Biol698/Math635/Biol498/Math430
Fall 2019
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Homework 3

Answer the following questions.

- Justify your answers.
- Explain your results.
- Provide the necessary calculations in a clear way.
- Provide the necessary supporting graphs and codes.
- Make sure the graphs are properly labeled and include the information (title and parameter values) necessary to understand your explanations.
- You may write your own code or adapt the template code provided in the course website.

1. Consider the following passive membrane equation

$$\tau \frac{dV}{dt} = -(V - E_L) + R\, I_{app}(t) \tag{1}$$

with $E_L = -60$ mV, $C = 1\ \mu$F/cm^2, $G_L = 0.1$ mS/cm^2, Write a Matlab code to solve eq. (1). Use $V(0) = -60$ mV and the following units for V, t and I_{app} respectively: $[V] =$ mV,$[t] =$ msec, $[I_{app}] = \mu$A/cm^2.

   (a) Calculate the time constant τ.

(b) Consider the following two time-independent applied currents $I_{app} = -0.5$ and $I_{app} = 0.5$. For each value of I_{app} ($I_{app} = 0.5$ and $I_{app} = -0.5$) compare the numerical and analytical solutions to the passive membrane equation (1) by (i) plotting superimposed graphs of these solutions, and (ii) plotting the error (absolute value of the difference between these two solutions).

(c) Consider a square pulse of current

$$I_{app}(t) = I_0\, Heav(t - t_i) * Heav(t_f - t)$$

with $I_0 = 0.5$, $t_i = 100$ msec and $t_f = 200$ msec. Plot both the numerical and analytical solutions to the passive membrane equation (1), and compare them as in **(a)**. Run your simulation for 400 msec.

(d) Consider the following current

$$I_{app}(t) = I_0\ \sin(2\,\pi\,\omega\,t/1000).$$

with $I_0 = 0.5$ nA. **(i)** Plot the numerical solution to the passive membrane equation (1) for $\omega = 1$, $\omega = 5$, $\omega = 10$, $\omega = 20$, and $\omega = 100$. **(ii)** Plot a graph relating the output frequency (y-axis) vs. the input frequency (x-axis). **(iii)** Plot a graph relating the amplitude of the output oscillations (y-axis) vs. the input frequency (x-axis).

2. Build an integrate-and-fire model using $V_{th} = -50\,mV$, $V_{rst} = -65\,mV$ and the parameters values for the passive membrane equation above.

(a) Calculate (analytically) the interspike-interval firing rate (r_{isi}) for $g_{sra} = 0$ (no spike-rate adaptation) and

i. $I_{app} = 0.5$
ii. $I_{app} = 1$
iii. $I_{app} = 1.01$
iv. $I_{app} = 2$

Plot the corresponding numerical solutions and compare your analytical and numerical results.

(b) Compute the numerical solutions and plot the corresponding graphs for for $I_{app} = 2$, $E_k = -85$, $\Delta g_{sra} = 0.1$ and

i. $\tau_{sra} = 10\,msec.$
ii. $\tau_{sra} = 100\,msec.$