\mathcal{E}_s}{N_0}\right)^{-1}\right)+\right.\\ &\left.\sum_{l=1}^{L-1}\frac{1}{l}T_l\left(\left(\frac{\mathcal{E}_s}{N_0}\right)^{-1}\right)T_{L-l}\left(-\left(\frac{\mathcal{E}_s}{N_0}\right)^{-1}\right)\right]. \quad (20)\end{aligned}$$

D. Asymptotic Behavior

We investigate the asymptotic behavior of the penalty per symbol P_Δ due to unknown CSI, for large block length L and high SNR, but at fixed fading rate.

1) Large Block Length: Since P_Δ is a nonincreasing sequence in L and $P_\Delta \geq 0$, it has a limit as block length L goes to infinity. By Szegö's theorem ([7], pp. 64-65), we have the asymptotic expression

$$\lim_{L\to\infty} P_\Delta = \lim_{L\to\infty} \frac{1}{L}\sum_{l=1}^{L}\log(1+\frac{\mathcal{E}_S}{N_0}\lambda_l) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\log\left(1+\frac{\mathcal{E}_S}{N_0}H(\omega)\right)d\omega, \quad (21)$$

where $H(\omega)$ is the power spectral density (PSD) function of the fading process h_k.

For Clarke's model with autocorrelation function (1), the corresponding PSD has the following form:

$$H_{\text{Clarke}}(\omega) = \begin{cases} \frac{1}{\pi f_D\sqrt{1-(\omega/2\pi f_D)^2}}, & |\omega| < 2\pi f_D \\ 0, & \text{otherwise.} \end{cases} \quad (22)$$

Using (22) in (21), the asymptotic P_Δ is found to be

$$\lim_{L\to\infty} P_{\Delta,\text{Clarke}} = 2f_D\phi\left(\frac{\mathcal{E}_S}{N_0\pi f_D}\right), \quad (23)$$

where the function ϕ is defined as

$$\phi(x) = \begin{cases} 0, & x=0 \\ \frac{\pi}{2}x+\log\frac{x}{2}+\sqrt{1-x^2}\log(\frac{1+\sqrt{1-x^2}}{x}), & 0<x\leq 1 \\ \frac{\pi}{2}x+\log\frac{x}{2}-2\sqrt{x^2-1}\tan^{-1}(\sqrt{\frac{x-1}{x+1}}), & x>1. \end{cases} \quad (24)$$

Expression (24) looks fairly complicated. For a simpler expression that provides more insight, we may replace the Clarke spectrum with the ideal uniform spectrum with the same cutoff frequency:

$$H_{\text{U}}(\omega) = \begin{cases} \frac{1}{2f_D}, & |\omega| < 2\pi f_D \\ 0, & \text{otherwise.} \end{cases} \quad (25)$$

Substitution of (25) into (21) leads to

$$\lim_{L\to\infty} P_{\Delta,\text{U}} = 2f_D\log\left(1+\frac{\mathcal{E}_S}{N_0}\frac{1}{2f_D}\right). \quad (26)$$

This result is pleasantly intuitive as it indicates that the information rate lost by not knowing the channel is the capacity of a Gaussian channel with bandwidth equal to the Doppler spread of the channel variation. From (15), in the extreme case that the bandwidth of channel variations matches the symbol rate, the attainable information rate without CSI drops to zero, confirming the intuition that it is impossible to separate between the effects of the symbols and channel on the observations.

2) High SNR: It can be easily seen from (9) that P_Δ is monotonically increasing with $\mathcal{E}_S/N_0$, and $P_\Delta \to \infty$ as $\mathcal{E}_S/N_0 \to \infty$. It is of interest to study the asymptotic behavior of the P_Δ/C_{AWGN} ratio for high SNR.

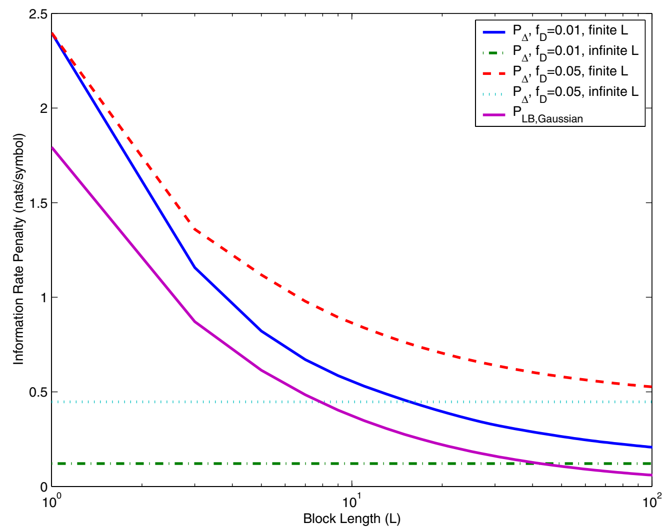

Fig. 1. Bounds on information rate penalty versus L for Clarke's Doppler spectrum ($\mathcal{E}_S/N_0 = 10$ dB).

For the uniform spectrum, and denoting $\widetilde{P}_{\Delta,\text{U}} = \lim_{L\to\infty} P_{\Delta,\text{U}}$, we have

$$\lim_{\mathcal{E}_S/N_0\to\infty}\frac{\widetilde{P}_{\Delta,\text{U}}}{C_{\text{AWGN}}} = \lim_{\mathcal{E}_S/N_0\to\infty}\frac{2f_D\log(1+\frac{\mathcal{E}_S}{N_0}\frac{1}{2f_D})}{\log(1+\frac{\mathcal{E}_S}{N_0})} = 2f_D.$$

With some manipulations, it can be shown that the same relation also holds for Clarke's spectrum. Now, since for the Rayleigh channel with known CSI in (18),

$$\lim_{\mathcal{E}_S/N_0\to\infty}\frac{C_{\text{Rayleigh}}}{C_{\text{AWGN}}} = 1,$$

we can readily see that for both uniform and Clarke spectra, the ratios of the upper bound in (15) and the lower bound in (19) to C_{Rayleigh} equal to $1-2f_D$. Therefore, for long blocks and and high SNR, the loss due to unknown CSI is proportional to twice the channel normalized fading rate.

IV. Numerical Results

Fig. 1 shows (9) and (11), the bounds on information rate penalty due to unknown CSI, versus block length L for SNR per symbol $\mathcal{E}_S/N_0 = 10$ dB, and for normalized fading rates $f_D = 0.01$ and 0.05. As expected, the penalty of not knowing the CSI decreases in opposite direction of the block length L, and P_Δ approaches the asymptotic bound in (23) as L goes to infinity. Conversely, short blocks do not allow observation of the channel over a period of time sufficient for learning the channel dynamics, and hence have the effect of increasing the penalty due to unknown channel.

Fig. 2 is a graph of the long block ($L\to\infty$) information rate penalties (23) and (26) as functions of $\mathcal{E}_S/N_0$, for various fading rates. It can be seen that the penalties due to unknown channel associated with the uniform spectrum are only slightly higher than those for Clarke's spectrum with the same fading

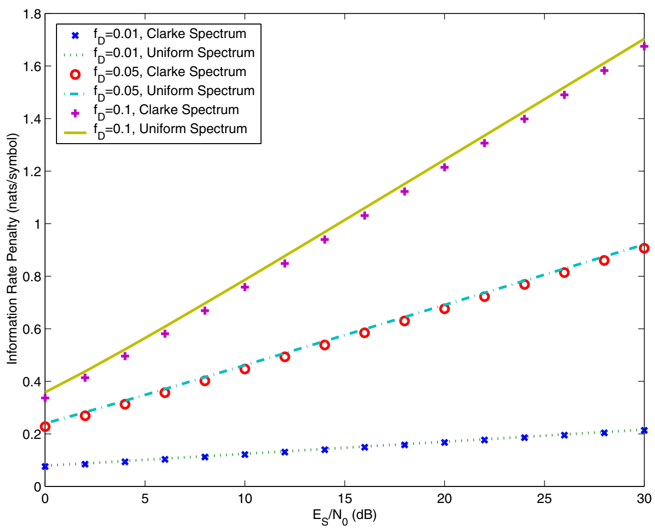

Fig. 2. Asymptotic P_Δ versus $\mathcal{E}_S/N_0$.

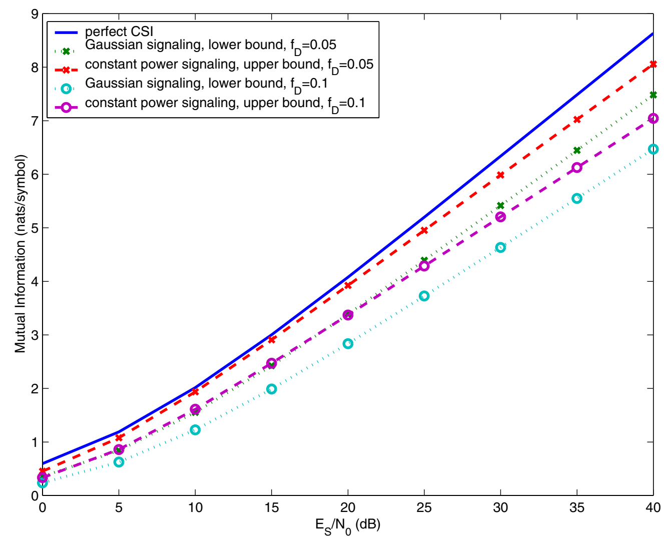

Fig. 3. Information rate bounds versus $\mathcal{E}_S/N_0$.

rate. The penalty increases linearly with the SNR per symbol in dB and with the fading rate.

In Fig. 3 the bounds (15) and (19) on the achievable information rates without CSI are plotted. The curves are shown as functions of SNR for normalized fading rates $f_D = 0.05$ and 0.1. Uniform spectra and long blocks are assumed ((26) holds). Since the upper bound (20) is too loose for long blocks, we include (18), which serves as a trivial upper bound, for comparison. It can be observed that in the high SNR regime, the achievable information rate has a slope of $0.7(1-2f_D)$ nats/symbol/3dB ($(1-2f_D)$ bits/symbol/3dB), as opposed to 0.7 nats/symbol/3dB (1 bit/symbol/3dB) in the presence of perfect CSI.

V. Conclusions

We derived bounds on the information rates achievable over time varying, Rayleigh fading channels. In particular, the penalty on the information rates due to unknown CSI was evaluated for the Clarke's Doppler spectrum and the uniform spectrum. Bounds on the achievable information rates were developed for constant power signaling and for Gaussian signaling. It was demonstrated that, for constant power signaling, when the block length is large and the SNR per symbol is high, the ratio of the penalty due to unknown CSI to the capacity with perfect CSI equals twice the normalized fading rate.

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