

# An Analytical Model for Measuring SIR in Ad-Hoc Wireless Networks

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## *Abstract -*

**This paper develops an analytical model for evaluation of the signal to interference ratio (SIR) in wireless ad-hoc networks. The effect of co-channel interference is determined using an ideal channel allocation algorithm where all neighbor nodes are prevented from transmitting on the same channel as the user. However, nodes that are more than one hop away are permitted to reuse the same channel as the user - thereby causing co-channel interference. A bound on the SIR is derived as a function of the number of nodes, network density, and parameters of a Markov chain model for the multiple user channel access protocol. The developed analytical expression is shown to be in agreement with computer simulations.**

## I. INTRODUCTION

In the last few years mobile ad-hoc networks for personal communication services have emerged as an alternative architecture to infrastructure based wireless networks (e.g. cellular networks). The distributed nature of ad-hoc networks makes them more robust to systemic failures, easier to deploy, and more flexible to reconfigure than infrastructure-based networks.

In ad-hoc networks, channel allocation algorithms are used to prevent the one-hop neighbors of a node from interfering with its transmissions. However, nodes that are more than one relay hop away are permitted to reuse the same channel as the user - thereby causing co-channel interference. The problem of co-channel interference in ad-hoc network has been previously addressed in the technical literature. In [2], the authors propose a channel assignment protocol so that the channel resource could be highly utilized in light of the co-channel interference problem. Further in [6], it was proven that in ad-hoc networks, interference from "hidden" terminals is a problem since the power needed for interrupting a packet reception is much lower than that of delivering a packet successfully.

Analytical models for evaluating interference in ad-hoc networks have also been studied in the literature. In [4], an analytical expression is derived, for a single link, for the radio interference from adjacent systems or users in ad-hoc networks. The analysis in this paper takes a different

approach from the one presented in [4]. The difference is that in [4] the mean interference power is derived based on geometry and path-loss for both inter-network (interference between different networks, such as IEEE 802.11b and Bluetooth, which operate in the same frequency band) and intra-network interferers (interference between nodes in the same network do to frequency reuse). This paper focuses on intra-network interferers by counting the maximum number of interferers based on an ideal channel access protocol ("hidden" terminals are not possible). With multiple user channel access techniques such as carrier sense multiple access/collision avoidance (CSMA/CA), neighbors of both the transmitter and the receiver nodes must be blocked from transmitting on the same channel as the transmitter node (provided that omni-directional antennas are used). Unlike landlines or wireless cellular systems with frequency reuse patterns, in wireless ad-hoc networks the maximum number of simultaneous transmissions ( $L$ ) supported by the network is also a function of the network density and of the radio range of each node in the network. If neighbors were not blocked, the number of possible transmissions would be equal to the number of nodes in the network ( $M$ ) divided by two ( $M / 2$ ). However, since neighbors must be blocked, this quantity is less than  $M / 2$  ( $L < M / 2$ ). Our goal in this paper is to develop an analytical model for interference that is derived from the maximum number of simultaneous transmissions.

The rest of this paper is organized as follows. First, in Section II, we present the system model that is used in our analysis, and derive the corresponding transition state diagram. In Section III we derive the expression for the SIR, while Section IV describes the results from two a computer simulation model that was developed to verify the derived SIR equation. Finally, Section V concludes the paper and highlights some of our current and future research work.

## II. SYSTEM MODEL

The system model used in this paper is derived from the multiple user channel access protocol employed in ad-hoc networks. As mentioned previously, a transmission between two nodes results in the blocking of the neighboring nodes, where a *neighbor* is defined as a node found within the transmission range of another node. Our network does not employ power control, and, without loss of

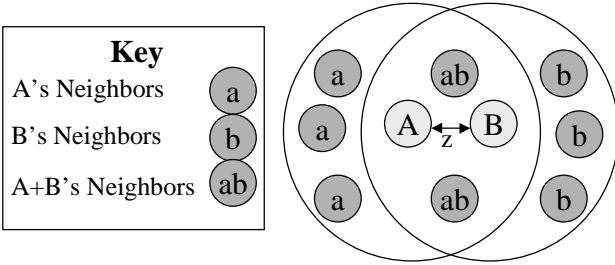


Fig. 1. Transmissions with Omni-Directional Antennas

generality, each node's transmission range is normalized to 1 km. To illustrate the concept of neighbor nodes, let us assume that node A (see Fig. 1) would like to send a packet to node B. In order to ensure interference free transmission, all of the RF neighbors of both nodes A and B must be blocked. The total number of blocked nodes (assuming no "hidden" terminals) is expressed in terms of number of RF neighbors of node A ( $N_A$ ), number of RF neighbors of node B ( $N_B$ ), and the number of shared RF neighbors ( $N_{AB}$ ):

$$\beta_{AB} = N_A + N_B - N_{AB} - 2. \quad (1)$$

In the example in Fig. 1,  $\beta_{AB} = 8$ . Note that more and more nodes get blocked with each additional simultaneous transmission. In the following we use a Markov Chain shown in Fig. 2, to model the blocking process of the RF neighbors. Each state ( $s$ ) in our model represents the actual number of current transmissions, and is associated with the following parameters:

- $\beta_s$  - Incremental number of blocked nodes in state  $s$
- $B_s$  - Total number of blocked nodes in state  $s$
- $C_s$  - Total number of communicating nodes in state  $s$  is  $2s$
- $T_s$  - Total number of involved nodes =  $B_s + C_s$
- $\lambda_s$  - Transmission initiation rate in state  $s$
- $\mu_s$  - Transmission service rate in state  $s$
- $\alpha_s$  - Destination blocking rate in state  $s$
- $L$  - Number of states (maximum number of transmissions)

In the following we assume that there are  $M$  nodes in the network at all times and the nodes are distributed uniformly over a 2-dimensional square plane with dimensions  $W \times W$ . Traffic is distributed uniformly over all the nodes and both the packet inter-arrival times and the packet transmission (processing) times are exponentially distributed with the packet arrival and service rates denoted by  $\lambda$  and  $\mu$  respectively. The packet queue size is assumed to be one;

that is if a node is blocked from transmitting, the packet is discarded. Finally, it is assumed that all the nodes in the network share a single RF channel.

The path-loss model assumed for this derivation is that of the simple exponential model:

$$P_{rcv}(d) = P_0 \left( \frac{d}{d_0} \right)^{-n} \quad (2)$$

where  $P_0$  is the power at the receiver at a small distance  $d_0$  and  $n$  is the path-loss exponent.

### III. DERIVATION OF SIR

The derivation of the SIR is accomplished in three steps. First we restate key results from [3]. Then we derive the bound on average interference power. We, then, use the bound to derive an expression for the average SIR.

#### A. Blocked Nodes and Simultaneous Transmissions

Since all the nodes in the network share a single channel, the RF neighbors of a TX/RX pair must be blocked from transmitting during the entire transmission sequence (like in CSMA/CA [1]). In [3] the average number of blocked neighbors  $\beta_1$  for a given TX/RX pair was derived, as a function of network density  $D$  ( $D = M / W^2$ ), to be:

$$\beta_1 = \frac{D}{P(z \leq 1)} \int_0^1 A(z) f_z(z) dz, \quad (3)$$

$A(z)$  is the area circumscribed by the two intersecting circles shown in Fig. 1 and can be shown to equal:

$$A(z) = 2\pi - 4 \int_{z/2}^1 \sqrt{1-x^2} dx. \quad (4)$$

$f_z(z)$  is the probability density function (PDF) of the distance  $z$  between two uniformly distributed nodes:

$$f_z(z) = \frac{2z}{W^2} \left( \frac{z^2}{W^2} - \frac{4z}{W} + \pi \right) \quad 0 \leq z \leq W \quad (5)$$

$P(z \leq 1)$  is the probability that a node is a RF neighbor (within 1 distance unit range):

$$P(z \leq 1) = \int_0^1 f_z(z) dz \quad (\text{assuming } W \geq 1) \quad (6)$$

It was also shown in [3] that based on linear regression of

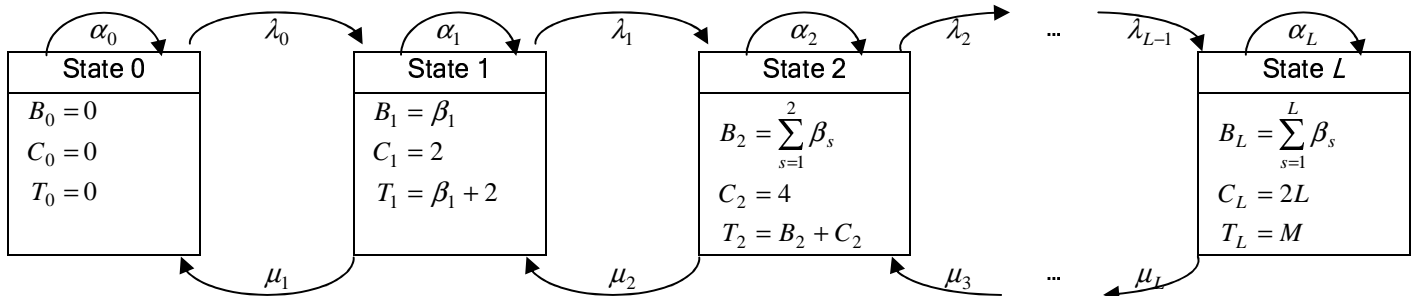


Fig. 2. Markov Chain Model of Multiple User Channel Access

simulation data, the number of blocked nodes at state  $s$  can be approximated as:

$$\tilde{\beta}_s = ms + b \text{ where } m = \frac{\beta_L - \beta_1}{L-1} = \frac{-\beta_1}{L-1} \text{ and } b = \frac{L\beta_1}{L-1} \quad (7)$$

Using the above expressions to solve the balanced equation of the Markov chain system model it was shown that the maximum number of simultaneous transmissions  $L$  can be written as:

$$L = \frac{2M}{\beta_1 + 4} \quad (8)$$

### B. Average Interference and Signal Power

As was stated previously, nodes that are within 1 km distance of either the transmitter or the receiver node are blocked from using the same channel as the transmitter. However, nodes that are outside this range are free to re-use this channel for their own transmissions. The interference power from these other nodes can be upper-bounded by integrating the received power over the network area which is outside the transmission range of the receiving node (greater than  $R=1$ ):

$$\begin{aligned} \bar{P}_i &\leq \int_R^Z P_{rcv}(r) f_z(r) | R \leq r \leq Z dr \text{ where } Z \equiv \sqrt{2W^2} \\ &\leq \frac{1}{P(R \leq r \leq Z)} \int_R^Z P_{rcv}(r) f_z(r) dr \end{aligned} \quad (9)$$

Note that  $Z$  is defined as the maximum distance between two nodes and therefore is equal to the diagonal distance of the  $WX$   $W$  square plane. Since the maximum number of simultaneous transmissions was shown to be  $L$ , the total interference power at a receiver can be calculated by multiplying the average interference power by the maximum number of interferers ( $L-1$ , this implies that the network is fully utilized):

$$\bar{P}_{iT} \leq (L-1)\bar{P}_i \quad (10)$$

The lower bound for SIR as a function of distance between the receiver and transmitter can now be written as:

$$SIR(z) (dB) \geq 10 \log_{10} \left( \frac{P_{rcv}(z)}{\bar{P}_i \cdot (L-1)} \right) \quad (11)$$

To find average SIR we must first determine the average signal power at the receiver. The average signal power at the receiver can be determined by using our path-loss equation and integrating it over all possible distances between the transmitter and the receiver:

$$\begin{aligned} \bar{P}_r &= \int_0^R P_{rcv}(r) f_z(r) | 0 \leq r \leq R dr \\ &= \frac{1}{P(0 \leq r \leq R)} \int_0^R P_{rcv}(r) f_z(r) dr \end{aligned} \quad (12)$$

Combining the above equations we obtain a lower bound (LB) on the average SIR:

$$\overline{SIR} (dB) \geq 10 \log_{10} \left( \frac{\bar{P}_r}{\bar{P}_i \cdot (L-1)} \right) \quad (13)$$

## IV. COMPUTER SIMULATIONS

Computer simulations were used to verify the system model and the SIR LB equation (13) that was derived in previous sections. All the simulations used a path-loss exponent of 3. The simulation results are given in Fig. 3 for 1,000, 5,000, and 10,000 node simulations. For each network size, the SIRs from the simulations and from the

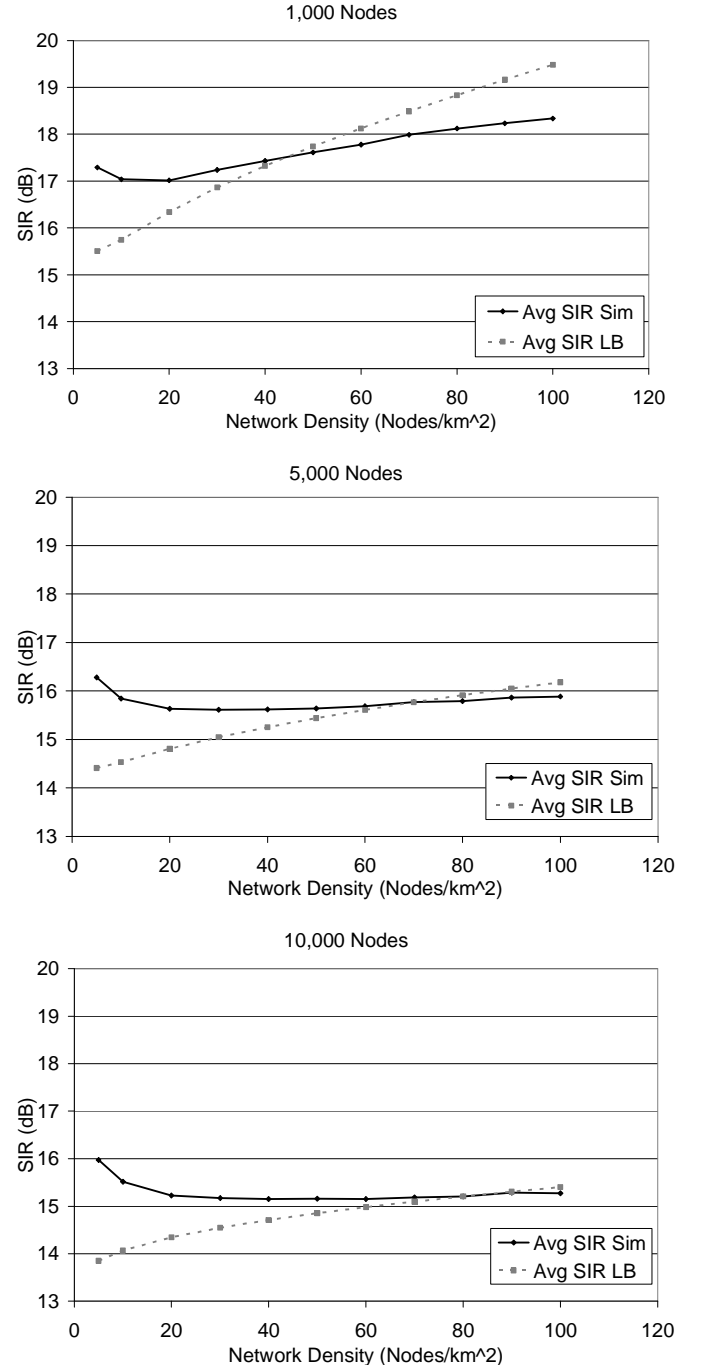


Fig. 3 1000, 5000, and 10000 nodes - SIR Versus Density

analytical model are plotted versus network density. Note that the SIR LB holds within about 2 dB as long as the network density does not exceed a threshold (this threshold is a function of network size). As the network density increases, there are few interferers since most of the nodes are within range of one another. As a reminder, a neighbor node is a node which is within the radio range of the transmitter or receiver and therefore can hear the transmitter or receiver nodes' control messages (RTS/CTS). Conversely, an interferer node is outside the radio range of the transmitter and receiver and therefore can not hear the transmitter and receiver nodes' reservation control messages. At the upper limit the network density becomes large, the number of neighbors  $\beta_1$  becomes equal to  $M - 2$ , and the number of interferers becomes zero. In turn the maximum number of simultaneous transmissions  $L$  becomes equal to one (no interferers).

For the 1,000-node network the SIR LB holds up to network density of 50 Nodes/km<sup>2</sup>. At 60 Nodes/km<sup>2</sup>, the maximum number of simultaneous transmissions  $L$  can be found using (8) to be 7.458. However from the simulation, the measured average of the maximum number of simultaneous transmissions  $L_{sim}$  was 8.792. Since (13) uses  $L$  from (8) which is smaller than the measured result, the SIR LB becomes larger than the measured SIR. If we plug in  $L_{sim}$  into (13), the result is 17.309 dB which is less than the measured simulation result of 18.125 dB (bound holds). The same is true for network density of 100 nodes/km<sup>2</sup>. In this case  $L=4.515$  and  $L_{sim}=6.140$ . Similarly, if we plug  $L_{sim}$  into (13), the result is 17.832 dB which is less than the measured simulation result of 18.335 dB (again bound holds).

For larger networks the difference between  $L$  and  $L_{sim}$  diverges slower (as a function of network density) than for smaller networks. For the 5,000-node network, the SIR LB holds until the density of 70 Nodes/km<sup>2</sup> and for the 10,000-node network, the SIR LB holds until the density of 80 Nodes/km<sup>2</sup>.

## V. CONCLUSION

In this paper an analytical model to study, evaluate and quantify the SIR in ad-hoc networks was introduced. First we reviewed expressions, derived in [3] for the average number of blocked nodes and for the maximum number of simultaneous transmissions (on a single channel). Then, we used these expressions to derive the SIR LB equation (13).

In our current research we are using the SIR LB equation as an analytical tool to study how SIR can affect link performance in ad-hoc networks. Furthermore we are working on the expansion of the presented results by refining the SIR LB equation for different assumptions (e.g. infinite queue, finite queue, power control).

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