The Power Spectral Density of a Time Hopping UWB Signal: A Survey

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Abstract— **We discuss in this publication the power spectral density (PSD) of some common time hopping impulse radio signals. Both analytical expressions and simulation results for the PSD are provided. The impact of the PSD on the transmitted power is analyzed in respect to the Federal Committee on Communications regulations on UWB emissions.**

I. Introduction

Ultra-wideband (UWB) communication is a promising approach to reuse from other systems occupied frequencies. The basic principle is to maintain a very low power spectral density by spreading the necessary signal power over a wide bandwidth. This can be achieved by any modulation approach that provides processing gain, e.g. DS-CDMA. However, we focus here on impulse radio, which was the first approach proposed for UWB. As impulse radio uses very short pulses, in the ideal case the transmitted power is equally distributed over a very large bandwidth. The signal has a low duty cycle and the spectral density of such a signal is low enough to reuse frequencies used by other systems [1]. The information can be transmitted , for example, by pulse position modulation (PPM) or pulse amplitude modulation (PAM), both of which we will discuss in this publication.

Although one pulse per symbol can be sufficient to represent the information, we assume here that one pulse modulated by the same data is repeated several times with different time shifts defined by a time hopping (TH) sequence. This can be motivated by the following arguments:

- The signal would be too peaky otherwise.
- The time shifted pulses are more difficult to detect and to intercept.
- The mutual interference of two impulse radios transmitting at the same time is limited by the time hopped pulse repetitions, [2].
- The stability of the clocks in the receiver and the transmitter can limit the time between two pulses, as the synchronization has to be maintained.

The FCC has published emission regulations for the PSD of UWB devices to limit the interference with narrowband systems operating in the same spectrum [3]. Therefore, the PSD is an important part of the system design.

In this paper, general analytical expressions for different kinds of time hopping are presented. Numerical simulation results are presented to illustrate the analysis. As described in Appendix A these simulations are based on both a realistic signal model and a procedure to estimate the PSD according to the FCC recommendations for measurements. The paper is organized as follows: In Section II we discuss the PSD of PPM and PAM modulated signals with finite deterministic time hopping patterns. In Section III we cover random time hopping and in Section IV we extend the analysis to signals with timing jitter. We conclude the paper by interpreting and comparing those results.

II. Analysis of the Power Spectral Density for finite Time Hopping Sequences

The time hopping impulse radio signal here considered is a stream of narrow pulses, which are either shifted in time (PPM) or amplitude modulated (PAM) or both by the data. The same modulated pulse is repeated R times at different time shifts, which are determined by a time hopping sequence described by the vector $\vec{c} = (c_0, \ldots, c_{P-1})$. The elements c_i are positive integers smaller than q, and the time hopping sequence is repeated several times during the transmission. The transmitted signal $S(t)$ can be written as:

$$S(t) = \sum_{l=-\infty}^{\infty} \frac{1}{\sqrt{R}} \sum_{i=0}^{R-1} A_l \cdot g_{Pulse}\big(t - l \cdot T_S - B_l \cdot T_{PPM} - i \cdot T_F - c_{(l \cdot R+i)_P} \cdot T_{TH}\big), \quad (1)$$

where $(l \cdot R + i)_P \equiv (l \cdot R + i) \ mod \ P$, and $g_{Pulse}(t)$ is the basic transmitted pulse. A_l and B_l represent the data modulation, which takes place once per symbol time T_S. Thus A_l and B_l are stochastic processes. In each frame of the duration T_F, only one pulse is transmitted. The PPM modulation index is T_{PPM}, and T_{TH} is the time hopping index. When M, N is the smallest pair of integers for which $N \cdot R = M \cdot P$ is valid, the periodic repeated sequence can be represented by N subsequences. So it is possible to describe the time hopping as a filter with a deterministic, varying discrete impulse response $g_l(t)$. For convenience, the pulse excitation process $M(t)$, which includes both the time hopping and the data modulation, is introduced here. The radiated signal is then given by $S(t) = M(t) * g_{Pulse}(t)$, where the operation is convolution, and the total PSD by $\mathbf{S}_{SS}(f) = |G_{Pulse}(f)|^2 \cdot \mathbf{S}_{MM}(f)$. All definitions are summarized below:

$$M(t) = \sum_{k=-\infty}^{\infty} \sum_{l=0}^{N-1} D(t - l \cdot T_S - k \cdot N \cdot T_S) * g_l(t)$$

$$D(t - l \cdot T_S) = A_l \cdot \delta(t - l \cdot T_S - B_l \cdot T_{PPM})$$

$$g_l(t) = \frac{1}{\sqrt{R}} \sum_{i=0}^{R-1} \delta(t - c_{(l \cdot R+i)_P} \cdot T_{TH}).$$

To derive now the PSD of the excitation process $M(t)$, it must be realized that its autocorrelation function (ACF) $\varphi_{MM}(t +$

$\tau, t)$ is periodic in time with NT_S. Thus the time-average autocorrelation function has to be derived [4]:

$$\hat{\varphi}_{MM}(\tau) = \frac{1}{NT_S}\int_0^{NT_S} E\{M(t)M(t+\tau)\}dt. \quad (2)$$

The power spectral density $\mathbf{S}_{MM}(f)$ is then obtained by the Fourier transform of the time-average autocorrelation function. The authors of this publication presented a derivation of this expression in [5] and a similar expression can be found in [6], too.

$$\mathbf{S}_{MM}(f) = \frac{1}{NT_S}\cdot\Bigg(\sum_{l=0}^{N-1} G_l^*(f)G_l(f)E_{l=l'}\{A_lA_{l'}\} \quad (3)$$
$$+ E_{l\neq l'}\{A_lA_{l'}\}E_{l\neq l'}\{e^{-j2\pi fT_{PPM}(B_{l'}-B_l)}\}$$
$$\cdot\Big(\frac{1}{NT_S}\sum_{k=-\infty}^{\infty}\sum_{l=0}^{N-1}\sum_{l'=0}^{N-1}\big(G_l^*(f)G_{l'}(f)$$
$$\cdot e^{-j2\pi fT_S(l-l')}\;\delta(f-\frac{k}{NT_S})\big) - \sum_{l=0}^{N-1}G_l^*(f)G_l(f)\Big)\Bigg),$$

where $G_l(f)$ is the Fourier transform of the l-th time hopping filter $g_l(t)$. The different expectation values describe the properties of the modulation. The data is assumed to be uncorrelated, thus $E\{A_lA_{l'}\}$ and $E\{e^{-j2\pi fT_{PPM}(B_{l'}-B_l)}\}$ depend only if l equals l' or not. We now consider two special cases:

A. The PSD of a Binary PPM Signal

In a binary PPM signal, B_l is either 0 or 1, and A_l is identical 1. The PSD of a 2PPM signal can be derived by the general expression (3), if the expectation values are evaluated:

$$E_{l=l'}\{A_lA_{l'}\} = E_{l\neq l'}\{A_lA_{l'}\} = 1 \qquad \forall l, l'$$
$$E_{l\neq l'}\{e^{-j2\pi fT_{PPM}(B_{l'}-B_l)}\} = \frac{1}{2}(1+\cos(2\pi fT_{PPM})) \quad \forall l\neq l'$$

The PSD has then the following analytical expression:

$$\mathbf{S}_{\text{2PPM}}(f) = \frac{1}{NT_S}\cdot\frac{1}{2}(1-\cos(2\pi fT_{PPM}))\sum_{l=0}^{N-1}\left|G_l(f)\right|^2$$
$$+\frac{1}{(NT_S)^2}\cdot\frac{1}{2}(1+\cos(2\pi fT_{PPM}))$$
$$\cdot\sum_{k=-\infty}^{\infty}\sum_{l=0}^{N-1}\sum_{l'=0}^{N-1}\Big(G_l^*(f)\cdot G_{l'}(f)$$
$$\cdot e^{-j2\pi fT_S(l'-l)}\;\delta(f-\frac{k}{NT_S})\Big). \quad (4)$$

It contains two terms: the first, continuous, and the second, discrete (so called spectral lines). Figure 1 contains two plots obtained by a simulation as described in Appendix A. The first plot shows the PSD of such a signal and the FCC emission limits for outdoor or hand-held devices. The here presented expressions are not limited to any particular pulse shape. However, the plots have been generated by signals with the 7^{th} derivative of the Gaussian pulse and the parameter $\sigma = 64$ps. This shape is optimized in respect to the FCC regulation for outdoor devices

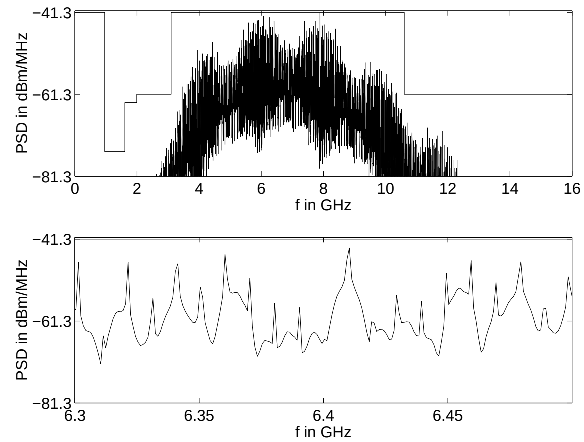

Fig. 1: PSD of a 2PPM UWB Signal

[7] and a plot of the shape can be found in Figure 6.

One is able to distinguish between the discrete part and the continuous, if an extract of the signal bandwidth is considered as the second plot illustrates. The spectral lines appear at distances of $\frac{1}{NT_S}$. As the FCC requires a measurement resolution of 1MHz, [3], the discrete part of the PSD (represented by Dirac pulses) has a finite ordinate value instead of infinity. Obviously, the average power must be decreased to prevent violations of the regulations by the discrete part. Table I in section V illustrates this convincingly.

B. The PSD of a Binary PAM Signal

In an antipodal 2PAM signal, A_l is either -1 or 1 and, B_l is identical 0. The expectation values for 2PAM modulated pulses are:

$$E_{l=l'}\{A_lA_{l'}\} = 1 \qquad \forall l = l'$$
$$E_{l\neq l'}\{A_lA_{l'}\} = 0 \qquad \forall l \neq l'$$
$$E_{l\neq l'}\{e^{-j2\pi fT_{PPM}(B_{l'}-B_l)}\} = 1 \qquad \forall l, l'$$

Inserting them in the general expression (3) yields the PSD of a PAM modulated UWB signal:

$$\mathbf{S}_{\text{2PAM}}(f) = \frac{1}{NT_S}\cdot\sum_{l=0}^{N-1}\left|G_l(f)\right|^2. \quad (5)$$

The PSD of a 2PAM signal is continuous, because the autocorrelation function equals 0 for $|\tau| > T_S$. But the PSD is still influenced by the time hopping sequence, as Figure 2 illustrates. Comparing the second plot of this figure with the one for binary PPM, illustrates that the continuous part of the PSD is basically the same for both modulations, except that in the PPM scheme it is multiplied by $\frac{1}{2}(1-\cos(2\pi fT_{PPM}))$ and additionally attenuated to prevent violations of the FCC regulations.

It has to be emphasized here, that this analysis is based on the repetition of the same data modulated pulse. As the time hopped repetition causes fluctuations of transmission power from some narrow frequency bands to other frequency bands, the total transmitted power has to be reduced to not violate the FCC regulations.

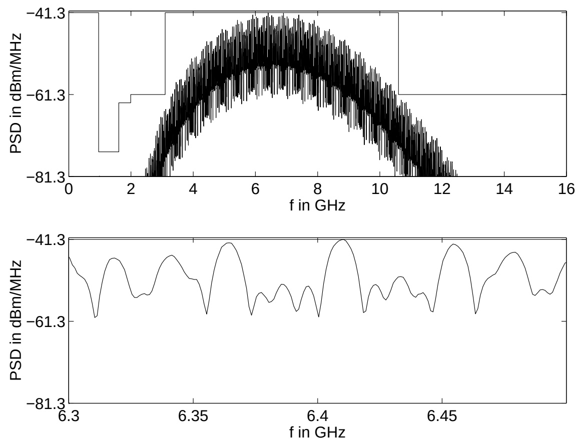

Fig. 2: PSD of a 2PAM UWB Signal

III. Analysis of the Power Spectral Density of Time Hopping Sequences with infinite length

In the preceeding section we have realized that for PPM the spectral lines of the discrete part appear at distances of $\frac{1}{NT_S}$. Any kind of spectral analyzer has only a finite resolution in the frequency domain; for example the FCC recommends a resolution of 1 MHz. It is appealing to try to reduce the effect of the time hopped repetitions by increasing the sequence length P, because the spectral lines in the PSD of a pure PPM signal are then separated by a decreasing distance. If this distance becomes smaller than the resolution, one would be unable to distinguish between the spectral lines and the continuous part. Furthermore, the continuous part of the spectrum is the average over an increasing amount of different frequency responses $G_l(f)$, so this part could get close to a flat frequency response. We note, that the time hopping sequence has to be known by the receiver. Thus, the desired receiver has to acquire the sequence at the beginning of the transmission. It is important to simplify the acquisition process. That limits the length of the time hopping sequence.

But, more important is the fact, that even for very long sequences, the time hopping still influences the PSD in a dominant way. To illustrate this, sequences of infinite length are considered in the following. Thus the time hopping C_i in the i-th frame of a symbol is treated as a random variable. So the filter excitation signal $M(t)$ can be written as:

$$\begin{aligned} M(t) &= \sum_{l=-\infty}^{\infty} \frac{1}{\sqrt{R}} \sum_{k=0}^{R-1} A_l \cdot \delta(t - l \cdot T_S - B_l \cdot T_{PPM} \\ &\quad - k \cdot T_F - C_{lR+k} \cdot T_{TH}). \end{aligned} \tag{6}$$

In contrast to equation (1), $C_i \in \{0, \ldots, q-1\}$ is not an element of a determined sequence, but a realization of a stochastic process.

The autocorrelation function of the process $\varphi_{MM}(t+\tau, t)$ is now periodic with T_S. The derivation of the PSD of this excitation process is provided in [5]. Alternatively, the expression can be obtained by setting the jitter I of the PSD–expression in the next Section to zero. The derivation of this expression is provided in Appendix B of this article.

$$\begin{aligned} &\mathbf{S}_{MM}(f) = \\ &\frac{1}{T_S} E_{l=l'}\{A_l A_{l'}\}\Big(1 + \sum_{l=-\infty}^{\infty} \mathrm{sinc}^2(\pi q T_{TH}(f - \frac{l}{T_{TH}})) \\ &\cdot \big(R \sum_{k=-\infty}^{\infty} \mathrm{sinc}^2(\pi R T_F(f - \frac{k}{T_F})) \ - \ 1\big)\Big) \\ &+ \frac{R}{T_S} E_{l\neq l'}\{A_l A_{l'}\} E_{l\neq l'}\{e^{-j2\pi f(B_{l'}-B_l)T_{PPM}}\} \\ &\cdot \sum_{l=-\infty}^{\infty} \mathrm{sinc}^2(\pi q T_{TH}(f - \frac{l}{T_{TH}})) \sum_{k=-\infty}^{\infty} \mathrm{sinc}^2(\pi R T_F(f - \frac{k}{T_F})) \\ &\cdot \big(\frac{1}{T_S} \sum_{j=-\infty}^{\infty} \delta(f - \frac{j}{T_S}) - 1\big). \end{aligned} \tag{7}$$

The sinc function, $\mathrm{sinc}(x) = \frac{\sin(x)}{x}$, is used in this and the following expressions. The general PSD has still continuous and discrete parts. The time hopping index, the frame duration and the symbol time are parameters of this PSD. In particular, we find spectral lines at distances of $\frac{1}{T_S}$, and undesired concentrations of transmission power at distances of $\frac{1}{T_F}$ and $\frac{1}{T_{TH}}$. Moreover, those concentrations in some narrow bands are linear with the number of repetitions R. In fact the subbands get narrower but higher, with an increasing number of repetitions.

A. PSD of a PPM Signal with Random Time Hopping

By substituting the expectation values in the general PSD (7) in the same way as for the finite sequence in Section II, the PSD of binary PPM with infinite time hopping is given by:

$$\begin{aligned} &\mathbf{S}_{2PPM}(f) = \\ &\frac{1}{T_S} \cdot \Bigg(1 + \sum_{l=-\infty}^{\infty} \mathrm{sinc}^2(q\pi T_{TH}(f - \frac{l}{T_{TH}})) \\ &\cdot \Big(\frac{1}{2}(1-\cos(2\pi T_{PPM} f)) \cdot R \cdot \sum_{k=-\infty}^{\infty} \mathrm{sinc}^2(R\pi T_F(f - \frac{k}{T_F})) - 1\Big) \\ &+ \frac{1}{2}(1+\cos(2\pi T_{PPM} f)) \cdot \frac{R}{T_S} \cdot \sum_{j=-\infty}^{\infty} \delta(f - \frac{j}{T_S}) \\ &\cdot \sum_{l=-\infty}^{\infty} \mathrm{sinc}^2(q\pi T_{TH}(f - \frac{l}{T_{TH}})) \cdot \sum_{k=-\infty}^{\infty} \mathrm{sinc}^2(R\pi T_F(f - \frac{k}{T_F}))\Bigg). \end{aligned} \tag{8}$$

The PSD still contains a discrete part. Figure 3 is again a result of the simulation model described in Appendix A. Note that this results are also valid if the time hopping is deterministic, but the sequence length longer than the observation interval used to derive the spectral expectation. The distances between the discrete lines equals in this particular case $\frac{1}{T_F}$, as the symbol time T_S is a multiple of the frame duration T_F. Sinc functions at multiples of $\frac{1}{T_F}$ and $\frac{1}{T_{TH}}$ are clearly noticeable in the plot covering the complete spectrum and the one covering the extract.

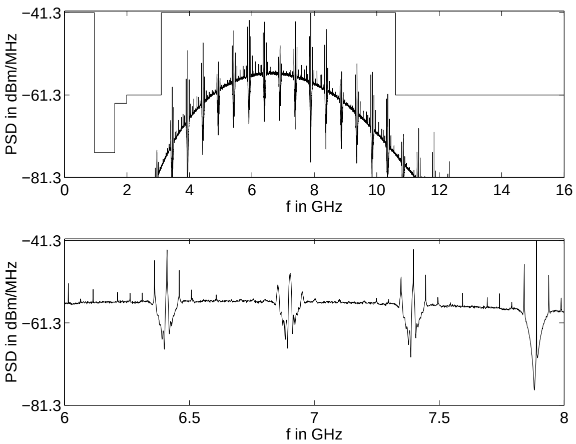

Fig. 3: PSD of a 2PPM Signal with random Time Hopping

B. PSD of a PAM Signal with Random Time Hopping

The PSD of a 2PAM signal can be derived similarly to the PPM case. The analytical expression is:

$$\mathbf{S}_{2PAM}(f) = \frac{1}{T_S} \cdot \left(1 + \sum_{l=-\infty}^{\infty} \operatorname{sinc}^2(q\pi T_{TH}(f - \frac{l}{T_{TH}})) \cdot \left(R \sum_{m=-\infty}^{\infty} \operatorname{sinc}^2(R\pi T_F(f - \frac{m}{T_F})) - 1\right)\right). \quad (9)$$

The PSD is continuous but has still significant concentrations of power at multiples of $\frac{1}{T_{TH}}$, which are linear in the number of repetitions as mentioned before. Figure 4 illustrates this.

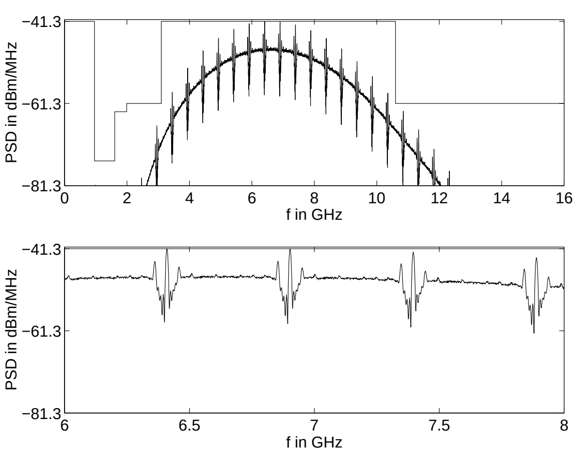

Fig. 4: PSD of a 2PAM Signal with random Time Hopping

In conclusion, we find that increasing the length of the TH–sequence is not a sufficient method for achieving a "smooth" PSD. Table I especially illustrates that the length of the sequence is of limited impact with respect to available transmission power when the FCC–regulations have to be met. That is in contrast to the conclusions of [8].

The PSD still has undesired concentrations of transmission power, which are due to the sinc–functions, at distances of $\frac{1}{T_F}$ and $\frac{1}{T_{TH}}$. To avoid this effect, one could choose those parameters in such a manner that those concentrations are positioned at frequencies where the frequency response of the considered pulse are close to zero. However, as the bandwidth of the pulse is roughly given by $\frac{1}{T_{Pulse}}$, with T_{Pulse} describing the pulse duration, the time hopping index T_{TH} has then to be smaller than the pulse duration T_{Pulse}. This should be difficult to implement, in particular for UWB approaches.

IV. The PSD of Time Hopping Signals with Infinite Sequence Length and Timing Jitter

In this section we want to extend our discussion of the PSD of TH impulse radio to include timing jitter. We consider here for each pulse independent timing jitter, which is uniformly distributed over the interval $[-\frac{I}{2}, \frac{I}{2}]$. Therefore, the excitation process is described by the following expression:

$$M(t) = \sum_{l=-\infty}^{\infty} \frac{1}{\sqrt{R}} \sum_{k=0}^{R-1} A_l \cdot \delta(t - l \cdot T_S - B_l \cdot T_{PPM} - k \cdot T_F - C_k \cdot T_{TH} - \Delta_k). \quad (10)$$

Here, Δ_k represents the timing jitter process. In [9] is stated that such a timing jitter would help to "smooth" out the PSD. To discuss this we consider the PSD of this signal. The derivation of the appropriate analytical expression can be found in Appendix B.

$$\begin{aligned} \mathbf{S}_{MM}(f) = {} & \frac{1}{T_S} E_{l=l'}\{A_l A_{l'}\} \Big(1 + \operatorname{sinc}^2(\pi f I) \quad (11) \\ & \cdot \sum_{l=-\infty}^{\infty} \operatorname{sinc}^2(qT_{TH}\pi(f - \frac{l}{T_{TH}})) \\ & \cdot \Big(R \sum_{k=-\infty}^{\infty} \operatorname{sinc}^2(RT_F\pi(f - \frac{k}{T_F})) - 1\Big)\Big) \\ & + \frac{R}{T_S} E_{l\neq l'}\{A_l A_{l'}\} E_{l\neq l'}\{e^{-j2\pi(B_{l'}-B_l)T_{PPM}}\} \operatorname{sinc}^2(\pi f I) \\ & \cdot \sum_{l=-\infty}^{\infty} \operatorname{sinc}^2(qT_{TH}\pi(f - \frac{l}{T_{TH}})) \sum_{k=-\infty}^{\infty} \operatorname{sinc}^2(RT_F\pi(f - \frac{k}{T_F})) \\ & \cdot \Big(\frac{1}{T_S} \sum_{j=-\infty}^{\infty} \delta(f - \frac{j}{T_S}) - 1\Big) \end{aligned}$$

Expression (11) is similar to the one without jitter (7). The jitter introduces a multiplication of the undesired power concentrations and the spectral lines with another sinc function. The term $\operatorname{sinc}^2(\pi f I)$ concentrates those troubling parts of the spectrum at low frequencies. As the expressions are similar to the ones of section III, apart of the additional sinc function, and for conciseness, we do not consider the expression for PAM or PPM in detail, and continue directly with discussing the results.

The upper plot of Figure 5 shows the PSD of a PPM signal, and the lower one of a PAM signal with timing jitter. In both cases the interval of the timing jitter equals 92 ps. Comparing those plots with the ones of Figure 3 and 4 we realize that the

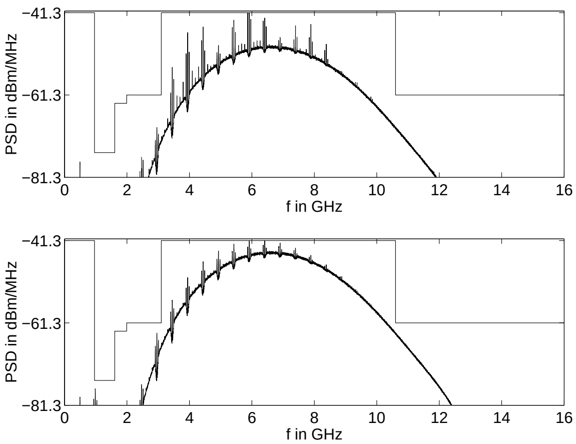

Fig. 5: PSD of a PPM Signal and a PAM Signal with random Time Hopping and independent Timing Jitter

$\mathrm{sinc}^2(\pi f I)$ makes the PSD "smooth" at high frequencies. In this sense the jitter helps achieving a good PSD. An increasing timing jitter concentrates the spectral lines etc. more and more at low frequencies, where the frequency response of the pulse vanishes, anyway.

However, as the considered jitter is independent uniformly distributed, it increases the BER performance due to clocking errors between the transmitted pulses and the receiver. Figure 6 shows the considered pulse shape (upper plot) and its autocorrelation function. As we consider real signals, the ACF of the

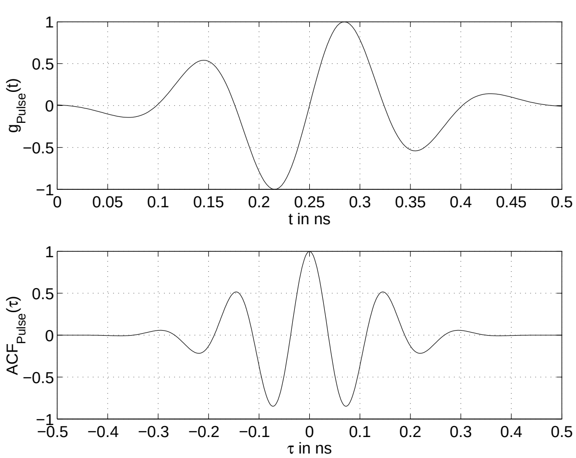

Fig. 6: Considered Pulse shape and its Autocorrelation Function

pulse shape is identical with the output of a matched filter at the receiver with the timing error τ. Therefore, the ACF plot enables us to estimate the effect of the timing error on the BER performance. We can infer from this plot that a timing error of say ± 50 ps will have a serious impact on the BER. Thus, the jitter interval of 92 ps considered in Figure 5 will significantly increase the BER. A larger jitter interval, which would lead to a PSD without noticeable undesired effects, would increase the BER even more or even make data transmission impossible. The same observation can be made for TH impulse radio with finite TH–sequences. Therefore, the idea of time jitter improving the spectrum has to be considered with care.

V. Conclusion

Scheme		total power in dBm
PAM	finite, $P = 5$	-13.08
	infinite	-13.16
	infinite, jitter	-9.28
PPM	finite, $P = 5$	-21.01
	infinite	-20.99
	infinite, jitter	-14.60
optimal case		-6.81

Table I: List of total available Transmission Power for the Signals under consideration

Table I lists the total available transmission power of the TH–signals under the constraint of meeting the FCC regulations. The "optimal case" is the situation in which the PSD is determined just by the Fourier transform of the single pulse. That is true for example for binary PAM and one pulse per symbol, $R = 1$. Thus we can infer from the table above that the time hopped repetitions make necessary to attenuate the transmitted power. Let us note that we based our discussion on a particular set of parameters, which can be found in Appendix A, and a time hopping scheme as described for example in [1], [2], [6], [8]. We have presented analytical expressions for impulse radio with finite TH–sequences and infinite TH–sequences with and without timing jitter. Further, those expressions have been illustrated with PSD–plots based on realistic simulations.

We have demonstrated that increasing the length of the TH–sequence without choosing the other parameters in a particularly advantageous way does not yield a better PSD. Moreover, we have discussed that a timing jitter, which helps to "smooth" the PSD, increases the BER of the impulse radio link. However, the work presented here should allow to discuss the spectral properties of different impulse radios in the UWB context.

Appendix

A. Simulation Model and Parameters of the Signal

The PSD plots presented here are all obtained using the Bartlett periodogram method, [10]. The resolution in the frequency domain is 1MHz and the number of the single FFTs that are used for the spectrum estimation is chosen such that the duration of the total considered signal is 1ms (corresponding to a video bandwidth of roughly 1kHz). Therefore the parameters are chosen to yield spectral estimations realistic with respect to the FCC recommendations. The parameters of the signals considered in the plots are listed in the Table II.

Parameter	Value
PPM-Modulation index, T_{PPM}	0.504 ns
TH-index, T_{TH}	2.03 ns
Frame duration, T_F	20.28 ns
Symbol time, T_S	101.29 ns
Hopping alphabet size, q	5
Number of repetitions, R	5
Timing jitter interval, I	0.92 ns

Table II: List of the Parameters of the Simulated Signals

B. PSD of a TH Signal with Infinite Sequence Length and Timing Jitter

Here we want to derive the PSD of the modulation process of a time hopping signal with infinite sequence length and independent uniformly distributed timing jitter. Let us start with the ACF of this process:

$$\varphi_{MM}(t+\tau,t) = \tag{12}$$

$$E\Big\{\frac{1}{R}\sum_{l=-\infty}^{\infty}A_l\sum_{k=0}^{R-1}\delta(t-lT_S-B_lT_{PPM}-kT_F-C_kT_{TH}-\Delta_k)$$

$$\cdot\sum_{l'=-\infty}^{\infty}A_{l'}\sum_{k'=0}^{R-1}\delta(t+\tau-l'T_S-B_{l'}T_{PPM}-k'T_F-C_{k'}T_{TH}-\Delta_{k'})\Big\}$$

Now we realize that this ACF is periodic with T_S. Thus, we have to evaluate the time–average autocorrelation function by averaging over $[0,T_S]$. Therefore, we have to take only $l=0$ into account:

$$\hat{\varphi}_{MM}(\tau) = \frac{1}{T_S}\int_0^{T_S}\varphi_{MM}(t+\tau,t)dt \tag{13}$$

$$= E\Big\{\frac{1}{R\cdot T_S}\sum_{l'=-\infty}^{\infty}A_lA_{l'}\sum_{k=0}^{R-1}\sum_{k'=0}^{R-1}\delta(\tau-l'T_S$$

$$-(B_{l'}-B_l)T_{PPM}-(C_{k'}-C_k)T_{TH}-(\Delta_{k'}-\Delta_k))\}$$

Let us note that here A_l, B_l, C_k and Δ_k are stochastic processes. Now, we distinguish the parts of the sums in which $l=l'=0$ and $k=k'$ from the ones in which $l'\neq l$ or $k'\neq k$. Furthermore, we explicitly evaluate the expectations over C_k and Δ_k taking into account that they are independent and uniformly distributed for $k\neq k'$ or $l\neq l'$:

$$\hat{\varphi}_{MM}(\tau) = \frac{1}{T_S}E_{l=l'}\{A_lA_{l'}\}\Bigg(\delta(\tau) \tag{14}$$

$$+\frac{1}{Rq^2I^2}\sum_{k=0}^{R-1}\sum_{k'=0}^{R-1}\sum_{c_k=0}^{q-1}\sum_{c_{k'}=0}^{q-1}\int_{-\frac{I}{2}}^{\frac{I}{2}}\int_{-\frac{I}{2}}^{\frac{I}{2}}E_{l=l'}\Big\{\delta\big(\tau-(B_{l'}-B_l)T_{PPM}$$

$$-(k'-k)T_F-(c_{k'}-c_k)T_{TH}-(\Delta_{k'}-\Delta_k)\big)\Big\}d\Delta_{k'}d\Delta_k$$

$$-\frac{1}{q^2I^2}\sum_{c_k=0}^{q-1}\sum_{c_{k'}=0}^{q-1}\int_{-\frac{I}{2}}^{\frac{I}{2}}\int_{-\frac{I}{2}}^{\frac{I}{2}}E_{l=l'}\Big\{\delta\big(\tau-(B_{l'}-B_l)T_{PPM}$$

$$-(c_{k'}-c_k)T_{TH}-(\Delta_{k'}-\Delta_k)\big)\Big\}d\Delta_{k'}d\Delta_k\Bigg)$$

$$+\frac{1}{T_SRI^2q^2}E_{l\neq l'}\{A_lA_{l'}\}\Bigg(\sum_{l'=-\infty}^{\infty}\sum_{k=0}^{R-1}\sum_{k'=0}^{R-1}\sum_{c_k=0}^{q-1}\sum_{c_{k'}=0}^{q-1}\int_{-\frac{I}{2}}^{\frac{I}{2}}\int_{-\frac{I}{2}}^{\frac{I}{2}}$$

$$E_{l=l'}\Big\{\delta\big(\tau-l'T_S-(B_{l'}-B_l)T_{PPM}-(k'-k)T_F-(c_{k'}-c_k)T_{TH}$$

$$-(\Delta_{k'}-\Delta_k)\big)\Big\}d\Delta_{k'}d\Delta_k$$

$$-\sum_{k=0}^{R-1}\sum_{k'=0}^{R-1}\sum_{c_k=0}^{q-1}\sum_{c_{k'}=0}^{q-1}\int_{-\frac{I}{2}}^{\frac{I}{2}}\int_{-\frac{I}{2}}^{\frac{I}{2}}E_{l=l'}\Big\{\delta\big(\tau-(B_{l'}-B_l)T_{PPM}$$

$$-(k'-k)T_F-(c_{k'}-c_k)T_{TH}-(\Delta_{k'}-\Delta_k)\big)\Big\}d\Delta_{k'}d\Delta_k\Bigg)$$

The Fourier transform of the expression (14) is the PSD of the considered signal.

$$\mathbf{S}_{MM}(f) = \frac{1}{T_S}E_{l=l'}\{A_lA_{l'}\}\Bigg(1+\mathrm{sinc}^2(\pi fI) \tag{15}$$

$$\cdot\sum_{l=-\infty}^{\infty}\mathrm{sinc}^2(qT_{TH}\pi(f-\frac{l}{T_{TH}}))\cdot\Big(R\sum_{k=-\infty}^{\infty}\mathrm{sinc}^2(RT_F\pi(f-\frac{k}{T_F}))-1\Big)\Bigg)$$

$$+\frac{R}{T_S}E_{l\neq l'}\{A_lA_{l'}\}E_{l\neq l'}\{e^{-j2\pi(B_{l'}-B_l)T_{PPM}}\}\mathrm{sinc}^2(\pi fI)$$

$$\cdot\sum_{l=-\infty}^{\infty}\mathrm{sinc}^2(qT_{TH}\pi(f-\frac{l}{T_{TH}}))\sum_{k=-\infty}^{\infty}\mathrm{sinc}^2(RT_F\pi(f-\frac{k}{T_F}))$$

$$\cdot\big(\frac{1}{T_S}\sum_{j=-\infty}^{\infty}\delta(f-\frac{j}{T_S})-1\big)$$

To evaluate the Fourier transform of the expression (14) the following relations have been used:

$$\mathcal{F}\{\sum_{l=-\infty}^{\infty}\delta(t-lT)\} = \frac{1}{T}\sum_{l=-\infty}^{\infty}\delta(f-\frac{l}{T})$$

$$\mathcal{F}\{\frac{1}{q^2}\sum_{k=0}^{R-1}\sum_{k'=0}^{R-1}\delta(t-(k'-k)T)\} = \sum_{k=-\infty}^{\infty}\mathrm{sinc}^2(RT\pi(f-\frac{k}{T}))$$

$$\mathcal{F}\{\frac{1}{I^2}\int_{-\frac{I}{2}}^{\frac{I}{2}}\int_{\frac{I}{2}}^{\frac{I}{2}}\delta(t-(\Delta_{k'}-\Delta_k))d\Delta_{k'}d\Delta_{k'}\} = \mathrm{sinc}^2(I\pi f)$$

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