Symbol Error Probability and Bit Error Probability for Optimum Combining with MPSK Modulation

Debang Lao and Alexander M. Haimovich
Corresponding Address:
Alexander M. Haimovich
Department of Electrical and Computer Engineering
New Jersey Institute of Technology
Newark, New Jersey 07102, USA
Tel: (973) 596-3534 Fax: (973) 596-8473
E-mail: haimovic@njit.edu

This work was supported by AFOSR Grant F49620-00-1-0107 and New Jersey Center for Wireless Telecommunications.

Abstract

New expressions are derived for the exact symbol error probability and bit error probability for OC with multiple phase-shift keying. The expressions are for any numbers of equal power co-channel interferers and receive branches. It is assumed that the aggregate interference and noise is Gaussian and that both the desired signal and interference are subject to flat Rayleigh fading. The new expressions have low computational complexity as they contain only a single integral form with finite limits and finite integrand.

Index Terms

Receive diversity, optimum combining, interference suppression, fading channels, error probability performance.

I. INTRODUCTION

Optimum combining (OC) is a well-known method to combat fading and suppress co-channel interference in wireless communication systems with receive diversity. It combines the outputs of the receive branches in an optimum way and achieves the maximum output signal-to-interference plus noise ratio (SINR).

Performance analysis of OC has been an active research area. Analysis for the case of a single interference source with binary phase-shift keying (BPSK) modulation can be found in [1], [2] and [3]. The performance of systems with more than one interferer has been studied extensively through the use of Monte Carlo simulations [1], upper bounds [4], approximate expressions [5], and exact expressions with integral forms [6], [7]. Closed-form expressions of BEP for the number of interferers no less than the number of receive branches and negligible thermal noise with BPSK modulation were developed in [8]. For arbitrary numbers of interferers and receive branches, closed-form expressions of BEP were derived in [9] and [10] for BPSK modulation.

An expression for symbol error probability (SEP) for multiple phase-shift keying (M-PSK) was derived in [7]. The expression was exact and it applied to any number of interferers and receive branches. It involved multiple-fold integration. A simpler and elegant SEP expression was derived in recent work [11] for the same case. The expression contained integration over an integrand, which included the incomplete Gamma function, itself an integral form.

In this paper, we derive expressions for both SEP and BEP for M-PSK, with any number of receive branches and interferers. The expressions involve only a single integration over elementary functions. With these expressions, it takes much less time to evaluate the SEP and BEP than it would take to carry out Monte Carlo simulations or to evaluate multiple-fold integrals.

The paper is organized as follows. Following the system model in Section II, we develop the expressions for SEP and BEP in Section III. Numerical results are shown in Section IV and finally, conclusions are drawn in Section V.

II. SYSTEM MODEL

Consider a wireless communication system with N independent receive branches and $L+1$ users. Of the users, one is the desired user and it transmits signals with power P_s. The other L sources are considered interference sources. Assuming perfect carrier demodulation and synchronization, the sampled output of the matched filter for the j-th branch is

$$r_j = \sqrt{P_s} c_j s + \sum_{i=1}^{L} \sqrt{P_I} c_{i,j} s_i + n_j, \; j = 1, 2, \cdots, N, \tag{1}$$

where c_j and s are respectively, the channel gain and M-PSK symbol of the desired user; $c_{i,j}$ and s_i are respectively, the i-th interferer's channel gain and symbol; P_I is the interference power (assumed equal for all interference sources). The term n_j represents additive white Gaussian noise (AWGN). The channel gains c_j and $c_{i,j}$ are assumed to be independently and identically distributed (i.i.d.), zero-mean, circularly symmetric, complex Gaussian random variables (Rayleigh fading), with variance $1/2$ per dimension. The signal model in vector notation is

$$\mathbf{r} = \sqrt{P_s} \mathbf{c} s + \sqrt{P_I} \sum_{i=1}^{L} \mathbf{c}_i s_i + \mathbf{n}, \tag{2}$$

where $\mathbf{r} = \left[r_1, r_2, \cdots, r_N \right]^T$, $\mathbf{c}$, $\mathbf{c}_i$ and $\mathbf{n}$ are defined similarly, and the superscript T denotes vector transposition.

Define the interference plus noise vector as $\mathbf{z} = \sqrt{P_I} \sum_{i=1}^{L} \mathbf{c}_i s_i + \mathbf{n}$. Assume the interference signal s_i is Gaussian distributed with zero-mean and unit variance. Then conditioned on the vectors $\mathbf{c}_i$'s, $\mathbf{z}$ has a multivariate complex-Gaussian distribution with zero-mean and covariance matrix

$$\mathbf{R} = E\left[\mathbf{z}\mathbf{z}^H \right] = P_I \sum_{i=1}^{L} \mathbf{c}_i \mathbf{c}_i^H + \sigma^2 \mathbf{I}_N, \tag{3}$$

where the superscript H denotes the Hermitian transposition, σ^2 is the power of the additive white Gaussian noise, and $\mathbf{I}_N$ is an identity matrix of rank N.

Define $N_{\text{max}} = \max(N, L)$ and $N_{\text{min}} = \min(N, L)$. We sort the N eigenvalues of the interference plus noise covariance matrix $\mathbf{R}$ in descending order as $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_N \geq \sigma^2$. It is well known that $\lambda_i = \sigma^2$ for $i = N_{\text{min}} + 1, N_{\text{min}} + 2, \cdots, N$. For notational convenience, we denote the other N_{min} non-trivial eigenvalues as $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \cdots, \lambda_{N_{\text{min}}}]^T$. The joint probability density function of the N_{min} random eigenvalues is [7]

$$p_{\boldsymbol{\lambda}}(\boldsymbol{\lambda}) = K_0 \left[\prod_{i=1}^{N_{\text{min}}} \exp\left(-\frac{\lambda_i - \sigma^2}{P_I}\right) \left(\frac{\lambda_i - \sigma^2}{P_I}\right)^{N_{\text{max}} - N_{\text{min}}} \right] \left[\prod_{1 \leq i < j \leq N_{\text{min}}} (\lambda_i - \lambda_j)^2 \right] \tag{4}$$

for $\infty > \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{N_{\text{min}}} \geq \sigma^2$, where

$$K_0 = \frac{1}{\prod_{i=1}^{N_{\text{min}}} (N_{\text{max}} - i)! \prod_{i=1}^{N_{\text{min}}} (N_{\text{min}} - i)!} \frac{1}{P_I^{N_{\text{min}}^2}}. \tag{5}$$

With the OC detector, the received signal vector $\mathbf{r}$ is weighted and combined to obtain the output signal. The weight vector that yields the maximum SINR is ([1], [12]) $\mathbf{w} = \mathbf{R}^{-1}\mathbf{c}$. The output of the combiner is

$$\mathbf{w}^H \mathbf{r} = \sqrt{P_s} \mathbf{c}^H \mathbf{R}^{-1} \mathbf{c} s + \mathbf{c}^H \mathbf{R}^{-1} \mathbf{z}. \tag{6}$$

The first term $\sqrt{P_s}\mathbf{c}^H\mathbf{R}^{-1}\mathbf{c}s$ corresponds to the desired signal, while the second term $\mathbf{c}^H\mathbf{R}^{-1}\mathbf{z}$ corresponds to interference plus noise. The latter is Gaussian distributed conditioned on the channel vectors $\mathbf{c}$ and $\mathbf{c}_i$. The signal model of (6) is similar to that of an AWGN channel with noise variance $E_{s_i,\mathbf{n}}\left[\left|\mathbf{c}^H\mathbf{R}^{-1}\mathbf{z}\right|^2\right]$, with the expectation taken over the interfering signal s_i and AWGN $\mathbf{n}$. The instantaneous output SINR γ_t is

$$\gamma_t = P_s \mathbf{c}^H \mathbf{R}^{-1} \mathbf{c}. \tag{7}$$

III. EXPRESSIONS FOR SEP AND BEP

In this and the next section, we carry out the theoretical analysis of the SEP and BEP for OC with M-PSK modulation in the presence of any number of interference sources and receive branches when both the desired signal and interference are subject to Rayleigh fading.

A. Expression for SEP

For M-PSK, the SEP conditioned on the output SINR γ_t can be written as [12, Eq. (8.22)]

$$P_{\text{sym}}(E|\gamma_t) = \frac{1}{\pi} \int_0^{(M-1)\pi/M} \exp\left\{-\gamma_t \frac{\sin^2(\pi/M)}{\sin^2\theta}\right\} d\theta, \tag{8}$$

where M is the number of symbols of the M-PSK modulation. The SEP is conditioned on channel realizations through γ_t. In order to get the ensemble average SEP $P_{\text{sym}}(E)$ for OC, we need to average $P_{\text{sym}}(E|\gamma_t)$ over the distribution of γ_t,

$$P_{\text{sym}}(E) = \int_0^\infty P_{\text{sym}}(E|\gamma_t)\, p_{\gamma_t}(\gamma_t)\, d\gamma_t, \tag{9}$$

where $p_{\gamma_t}(\gamma_t)$ is the probability density function (PDF) of the SINR γ_t. Let $p_{\gamma_t|\boldsymbol{\lambda}}(\gamma_t|\boldsymbol{\lambda})$ represent the PDF of γ_t conditioned on the non-trivial eigenvalues $\boldsymbol{\lambda}= [\lambda_1, \lambda_2, \cdots, \lambda_{N_{\text{min}}}]$. The PDF $p_{\gamma_t}(\gamma_t)$ can be obtained by averaging $p_{\gamma_t|\boldsymbol{\lambda}}(\gamma_t|\boldsymbol{\lambda})$ over $\boldsymbol{\lambda}$:

$$p_{\gamma_t}(\gamma_t) = \int \cdots \int p_{\gamma_t|\boldsymbol{\lambda}}(\gamma_t|\boldsymbol{\lambda})\, p_{\boldsymbol{\lambda}}(\boldsymbol{\lambda}) d\boldsymbol{\lambda}. \tag{10}$$

Since $\boldsymbol{\lambda}$ is a vector, the above integration is multiple-fold.

Substituting (8) and (10) in (9), after some manipulations similar to those in [12], we have

$$P_{\text{sym}}(E) = \frac{1}{\pi} \int \cdots \int \left[\int_0^{(M-1)\pi/M} M_{\gamma_t|\boldsymbol{\lambda}}\left(-\frac{\sin^2(\pi/M)}{\sin^2\theta} \right) d\theta \right] p_{\boldsymbol{\lambda}}(\boldsymbol{\lambda}) d\boldsymbol{\lambda}, \tag{11}$$

where $M_{\gamma_t|\boldsymbol{\lambda}}(\cdot)$ is the moment generating function (MGF) of the SINR γ_t conditioned on eigenvalues $\boldsymbol{\lambda}$. For the Rayleigh fading channel, the MGF given by [12, Eq. 10.52] for $L < N$ can be generalized easily to any numbers of L and N as

$$M_{\gamma_t|\boldsymbol{\lambda}}(s) = \left(\frac{1}{1 - \frac{P_s}{\sigma^2} s} \right)^{N - N_{\text{min}}} \prod_{i=1}^{N_{\text{min}}} \frac{1}{1 - \frac{P_s}{\lambda_i} s}. \tag{12}$$

B. Expression for BEP

The expressions of BEP for M-PSK modulation with Gray code bit mapping over AWGN channel can be found in ([13], [12, Eq. (8.30)]). From these expression for AWGN, and similarly to the derivations from (8) to (11), we can obtain the BEP for OC as

$$P_{\text{bit}}(E) = \begin{cases} P_0 & M = 2 \\ \frac{1}{2}(P_1 + 2P_2 + P_3) & M = 4 \\ \frac{1}{3}(P_1 + 2P_2 + P_3 + 2P_4 + 3P_5 + 2P_6 + P_7) & M = 8 \\ \frac{1}{2}\left(\sum_{k=1}^{8} P_k + \sum_{k=2}^{5} P_k + P_5 + 2P_6 + P_7 \right) & M = 16 \end{cases}, \tag{13}$$

where

$$P_k \;=\; \frac{1}{2} C\left(\pi\left[1 - (2k-1)/M\right], \sin^2\left[(2k-1)\pi/M\right] \right)$$

$$-\frac{1}{2}C\left(\pi\left[1-(2k+1)/M\right],\sin^2\left[(2k+1)\pi/M\right]\right), \tag{14}$$

and

$$C(\phi,\xi)=\frac{1}{\pi}\int\cdots\int\left[\int_0^{\phi}M_{\gamma_t|\boldsymbol{\lambda}}\left(-\frac{\xi}{\sin^2\theta}\right)d\theta\right]p_{\boldsymbol{\lambda}}(\boldsymbol{\lambda})\,d\boldsymbol{\lambda}. \tag{15}$$

Note that the SEP in (11) can be expressed as

$$P_{\text{sym}}(E)=C\left((M-1)\pi/M,\sin^2(\pi/M)\right). \tag{16}$$

In Appendix A we show that $C(\phi,\xi)$ can be evaluated as

$$C(\phi,\xi)=\left(\frac{1}{\xi\beta}\right)^{N_{\text{min}}-1}\sum_{p=0}^{N_{\text{min}}-1}\left(\frac{\beta}{\gamma}\right)^{p}\sum_{q=0}^{N_{\text{min}}-1}(-1)^{N_{\text{min}}-1+q}H_{p,q}\Upsilon_q, \tag{17}$$

where $\gamma=P_s/\sigma^2$ is the symbol signal-to-noise ratio (SNR), and $\beta=P_s/P_I$ is the signal-to-interference ratio (SIR); $H_{p,q}$ and Υ_q are defined below:

- $H_{p,q}$ is a sequence indexed by p and q. For $0\le p,q\le N_{\text{min}}-1$,

$$\begin{aligned}H_{p,q}\;&=\;\frac{1}{\left[\prod_{i=1}^{N_{\text{min}}}(N_{\text{max}}-i)!\right]\left[\prod_{i=1}^{N_{\text{min}}}(N_{\text{min}}-i)!\right]}\\&\quad\times\sum_{\substack{m_1+\cdots+m_{N_{\text{min}}-1}=N_{\text{min}}-1-p\\ m_i\in\{0,1\}}}\;\sum_{\substack{n_1+\cdots+n_{N_{\text{min}}-1}=N_{\text{min}}-1-q\\ n_i\in\{0,1\}}}\det\mathbf{W},\end{aligned} \tag{18}$$

  where for $N_{\text{min}}=1, \det\mathbf{W}=1$; for $N_{\text{min}}>1$, $\det\mathbf{W}$ is the determinant of an $(N_{\text{min}}-1)\times(N_{\text{min}}-1)$ matrix whose i-th row and j-th column element is

$$W_{i,j}=(N_{\text{max}}-N_{\text{min}}+m_j+n_j+i+j-2)!.$$

- Υ_q is a sequence given by

$$\begin{aligned}\Upsilon_q\;&=\;\sum_{k=0}^{N_{\text{min}}}\binom{N}{k}(-\xi_1)^k\left[\sum_{i=1}^{N_{\text{min}}-k}F_{N_{\text{min}}-k-i}X_{q,i-1}+Y_{q,N_{\text{min}}-k}\right]\\&\quad+\sum_{k=N_{\text{min}}+1}^{N}\binom{N}{k}(-\xi_1)^k\left[-\sum_{i=0}^{k-N_{\text{min}}-1}G_{k-N_{\text{min}}-i}X_{q,-(i+1)}+Y_{q,N_{\text{min}}-k}\right],\end{aligned} \tag{19}$$

  where $\xi_1=\xi\gamma$. Other terms (F,X,Y and G) in (19) are defined as (m is an integer)

$$F_m\;=\;\frac{1}{\pi}\sum_{l=0}^{m}\binom{m}{l}\xi_1^{m-l}$$

$$\times\left[\frac{1}{2^{2l}}\begin{pmatrix}2l\\ l\end{pmatrix}\phi+\frac{(-1)^l}{2^{2l-1}}\sum_{k=0}^{l-1}(-1)^k\begin{pmatrix}2l\\ k\end{pmatrix}\frac{\sin\left(2l-2k\right)\phi}{2l-2k}\right] \tag{20}$$

$$X_{q,m}=\xi_1^m\sum_{l=0}^{N_{\min}-1-m}\begin{pmatrix}N_{\min}-1-m\\ l\end{pmatrix}\left(\frac{\beta}{\gamma}\right)^l\left(N_{\max}+q-l-1\right)! \tag{21}$$

$$\begin{aligned}Y_{q,m} &= \frac{1}{\pi}\xi_1^m\sqrt{\frac{1}{\xi\beta}}\int_0^{\frac{\pi}{2}}\frac{\left(\text{tg}\varphi+\frac{\beta}{\gamma}\right)^{N_{\min}-m}}{\sqrt{\text{tg}\varphi+\xi\beta+\frac{\beta}{\gamma}}}\text{arctg}\left(\sqrt{\frac{1}{\xi\beta}\left(\text{tg}\varphi+\xi\beta+\frac{\beta}{\gamma}\right)}\text{tg}\phi\right)\\ &\quad\times\exp\left(-\text{tg}\varphi\right)\left(\text{tg}\varphi\right)^{N_{\max}-N_{\min}+q+m}\sec^2\varphi d\varphi\end{aligned} \tag{22}$$

$$\begin{aligned}G_m &= \frac{1}{\pi}\frac{1}{\xi_1^m}\frac{1}{\left(1+\xi_1^{-1}\right)^{m-\frac{1}{2}}}\sum_{l=0}^{m-1}\begin{pmatrix}m-1\\ l\end{pmatrix}\begin{pmatrix}2l\\ l\end{pmatrix}\left(\frac{\xi_1^{-1}}{4}\right)^l\\ &\quad\times\left[\text{tg}^{-1}\left(\sqrt{1+\xi_1^{-1}}\text{tg}\phi\right)+\sqrt{1+\xi_1^{-1}}\frac{\text{tg}\phi}{2}\right.\\ &\quad\left.\times\sum_{j=1}^{l}\frac{4^j}{\begin{pmatrix}2j\\ j\end{pmatrix}j\left(1+\left(1+\xi_1^{-1}\right)\text{tg}^2\phi\right)^j}\right].\end{aligned} \tag{23}$$

By inspection of the terms that make up $C\left(\phi,\xi\right)$ in (17), it follows that the integration in (22) is the only one required to evaluate $C\left(\phi,\xi\right)$. With (16) and (17), we can calculate the SEP. With (13), (14) and (17), we can calculate the BEP. Although (17) and the related expressions appear involved, they consist of elementary functions and a single integral form, which can be readily computed numerically using Matlab or similar software.

These expressions are exact. But since the calculation of $Y_{q,m}$ in (22) involves integration, the actual accuracy of the final result will depend on the accuracy of the numerical integration.

IV. NUMERICAL RESULTS

In this section, we use numerical results to demonstrate the new exact SEP and BEP expressions. To facilitate the comparison, in all figures we represent both simulation results and analysis results. Analytical results were calculated using (16) (for SEP) and (13) (for BEP) and related expressions such as (14) and (17).

Fig. 1 shows the SEP versus symbol SNR $= P_s/\sigma^2$ for $N = 6$ branches, $L = 4$ interferers, and SIR $= P_s/P_I = 10$ dB. Fig. 2 shows the BEP versus the number of receive branches N for $L = 4$ interferers, bit SNR $= 10$ dB, and SIR $= 15$ dB. We can see $\log_{10}(\text{BEP})$ decreases linearly as the number of receive branches increases. In both figures, the interference signal s_i is generated as Gaussian distributed as assumed in Section II. It can be observed that analysis results match simulation results.

Fig. 3 shows BEP versus SIR for $N = 4$ branches, various numbers of interferers, and SNR $= 10$ dB. Both the desired signal and the interference signal are quadrature phase-shift keying (QPSK) symbols. It shows that though the interference signal is not Gaussian distributed, the analysis results are still very close to simulation results regardless of the number of interferers and the SIR levels. Similar conclusion was drawn in [10].

V. CONCLUSIONS

In this paper, we derived expressions of the exact SEP and BEP for OC with M-PSK modulation over a diversity channel with Rayleigh fading, with any number of diversity branches and interference sources. The interference sources were assumed to have equal power and the Gaussian assumption was invoked for the aggregate of interference plus noise. The computational complexity of the new expressions is relatively low as they contain only a single integration form. The theoretical results in the paper are amply demonstrated by simulations.

APPENDIX A

EVALUATION OF $C(\phi, \xi)$

In this appendix, we evaluate $C(\phi, \xi)$ defined in (15) to prove the relation in (17). We will present the procedure of the derivation but omit some details.

We start by substituting (12) in (15),

$$C(\phi, \xi) = \frac{1}{\pi} \int \cdots \int \left\{ \int_0^{\phi} \left(\frac{\sin^2\theta}{\sin^2\theta + \xi\gamma} \right)^{N-N_{\text{min}}} \left[\prod_{i=1}^{N_{\text{min}}} \left(\frac{\sin^2\theta}{\sin^2\theta + \xi \frac{P_s}{\lambda_i}} \right) \right] d\theta \right\} p_{\boldsymbol{\lambda}}(\boldsymbol{\lambda})\, d\boldsymbol{\lambda}, \tag{24}$$

where $\gamma = P_s/\sigma^2$ is the symbol SNR. The direct evaluation of (24) is computationally intensive even for small N_{min} since it involves a $(N_{\text{min}} + 1)$-fold integration. We will show that an expression for $C(\phi, \xi)$ can be obtained which involves only a single integration form.

Converting the product in (24) into a summation, we have

$$C(\phi,\xi)=\sum_{n=1}^{N_{\min}}\frac{1}{\pi}\int\cdots\int\left\{\int_0^{\phi}A_n(\boldsymbol{\lambda})\left(\frac{\sin^2\theta}{\sin^2\theta+\xi\gamma}\right)^{N-N_{\min}}\frac{\left(\sin^2\theta\right)^{N_{\min}}}{\sin^2\theta+\xi\frac{P_s}{\lambda_n}}d\theta\right\}p_{\boldsymbol{\lambda}}(\boldsymbol{\lambda})\,d\boldsymbol{\lambda} \tag{25}$$

where

$$A_n(\boldsymbol{\lambda})=\frac{\lambda_n^{N_{\min}-2}\prod_{i=1}^{N_{\min}}\lambda_i}{(\xi P_s)^{N_{\min}-1}}\prod_{i=1}^{n-1}\left(\frac{1}{\lambda_n-\lambda_i}\right)\prod_{i=n+1}^{N_{\min}}\left(\frac{1}{\lambda_n-\lambda_i}\right). \tag{26}$$

Starting with (25) and following similar procedure detailed in [10] and [9], we can express $C(\phi,\xi)$ as

$$C(\phi,\xi)=\left(\frac{1}{\xi\beta}\right)^{N_{\min}-1}\sum_{p=0}^{N_{\min}-1}\left(\frac{\beta}{\gamma}\right)^{p}\sum_{q=0}^{N_{\min}-1}(-1)^{N_{\min}-1+q}H_{p,q}\Upsilon_q, \tag{27}$$

where $\beta=P_s/P_I$ is the signal-to-interference ratio (SIR), $H_{p,q}$ is defined by (18). And Υ_q is a sequence defined by

$$\Upsilon_q=\int_0^{\infty}D\left(z_{N_{\min}}\right)f_q\left(z_{N_{\min}}\right)dz_{N_{\min}}, \tag{28}$$

where

$$\begin{aligned}
D\left(z_{N_{\min}}\right)&=\frac{1}{\pi}\int_0^{\phi}\left(\frac{\sin^2\theta}{\sin^2\theta+\xi_1}\right)^{N-N_{\min}}\frac{\left(\sin^2\theta\right)^{N_{\min}}}{\sin^2\theta+\xi_2}d\theta && (29)\\
\xi_1&=\xi\gamma && (30)\\
\xi_2&=\xi\frac{P_s}{P_I z_{N_{\min}}+\sigma^2} && (31)\\
f_q\left(z_{N_{\min}}\right)&=z_{N_{\min}}^{N_{\max}-N_{\min}+q}\left(z_{N_{\min}}+\frac{\sigma^2}{P_I}\right)^{N_{\min}-1}e^{-z_{N_{\min}}}. && (32)
\end{aligned}$$

A. Evaluation of $D\left(z_{N_{min}}\right)$

We first evaluate $D\left(z_{N_{\min}}\right)$, which involves the integration over variable θ. We want to express $D\left(z_{N_{\min}}\right)$ without integration.

From (29),

$$D\left(z_{N_{\min}}\right)=\frac{1}{\pi}\int_0^{\phi}\frac{\left(\sin^2\theta+\xi_1-\xi_1\right)^{N}}{\left(\sin^2\theta+\xi_1\right)^{N-N_{\min}}}\frac{1}{\sin^2\theta+\xi_2}d\theta. \tag{33}$$

Using the binomial expansion, we get

$$D\left(z_{N_{\min}}\right)=\sum_{k=0}^{N}\begin{pmatrix}N\\k\end{pmatrix}(-\xi_1)^k\frac{1}{\pi}\int_0^{\phi}\frac{1}{\sin^2\theta+\xi_2}\left(\sin^2\theta+\xi_1\right)^{N_{\min}-k}d\theta. \tag{34}$$

Separate the summation into two parts according to whether $N_{\min} - k$ is non-negative or negative. Then

$$D\left(z_{N_{\min}}\right) = \sum_{k=0}^{N_{\min}} \left(\begin{array}{c} N \\ k \end{array} \right) \left(-\xi_1\right)^k E_{N_{\min}-k}\left(\xi_1, \xi_2\right) + \sum_{k=N_{\min}+1}^{N} \left(\begin{array}{c} N \\ k \end{array} \right) \left(-\xi_1\right)^k U_{k-N_{\min}}\left(\xi_1, \xi_2\right), \tag{35}$$

where

$$\begin{aligned} E_m\left(\xi_1, \xi_2\right) &= \frac{1}{\pi} \int_0^{\phi} \frac{1}{\sin^2\theta + \xi_2} \left(\sin^2\theta + \xi_1\right)^m d\theta & (36) \\ U_m\left(\xi_1, \xi_2\right) &= \frac{1}{\pi} \int_0^{\phi} \frac{1}{\sin^2\theta + \xi_2} \frac{1}{\left(\sin^2\theta + \xi_1\right)^m} d\theta. & (37) \end{aligned}$$

1) Evaluation of $E_m\left(\xi_1, \xi_2\right)$: For $m = 0$,

$$\begin{aligned} E_0\left(\xi_2\right) &= \frac{1}{\pi} \int_0^{\phi} \frac{1}{\sin^2\theta + \xi_2} d\theta \\ &= \frac{1}{\pi} \frac{1}{\sqrt{\xi_2\left(\xi_2 + 1\right)}} \text{arctg}\left(\sqrt{\frac{\xi_2 + 1}{\xi_2}} \text{tg}\phi\right), & (38) \end{aligned}$$

where we use the result from [14, Eq. 2.562]. For $m \geq 1$, it can be shown that

$$E_m\left(\xi_1, \xi_2\right) = F_{m-1}\left(\xi_1\right) + \left(\xi_1 - \xi_2\right) E_{m-1}\left(\xi_1, \xi_2\right), \tag{39}$$

where

$$F_m\left(\xi_1\right) = \frac{1}{\pi} \int_0^{\phi} \left(\sin^2\theta + \xi_1\right)^m d\theta. \tag{40}$$

Using the binomial expansion and [14, Eq. 2.513.1], we obtain the expression for $F_m\left(\xi_1\right)$ shown in (20). Expanding (39) further, we have

$$E_m\left(\xi_1, \xi_2\right) = \sum_{i=1}^{m} \left(\xi_1 - \xi_2\right)^{i-1} F_{m-i}\left(\xi_1\right) + \left(\xi_1 - \xi_2\right)^m E_0\left(\xi_2\right), \tag{41}$$

which shows $E_m\left(\xi_1, \xi_2\right)$ can be evaluated from $F_{m-i}\left(\xi_1\right)$ and $E_0\left(\xi_2\right)$.

2) Evaluation of $U_m\left(\xi_1, \xi_2\right)$: Similarly to the evaluation of $E_m\left(\xi_1, \xi_2\right)$, we have

$$U_m\left(\xi_1, \xi_2\right) = -\sum_{i=0}^{m-1} \left(\frac{1}{\xi_1 - \xi_2}\right)^{i+1} G_{m-i}\left(\xi_1\right) + \left(\frac{1}{\xi_1 - \xi_2}\right)^m E_0\left(\xi_2\right), \tag{42}$$

where

$$G_m\left(\xi_1\right) = \frac{1}{\pi} \int_0^{\phi} \frac{1}{\left(\sin^2\theta + \xi_1\right)^m} d\theta. \tag{43}$$

Using Eq. (29) and (40) in [15], we get the expression for $G_m\left(\xi_1\right)$ shown in (23).

3) Summary for $D\left(z_{N_{min}}\right)$: Substituting (41) and (42) in (35), we obtain the expression for $D\left(z_{N_{\min}}\right)$ as

$$\begin{aligned} D\left(z_{N_{\min}}\right) &= \sum_{k=0}^{N_{\min}} \begin{pmatrix} N \\ k \end{pmatrix} \left(-\xi_1\right)^k \left[\sum_{i=1}^{N_{\min}-k} \left(\xi_1-\xi_2\right)^{i-1} F_{N_{\min}-k-i}\left(\xi_1\right) + \left(\xi_1-\xi_2\right)^{N_{\min}-k} E_0\left(\xi_2\right)\right] \\ &\quad + \sum_{k=N_{\min}+1}^{N} \begin{pmatrix} N \\ k \end{pmatrix} \left(-\xi_1\right)^k \left[-\sum_{i=0}^{k-N_{\min}-1} \left(\frac{1}{\xi_1-\xi_2}\right)^{i+1} G_{k-N_{\min}-i}\left(\xi_1\right)\right. \\ &\quad \left. + \left(\frac{1}{\xi_1-\xi_2}\right)^{k-N_{\min}} E_0\left(\xi_2\right)\right], \end{aligned} \tag{44}$$

which does not contain any integral forms.

B. Evaluation of Υ_q

Substitute (44) into (28), then

$$\begin{aligned} \Upsilon_q &= \sum_{k=0}^{N_{\min}} \begin{pmatrix} N \\ k \end{pmatrix} \left(-\xi_1\right)^k \left[\sum_{i=1}^{N_{\min}-k} F_{N_{\min}-k-i}\left(\xi_1\right) \int_0^\infty \left(\xi_1-\xi_2\right)^{i-1} f_q\left(z_{N_{\min}}\right) dz_{N_{\min}}\right. \\ &\quad \left. + \int_0^\infty \left(\xi_1-\xi_2\right)^{N_{\min}-k} E_0\left(\xi_2\right) f_q\left(z_{N_{\min}}\right) dz_{N_{\min}}\right] + \sum_{k=N_{\min}+1}^{N} \begin{pmatrix} N \\ k \end{pmatrix} \left(-\xi_1\right)^k \\ &\quad \times \left[-\sum_{i=0}^{k-N_{\min}-1} G_{k-N_{\min}-i}\left(\xi_1\right) \int_0^\infty \left(\frac{1}{\xi_1-\xi_2}\right)^{i+1} f_q\left(z_{N_{\min}}\right) dz_{N_{\min}}\right. \\ &\quad \left. + \int_0^\infty \left(\frac{1}{\xi_1-\xi_2}\right)^{k-N_{\min}} E_0\left(\xi_2\right) f_q\left(z_{N_{\min}}\right) dz_{N_{\min}}\right]. \end{aligned} \tag{45}$$

Substituting $f_q\left(z_{N_{\min}}\right)$ (from (32)), $E_0\left(\xi_2\right)$ (from (38)) and ξ_2 (from (31)) into (45), after some straightforward manipulations, we obtain Υ_q as shown in (19).

REFERENCES

[1] J. H. Winters, "Optimum combining in digital mobile radio with cochannel interference," *IEEE Transactions on Vehicular Technology*, vol. 33, pp. 144–155, August 1984.

[2] A. Shah, A. M. Haimovich, M. K. Simon, and M.-S. Alouini, "Exact bit-error probability for optimum combining with a Rayleigh fading Gaussian cochannel interference," *IEEE Transactions on Communications*, vol. 48, pp. 908–912, June 2000.

[3] V. A. Aalo and J. Zhang, "Performance of antenna array systems with optimum combining in a Rayleigh fading environment," *IEEE Communications Letters*, vol. 4, pp. 125–127, April 2000.

[4] J. H. Winters and J. Salz, “Upper bounds on the bit-error rate of optimum combining in wireless systems,” *IEEE Transactions on Communications*, vol. 46, pp. 1619–1624, December 1998.

[5] E. Villier, “Performance analysis of optimum combining with multiple interferes in flat Rayleigh fading,” *IEEE Transactions on Communications*, vol. 47, pp. 1503–1510, October 1999.

[6] J. Cui, D. D. Falconer, and A. U. Sheikh, “Analysis of BER for optimum combining with two co-channel interferers and maximal ratio combining with arbitrary number of interferers,” *Proc. IEEE Int. Symp. Personal indoor Mobil Commun. (PIMRC96)*, pp. 53–57, October 1996.

[7] M. Chiani, M. Z. Win, A. Zanella, and J. H. Winters, “Exact symbol error probability for optimum combining in the presence of multiple co-channel interferers and thermal noise,” *Global Telecommunications Conference*, vol. 2, pp. 1182–1186, 2001.

[8] A. Shah and A. M. Haimovich, “Performance analysis of optimum combining in wireless communications with Rayleigh fading and cochannel interference,” *IEEE Transactions on Communications*, vol. 46, pp. 473–479, April 1998.

[9] R.K. Mallik, M. Z. Win, and M. Chiani, “Exact analysis of optimum combining in interference and noise over a Rayleigh fading channel,” *IEEE International Conference on Communications (ICC ’02)*, vol. 3, pp. 1954–1958, April 2002.

[10] D. Lao and A. Haimovich, “Exact closed-form performance analysis of optimum combining with multiple co-channel interferers and Rayleigh fading,” *IEEE Transactions on Communications*, vol. 51, pp. 995–1003, June 2003.

[11] M. Z. Win, M. Chiani, and A. Zanella, “An analytical frame work for the performance evaluation of optimum combining for M-ary signals,” *Proceedings of the 2002 Conference on Information Sciences and Systems, Princeton University, NJ*, 2002.

[12] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channel: A Unified Approach to Performance Analysis*. New York, NY: John Wiley & Sons, 2000.

[13] P. J. Lee, “Computation of the bit error rate for coherent M-ary PSK with Gray code bit mapping,” *IEEE Transactions on Communications*, pp. 488–491, May 1986.

[14] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*. San Diego, CA: Academic Press, 1994.

[15] M. Z. Win, R. K. Mallik, G. Chrisikos, and J. H. Winters, “Canonical expressions for the error probability performance of M-ary modulation with hybrid selection/maximal-ratio combining in Rayleigh fading,” *IEEE Wireless Communications and Networking Conference*, vol. 1, pp. 266–270, 1999.

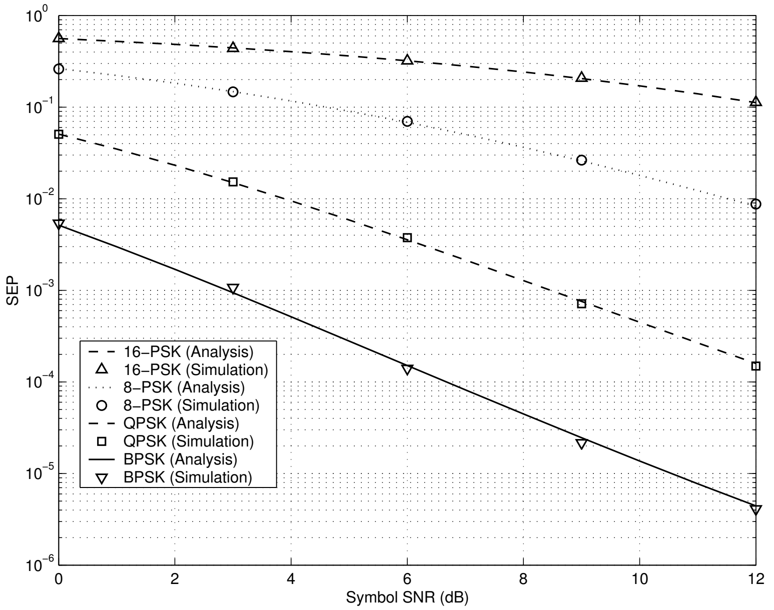

Fig. 1. SEP versus symbol SNR for $N = 6$ branches, $L = 4$ interferers, and SIR $= 10$ dB.

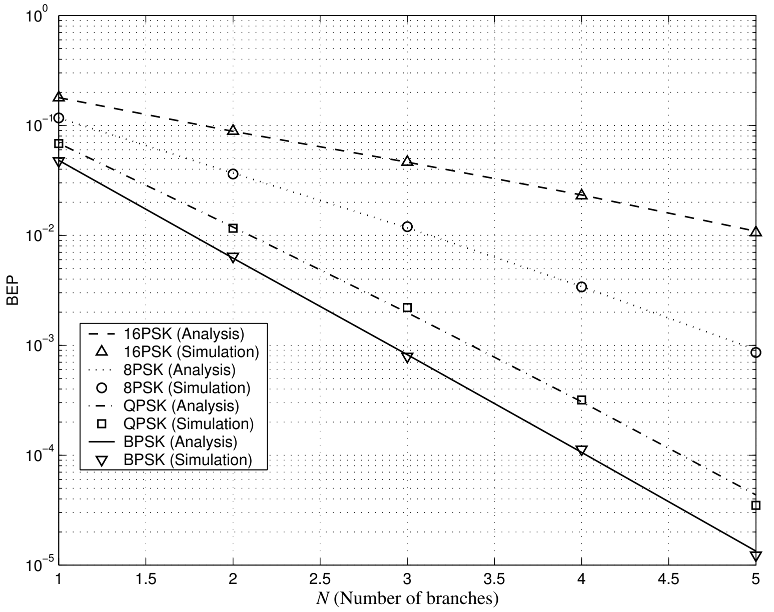

Fig. 2. BEP versus the number of receive branches N, $L = 4$ interferers, bit SNR $= 10$ dB, and SIR $= 15$ dB.

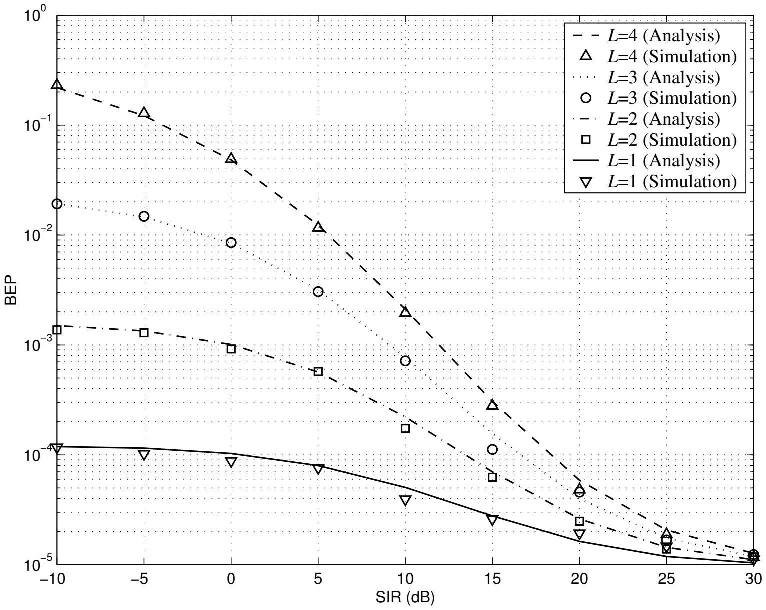

Fig. 3. BEP versus SIR for QPSK modulation, $N = 4$ branches, and SNR $= 10$ dB.