Biol635/Math635/Biol432/Math430
Fall 2021
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Homework 1

Answer the following questions.

- Justify your answers.
- Explain your results.
- Provide the necessary calculations in a clear way.
- Provide the codes you used (if any).

Consider the following *modified logistic equation with a threshold*. This equation has many applications. In particular, it can be used to describe the transition of a system from a resting state (V_{rest}) to and activated state (V_{act}) when they exist

$$\frac{dV}{dt} = F(V). \tag{1}$$

The existence of these states depend on the model parameters. In (1) t represents time, V is the dependent variable (e.g., voltage), and the function $F(V)$ is given by

$$F(V) = -r\,V\left(1 - \frac{V}{T}\right)\left(1 - \frac{V}{K}\right) + I_{app}. \tag{2}$$

The parameters r, K, T and I_{app} constants, $r, K, T > 0$ and $T < K$. For $I_{app} = 0$, eq (1) has three equilibria (V_{eq}): $V_{rest} = 0$, $V_{th} = T$ and $V_{sat} = K$ (resting, threshold and saturation equilibria, respectively). The parameter I_{app} represents the input given to the system (e.g., applied DC current). The three equilibria (V_{rest}, V_{th} and V_{sat}) depend on I_{app} and may cease to exist for certain ranges of values of I_{app}. The parameter r represents the rate constant (inverse of the time constant for the unbiased case $I_{app} = 0$). The units depend on the problem at hand. If the model is intended for the dynamics of a neuron, then the units of t are milliseconds (ms) and the units of V are millivolts (mV).

1. Write a code to solve numerically the ODE (1)-(2) (or adapt the template code provided in the course website). The simulation output for each set of parameter values must be

   (a) A graph of the solution $V(t)$.

   (b) The equilibrium value(s) $V_{eq} = \lim_{t\to\infty} V(t)$

   (c) A graph of F as a function of V.

   - Each simulation should be run long enough (large enough value of t) so that $V(t)$ reaches values close enough to V_{eq}, but not too long so the changes in $V(t)$ are clearly shown.
   - Plot the two graphs as two panels in the same graph.
   - Use correct labels for the axes.
   - The fonts should be large enough (suggested: "fontsize" = 24)

2. Consider the following parameter values: $r = 1$, $T = 0.25$, $K = 1$. Perform simulations as described above for $V(0) = 0.01$ and three values of I: $I = 0$, $I = 0.05$, $I = 0.1$.

3. (Graduate level) Consider the following parameter values and initial condition: $r = 1$, $T = 0.4$, $K = 1$, $I = [0, 0.2]$ with intervals $\Delta I = 0.02$ (11 values), and $X(0) = 0.25$

   (a) Simulate the model as described above in **ascending** order of the values of I_{app}. Plot the graph of V_{eq} as a function of I.

   (b) Simulate the model as described above in **descending** order of the values of I_{app}. Plot the graph of V_{eq} as a function of I_{app}.

4. (Graduate level) Consider the following parameter values and initial condition: $r = 1$, $T = 0.4$, $K = 1$, $I_{app} = [0, 0.2]$ with intervals $\Delta I = 0.02$ (11 values)

   (a) Simulate the model as described above in **ascending** order of the values of I. For $I = 0$ use $V(0) = 0$. For $I_{app} > 0$ set $V(0)$ equal to V_{eq} in the simulation for the previous value of I_{app}.

   (b) Simulate the model as described above in **descending** order of the values of I_{app}. For $I_{app} = 0.2$ set $V(0)$ equal to the value of V_{eq} computed in the previous simulation for $I_{app} = 0.2$. For $I_{app} < 0.2$ set $V(0)$ equal to V_{eq} in the previous simulation.

   (c) Plot a single graph with all the values of V_{eq} as a function of I_{app}.