# Link-Failure Probabilities for Practical Cooperative Relay Networks

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***Abstract*— In this paper, practical cooperative diversity schemes are proposed and investigated. The outage probability is used to evaluate four constant-power decode-and-forward cooperative schemes: Pre-Select One Relay, Best-Select Relay, Simple Relay, and ST-Coded Relay. Two new methods for improving these approaches, called simple distributed power allocation and $m$-Group ST-Coded Relay, are proposed. It is shown that the performance of Simple Relay and ST-Coded Relay is improved significantly by employing the proposed power allocation without an increase in implementation complexity. Similarly, $m$-Group ST-Coded Relay gains in lower complexity with only a slight degradation in performance. Performance of the various schemes in a network is evaluated through the link-failure probability.**

***Index Terms*— Cooperative diversity, decode-and-forward, link-failure probability, outage probability, relay networks**

## I. Introduction

Cooperative diversity is the name given to a set of techniques that exploit the potential of spatially dispersed user antennas to improve communications reliability. Several low-complexity two-stage relay strategies are proposed and studied in [1], including fixed relaying, selective relaying, and incremental relaying. Each of these relaying approaches can employ amplify-and-forward, where relay nodes amplify the received signals, or decode-and-forward, where relay nodes decode, re-encode, and re-transmit the messages. In [2], these algorithms are extended to large networks, and both repetition-based and space-time-coded cooperation are considered.

In this paper, we first investigate the outage probability performance of four selective decode-and-forward cooperative schemes: *Pre-Select One Relay*, *Best-Select Relay*, *Simple Relay*, and *ST-Coded Relay*. These schemes differ in the methods they employ for relay node selection and message processing. For Pre-Select One Relay and Best-Select Relay, only one node is selected as the relay node. However, in the first case the relay node is updated at the rate of slow fading, while it is updated at the rate of fast fading in the latter case. For Simple Relay and ST-Coded Relay, all decoded nodes are relay nodes and transmit simultaneously. However, in the first case signals are combined with uncontrolled (random) phases at the destination, while space-time codes are employed in the latter case. The diversity order of ST-Coded Relay is equal to the size of network [2], while the diversity order of the other schemes is two. Diversity order is defined as the exponent of the signal-to-noise ratio (SNR) term in the expression of the outage probability as $\mathrm{SNR} \to \infty$. Graphically, the diversity order can be evaluated from the slope of the outage probability versus SNR in logarithmic scale for sufficiently high SNR. In a diversity system, the diversity order and the coding gain are two important performance criteria. However, most existing research on cooperative diversity has focused on the diversity order [1], [2], while coding gain has not received much attention. By examining the performance of these four schemes, we observe that:

1) The slope of the outage probability of ST-Coded Relay is often much less than its diversity order in the range of outage probabilities of practical interest.
2) Although the diversity order of Best-Select Relay is much less than that of ST-Coded Relay, Best-Select Relay outperforms ST-Coded Relay across a range of outage probabilities.

These two observations suggest that focusing only on the diversity order is not sufficient in the study of cooperative diversity.

Studying the performance of these four schemes also motivates us to propose a simple, distributed power allocation as well as a new cooperative diversity scheme referred to as $m$-Group *ST-Coded Relay*. The simple, distributed power allocation is employed in Simple Relay and ST-Coded Relay, where each relay node reduces its power to a fraction of the source power. It is shown that the performance of Simple Relay and ST-Coded Relay are improved significantly, while the complexity is the same as that for the original constant-power schemes. In $m$-group ST-Coded Relay, relay nodes are divided into $m$ groups. Within each group Simple Relay is employed, while space-time codes are employed among groups. This scheme provides a tradeoff between performance and complexity.

## II. Basic Decode-and-Forward Schemes

In this section, we introduce four basic selective decode-and-forward cooperative schemes [2]. Consider a network with $N$ nodes. For a given source-destination $(s, d)$ pair, the remaining $N-2$ nodes can serve as potential relay nodes. Let $h_{i,j}$ denote the instantaneous channel gain between node $i$ and node $j$ with mean $g_{i,j}$. The channel gains are modeled as independently distributed exponential random variables. It is assumed that all nodes transmit with the same power, denoted as $p$. The noise power is normalized to be unity.

Similar to [2], selective decode-and-forward schemes consist of a two-stage transmission:

1) Stage one: source $s$ transmits, all other nodes including the destination $d$ and $N-2$ potential relay nodes listen. After receiving the signal, each node will decode the message. A node is called a *decoded node*, if its received SNR is above a given decoding threshold $\eta$.
2) Stage two: a relay set is determined from the decoded set. Various selection criteria result in various schemes as described later in this section. All nodes within the relay set will relay the decoded message to the destination node. Additional processing or coding may be added in each relay node before transmission depending on the various schemes. The destination node combines the received signal from two stages. If the resulting SNR at the destination after processing is above a threshold, the message is decoded correctly; otherwise, an outage is claimed.

More specifically, in the following sections, we investigate four basic decode-and-forward approaches, each using a different method for selecting relay nodes and for processing messages.

## A. Pre-Select One Relay

In Pre-Select One Relay, the potential relay node is only limited to one pre-specified node for a given $(s, d)$. The pre-specified node is selected based on the average (local mean) SNR, and updated at the rate of the slow fading. Once the potential relay node is specified, in each message delivery, two-stage selective decode-and-forward is employed.

The optimum pre-select relay node is the node that minimizes the outage probability. Let $r$ denote the pre-selected relay node. The outage probability is given by

$$\begin{aligned} P_o &= \Pr\{ph_{s,r} < \eta\}\Pr\{ph_{s,d} < \eta\} \\ &\quad + \Pr\{ph_{s,r} \geq \eta\}\Pr\{ph_{s,d} + ph_{r,d} < \eta\} \\ &= \left(1 - \exp\left(-\frac{\eta}{pg_{s,r}}\right)\right)\left(1 - \exp\left(-\frac{\eta}{pg_{s,d}}\right)\right) \\ &\quad + \exp\left(-\frac{\eta}{pg_{s,r}}\right)\left[1 - \exp\left(-\frac{\eta}{pg_{r,d}}\right)\frac{1}{1 - g_{s,d}/g_{r,d}}\right. \\ &\quad \left. - \exp\left(-\frac{\eta}{pg_{s,d}}\right)\frac{1}{1 - g_{r,d}/g_{s,d}}\right]. \end{aligned} \tag{1}$$

The optimum pre-select relay node is determined by evaluating the outage probabilities for all possible relay nodes. In the following, we show that the optimization criteria can be greatly simplified under high SNR conditions. When $pg_{s,r} \gg \eta$, $pg_{s,d} \gg \eta$, and $pg_{r,d} \gg \eta$, we have the following approximations

$$\Pr\{ph_{s,r} < \eta\} \approx \frac{\eta}{pg_{s,r}} \tag{2}$$

$$\Pr\{ph_{s,r} \geq \eta\} \approx 1 \tag{3}$$

$$\Pr\{ph_{s,d} < \eta\} \approx \frac{\eta}{pg_{s,d}} \tag{4}$$

$$\Pr\{ph_{s,d} + ph_{r,d} < \eta\} \approx \frac{\eta^2}{2p^2 g_{s,d} g_{r,d}} \tag{5}$$

It follows that $P_o \approx \frac{\eta^2}{p^2 g_{s,d}}\left(\frac{1}{g_{s,r}} + \frac{1}{2g_{r,d}}\right)$. Therefore, under high SNR conditions, a near-optimum pre-select relay node $r'$ is

$$r' = \arg\min_i \frac{1}{g_{s,i}} + \frac{1}{2g_{i,d}}\,. \tag{6}$$

## B. Best-Select Relay

In Best-Select Relay, the relay node is the decoded node with the highest *mean channel gain*[1] to the destination. Since the relay node is chosen from the decoded set, the relay node in Best-Select Relay is updated with the decoded set, which changes at the rate of the fast fading.

To implement Best-Select Relay, we propose the following centralized selection scheme: (1) For each transmission, each node in the decoded set feeds back, to the source node, its average SNR to the destination. (2) The source selects the relay node with the highest average SNR and broadcasts to all the nodes in the decoded set. (3) The chosen node relays the information to the destination. The main chanllenges with Best-Select Relay are the delay and overhead involved with relay selection.

## C. Simple Relay

In this scheme, all decoded nodes serve as relay nodes. They transmit the same information simultaneously, and the signals are combined with uncontrolled (and thus random) phases at the destination node. Two scenarios are considered, as follows: (1) The *s*ynchronous case, wherein the relative delays among the multiple relay nodes is much smaller than the symbol duration. In this case, the composite signal is Rayleigh-faded, with power equal to the sum of the powers of the component signals. The multiple relay nodes can be viewed as a super node with an effective channel gain. (2) The *n*on-synchronous case, wherein the relative delays among the multiple relay nodes are comparable to the symbol duration. In this case, an adaptive equalizer should be employed at the destination. A higher order of diversity can be achieved than that for the synchronous case. In this paper, we focus only on the synchronous case.

## D. ST-Coded Relay

Similar to Simple Relay, in ST-Coded Relay, all decoded nodes serve as relay nodes. However, all relay nodes transmit using a space-time code, so that the receiver output SNR is the SNR sum over all the relay nodes. Here again, we have synchronous and non-synchronous cases. The main challenge for the former is the overhead and complexity of the receiver. Since, for each transmission, only nodes in the decoded set relay the information, the receiver must know the decoded

[1]The mean channel gains instead of the instantaneous channel gains are used as the selection criterion, since in a real system it is usually assumed that only the mean channel gains are available at the transmitter.

set and the instantaneous channel gains (to the destination) of every node in that set [2]. For the non-synchronous case, in addition, new space-time codes must be designed.

### *E. Outage Performance*

In this section, we present the outage probability performance of these basic schemes. These results motivate the new approaches in Section III.

We examine a wireless network distributed in a grid with $4$ by $4$ nodes shown in Fig. 1. Each node is labeled from $0$ to $15$. Since outage probability performance depends on the source-destination pair, we examine two extreme cases:

1) Worst case: source$=0$, destination$=N-1$. In this case, the source and destination are far apart, and there are many potential relay nodes in between.
2) Best case: source$=5$, destination$=6$. In this case, the source and destination are the closest nodes, and there are potential relay nodes around them.

It is assumed that the instantaneous SNR is an exponentially distributed random variable with its mean determined by the distance power law with exponent equal to $4$. It is straightforward to extend the results to the case with shadow fading and other fading distributions.

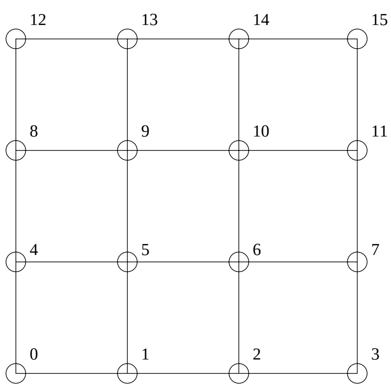


Fig. 1. Grid of 4 by 4 nodes

In Fig. 2 and 3, the outage probability versus average power performance of the basic schemes are plotted for the worst and best source-destination pair, respectively. The average power here is the average total consumed power over fading states for delivering one message. For example, for ST-Coded Relay and Simple Relay, at the second stage, all decoded nodes relay the message to the destination using the same transmission power as the source node. Thus, in these cases, the total consumed power is equal to the transmission power times the size of decoded set plus one. The total consumed power is then averaged over all fading states.

In Fig. 2 and 3, *Pre-Select One-Opt* refers to the optimum Pre-Select One Relay, while *Pre-Select One-Near* refers to the near optimum Pre-Select One Relay using the simplified criterion (6). As we can see, the performance of the near optimum Pre-Select One Relay is almost as good as the optimum Pre-Select One Relay.

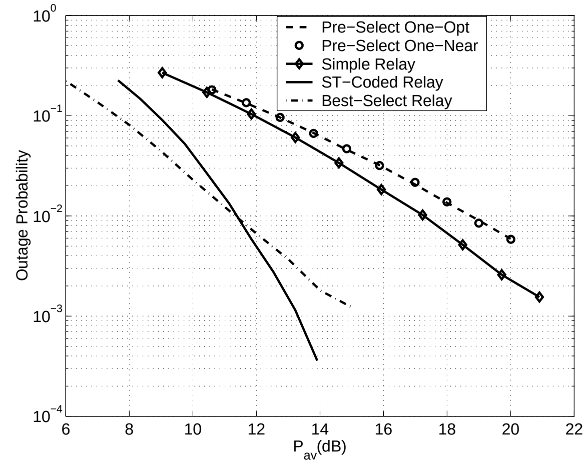


Fig. 2. The outage probability of the worst pair $(s=0, d=N-1)$ versus the average power for the basic schemes.

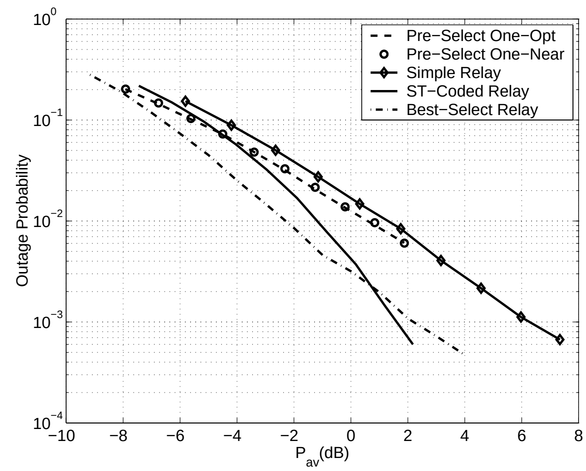


Fig. 3. The outage probability of the best pair $(s=5, d=6)$ versus the average power for the basic schemes.

The performance of Simple Relay is similar to the performance of the optimum Pre-Select One Relay. Since the optimum relay node in Pre-Select One Relay must be updated whenever the configuration of the network changes while the relay selection in Simple Relay is very simple, the Simple Relay scheme is more attractive than the Pre-Select One Relay.

It is can be seen that the slopes of the outage probabilities of Pre-Select One Relay, Simple Relay, and Best-Select Relay are similar, which confirms our conjecture that the diversity order of these three schemes is two. The slope of the outage probability of ST-Coded Relay is larger than the other three schemes. However, it is surprising to see that

1) The slope of the outage probability of ST-Coded Relay is much less than the size of the network $N=16$. As shown in [2], the diversity order of ST-Coded Relay is equal to the size of network. Since the diversity order is the slope of the outage probability as SNR goes to infinity, this observation shows that in the range of practical interests (outage probability from $10^{-1}$ to $10^{-4}$) the corresponding SNR is not high enough to

approach the diversity order. This is especially true in a cooperative diversity system where the relay-destination mean channel gains vary significantly.

2) Though the diversity order of Best-Select Relay is only two, it performs better than ST-Coded Relay for the outage probability between $10^{-1}$ and $10^{-2}$ in Fig. 2 and between $10^{-1}$ and $10^{-3}$ in Fig. 3. In Best-Select Relay, the relay node has the largest mean channel gain in the decoded set; while in ST-Coded Relay, all decoded nodes are selected and thus power is wasted in those nodes with small mean channel gains. We believe that the role of the mean channel gains of relay nodes is analogous to the role of coding gain of the space-time codes in a block fading channel. The proper definition of the coding gain in a cooperative diversity system is still under investigation.

## III. New Approaches

In this section, we propose new decode-and-forward cooperative approaches to improve the performance and/or to reduce the complexity and delay associated with the basic schemes.

### *A. Simple Distributed Power Allocation*

In this paper, a very simple distributed power allocation is proposed for ST-Coded Relay and Simple Relay and is shown to significantly improve the performance with the same complexity as the schemes with constant power.

In the four basic schemes presented in Section II, each node uses the same transmission power. For ST-Coded Relay and Simple Relay, every decoded node will relay the message in the second stage. Since the average size of the decoded set is usually much larger than one, the total energy in the second stage is much larger than the first stage. In [4], we propose equal power for the two relaying stages as a suboptimum allocation scheme for cooperative diversity in a network with uniformly distributed nodes. In the second stage, power is divided uniformly among selected relay nodes [3]. This suboptimum power allocation scheme requires a central device, which could be the source node, to acquire and distribute the knowledge of the size of the decoded set. The performance of the various power allocation schemes is presented in [4].

In this paper, motivated by the suboptimum power allocation scheme in [4], we propose a simple distributed power allocation scheme: each decoded node transmits with power equal to a fraction ($\rho \leq 1$) of the source power. The contant-power schemes presented in in Section II are schemes with $\rho = 1$. For Best-Select Relay and Pre-Select One Relay, only one relay node is employed, and thus the near optimum $\rho$ is equal to $1$ [4]. For Simple Relay and ST-Coded Relay, multiple relay nodes are employed, and thus the optimum $\rho$ should be less than one. The optimum value of $\rho$ for these two schemes can be found through simulation, or heuristically, is chosen to be inversly proportional to the average size of the decoded set.

As shown in Fig. 4, the outage probability performance of ST-Coded Relay with $\rho = 0.1$ is significantly better than constant-power ST-Coded Relay ($\rho = 1$) and Best-Select Relay for the worst source-destination pair. A similar result is observed for the outage probability of the best source-destination pair. The implementation complexity of this simple power allocation scheme is the same as the original constant-power scheme. Similar results are observed for Simple Relay.

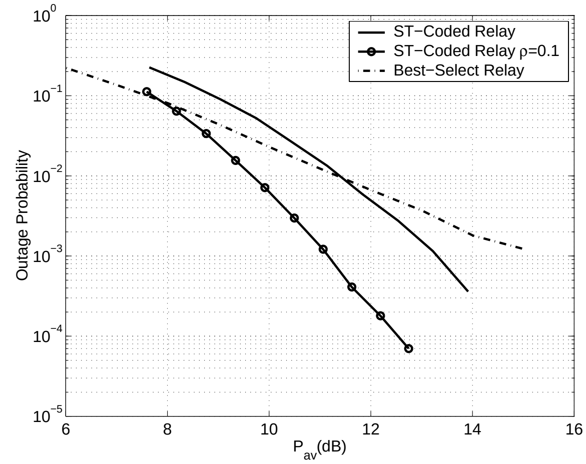


Fig. 4. The outage probability of the worst pair $(s = 0, d = N - 1)$ versus the average power for ST-Coded Relay with $\rho = 1$, ST-Coded Relay with $\rho = 0.1$, and Best-Select Relay.

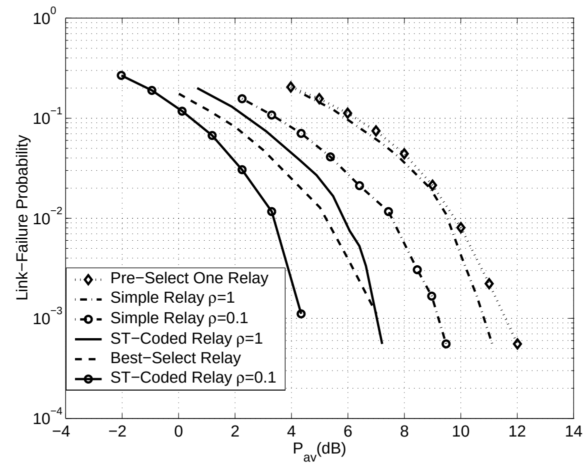


Fig. 5. The link-failure probability of a random network with $N = 10$ uniformly distributed nodes for various schemes with target outage probability equal to $0.01$.

We also present the *link-failure probability* of a random network for various schemes. A link is regarded as connected if its outage probability is below a specified target value, and is disconnected otherwise. The link-failure probability is defined as the ratio of the number of disconnected links over the total number of links in the network. In Fig, 5, the link-failure probability is plotted for a network with $N = 10$ uniformly distributed nodes in a square area. Here, the link-failure probability is averaged for $40$ randomly generated network configurations and the target outage probability is $0.01$. As we can see, with simple distributed power allocation, the link-failure probabilities both ST-Coded Relay and Simple

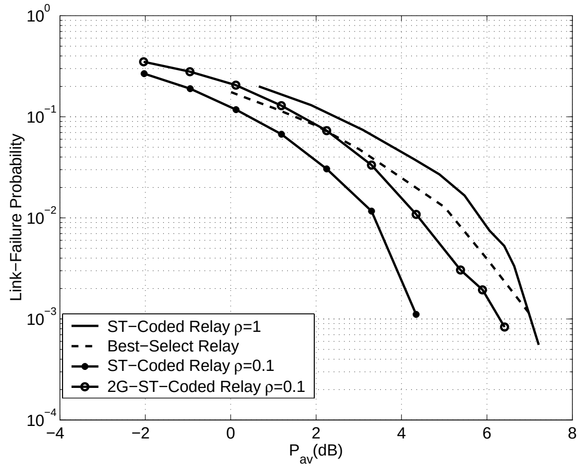


Fig. 6. The link-failure probability of a random network with $N = 10$ uniformly distributed nodes of $m = 2$-Group ST-Coded Relay with $\rho = 0.1$ with target outage probability equal to $0.01$.

Relay are significantly improved.

### B. $m$-Group ST-Coded Relay

In this section, we propose $m$-Group ST-Coded Relay to reduce the implementation complexity of ST-Coded Relay, as explained below. A distributed implementation of ST-Coded Relay is proposed in [2]. But when the number of relay nodes is large, the implementation complexity is very high. In order to do space-time processing at the destination node, the destination node must estimate the decoded set and the instantaneous channel gains for all relay nodes. The overhead for the estimation is high when the network is large. Further, high performance space-time codes apparently do not exist for cases with a large number of relays.

As shown in [2], the diversity order of a space-time-coded scheme in a wireless network is equal to the size of network. The diversity order is a quantity that characterizes the slope of the outage probability for high SNR. However, the corresponding SNR may be so high that it is out of the range of practical interest. As shown in Section II-E, for a range of outage probability from $10^{-1}$ to $10^{-4}$, the slope is far less than the size of network. This is due to the large variations in the relay-destination mean channel gains. Therefore, we propose the following $m$-Group ST-Coded Relay, whose diversity order is less than that of the ST-Coded Relay. While gaining greatly reduced complexity, we expect only a slight degradation in the outage probability performance.

In the $m$-Group ST-Coded Relay approach, the decoded set is divided into $m$ groups. Each node inside the decoded set decides which group it belongs to randomly and independently. ST-Coded Relay is employed among groups, while Simple Relay is employed inside each group. At the destination, the receiver only needs to estimate the effective channel gain of each group. Simple Relay can be viewed as the special case $m = 1$. In general, $m$-Group ST-Coded Relay provides a useful performance-complexity tradeoff between Simple Relay and ST-Coded Relay. Moreover, by applying the simple power allocation scheme proposed in Section III-A, we expect that a small number of groups can achieve a good performance.

In Fig. 6, the link-failure probability of a Two-Group ST-Coded Relay with $\rho = 0.1$ is plotted against ST-Coded Relay with $\rho = 0.1$, ST-Coded Relay with $\rho = 1$, and Best-Select Relay for a random network with $N = 10$ uniformly distributed nodes. As we can see, the performance of Two-Group ST-Coded Relay with $\rho = 0.1$ is between the performance of ST-Coded Relay with $\rho = 0.1$ and ST-Coded Relay with $\rho = 1$.

## IV. Conclusion

In this paper, we propose and investigate practical decode-and-forward cooperative diversity schemes in a wireless network. We first examine the outage probability performance of four constant-power schemes: Pre-Select One Relay, Best-Select Relay, Simple Relay, and ST-Coded Relay. It is observed that the slope of the outage probability of ST-Coded Relay is much less than its diversity order, that is, the size of the network [2], in the range of outage probabilities of practical interest. This is due to the large variations in the relay-destination mean channel gains. Although ST-Code Relay has a steeper slope than Best-Select Relay, it performs worse than Best-Select Relay in a range of outage probabilities. This observation suggests a study of effective ”coding gain” in cooperative diversity schemes.

Two new methods for improving the above approaches, called simple distributed power allocation and $m$-Group ST-Coded Relay, are proposed. Simple Relay and ST-Coded Relay with this simple distributed power allocation outperform the original constant-power schemes significantly without an increase in the implementation complexity. The $m$-Group ST-Coded Relay gains in lower complexity with only a slight degradation in performance. When combined with simple power allocation, Two-Group ST-Coded Relay outperforms the ST-Coded Relay with constant power. Since only two-dimensional space-time codes are employed, the implementation complexity of Two-Group ST-Coded Relay is very low, which makes this scheme very attractive.

## References


[1] J. Laneman, D. Tse, and G. Wornell, “Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behavior,” to appear in *IEEE Trans. Inform. Theory*.

[2] J. Laneman and G. Wornell, “Distributed Space-Time Coded Protocols for Exploiting Cooperative Diversity in Wireless Networks,” *IEEE Trans. Inform. Theory*, pp. 2415-2425, October 2003.

[3] J. Luo, R. Blum, L. Cimini, L. Greenstein, and A. Haimovich, “Power Allocation in a Transmit Diversity System with Mean Channel Gain Information,” submitted to *IEEE Communications Letters*.

[4] J. Luo, R. Blum, L. Cimini, L. Greenstein, and A. Haimovich, “Decode-and-forward cooperative diversity with power allocation in wireless network,” to be submitted to *Globecom* 2005.