

Biol698/Math635/Biol498/Math430
Fall 2019

Homework 1

Consider the following modified logistic equation with a threshold that can be used to describe the transition from a resting state (V_{rest}) to an activated state (V_{act}), if they exist

$$\frac{dV}{dt} = F(V), \quad (1)$$

where t represents time, V is the dependent variable (e.g., voltage), and the function $F(V)$ is given by

$$F(V) = -r V \left(1 - \frac{V}{T}\right) \left(1 - \frac{V}{K}\right) + I_{app}, \quad (2)$$

The parameters r, K, T and I_{app} constants, $r, K, T > 0$ and $T < K$. T and K represent the threshold (V_{th}) and saturation (V_{sat}) V levels for the unbiased case (I_{app}). The parameter I_{app} represents the input given to the system (e.g., applied DC current). Both V_{th} and V_{sat} depend on I_{app} . As mentioned above, for $I_{app} = 0$, $V_{th} = T$ and $V_{sat} = K$. The parameter r represents the rate constant (inverse of the time constant for the unbiased case).

1. Write a code to solve numerically the ODE (1)-(2) (or adapt the template code provided in the course website). The simulation output for each set of parameter values must be
 - (a) A graph of the solution $V(t)$
 - (b) The equilibrium value(s) $V_{eq} = \lim_{t \rightarrow \infty} V(t)$
 - (c) A graph of F as a function of V .
 - Each simulation should be run long enough (large enough value of t) so that $V(t)$ reaches values close enough to V_{eq} , but not too long so the changes in $V(t)$ are clearly shown.
 - Plot the two graphs as two panels in the same graph.

- The axis should be labeled correctly.
 - The fonts should be large enough (suggested: “fontsize” = 24)
2. Consider the following parameter values: $r = 1$, $T = 0.25$, $K = 1$. Perform simulations as described above for $V(0) = 0.01$ and three values of I : $I = 0$, $I = 0.05$, $I = 0.1$.
 3. (Graduate level) Consider the following parameter values and initial condition: $r = 1$, $T = 0.4$, $K = 1$, $I = [0, 0.2]$ with intervals $\Delta I = 0.02$ (11 values), and $X(0) = 0.25$
 - (a) Simulate the model as described above in **ascending** order of the values of I_{app} . Plot the graph of V_{eq} as a function of I .
 - (b) Simulate the model as described above in **descending** order of the values of I_{app} . Plot the graph of V_{eq} as a function of I_{app} .
 4. (Graduate level) Consider the following parameter values and initial condition: $r = 1$, $T = 0.4$, $K = 1$, $I_{app} = [0, 0.2]$ with intervals $\Delta I = 0.02$ (11 values)
 - (a) Simulate the model as described above in **ascending** order of the values of I . For $I = 0$ use $V(0) = 0$. For $I_{app} > 0$ set $V(0)$ equal to V_{eq} in the simulation for the previous value of I_{app} .
 - (b) Simulate the model as described above in **descending** order of the values of I_{app} . For $I_{app} = 0.2$ set $V(0)$ equal to the value of V_{eq} computed in the previous simulation for $I_{app} = 0.2$. For $I_{app} < 0.2$ set $V(0)$ equal to V_{eq} in the previous simulation.
 - (c) Plot a single graph with all the values of V_{eq} as a function of I_{app} .