Biol698/Math635/Math430
Fall 2018
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Homework 1

Consider the following modified logistic equation with a threshold that can be used to describe the transition from a resting state (V_{rest}) to an activated state (V_{act}), if they exist,

$$\frac{dV}{dt} = F(V), \tag{1}$$

where t represents time and the function $F(V)$ is given by

$$F(V) = -r\,V\,\left(1 - \frac{V}{T}\right)\left(1 - \frac{V}{K}\right) + I_{app}. \tag{2}$$

The parameters r, K, T and I are constants satisfying $r, K, T > 0$ and $T < K$. T and K represent the threshold (V_{th}) and saturation (V_{sat}) V levels for the unbiased case ($I_{app} = 0$). The parameter I_{app} represents the current applied to the system. V_{th} and V_{sat} depend on I_{app}. For $I_{app} = 0$, $V_{th} = T$ and $V_{sat} = K$.

1. Write a code to solve numerically the ODE (1) or adapt the template code provided in the course website. The simulation output for each set of parameter values must be

   (a) A graph of the solution $V(t)$

   (b) The equilibrium value(s) $V_{eq} = \lim_{t\to\infty} V(t)$

   (c) A graph of F as a function of V.

   - Each simulation should be run long enough (large enough value of t) so that $V(t)$ reaches values close enough to V_{eq}, but not too long so the changes in $V(t)$ are clearly shown.
   - Plot the two graphs as two panels of the same graph.
   - The axis should be labeled correctly.
   - The fonts should be large enough (suggested: “fontsize” = 24)

2. Consider the following parameter values: $r = 1$, $T = 0.25$, $K = 1$. Perform simulations as described above for $V(0) = 0.01$ an three values of I_{app}: $I_{app} = 0$, $I_{app} = 0.05$ and $I_{app} = 0.1$. For each one of them compute (if possible) V_{eq} and V_{sat}.

3. (Graduate level) Consider the following parameter values and initial condition: $r = 1$, $T = 0.4$, $K = 1$, $I_{app} = [0, 0.2]$ with intervals $\Delta I_{app} = 0.02$ (11 values), and $V(0) = 0.25$

   (a) Simulate the model as described above in **ascending** order of the values of I_{app}. Plot the graph of V_{eq} as a function of I_{app}.

   (b) Simulate the model as described above in **descending** order of the values of I_{app}. Plot the graph of V_{eq} as a function of I_{app}.

4. (Graduate level) Consider the following parameter values and initial condition: $r = 1$, $T = 0.4$, $K = 1$, $I_{app} = [0, 0.2]$ with intervals $\Delta I_{app} = 0.02$ (11 values)

   (a) Simulate the model as described above in **ascending** order of the values of I_{app}. For $I_{app} = 0$ use $V(0) = 0$. For $I_{app} > 0$ set $V(0)$ equal to V_{eq} in the simulation for the previous value of I_{app}.

   (b) Simulate the model as described above in **descending** order of the values of I_{app}. For $I_{app} = 0.2$ set $V(0)$ equal to the value of V_{eq} computed in the previous simulation for $I_{app} = 0.2$. For $I_{app} < 0.2$ set $V(0)$ equal to V_{eq} in the previous simulation.

   (c) Plot a single graph with all the values of V_{eq} as a function of I_{app}.