

Biol635 / Math635 / Biol432 / Math430  
Fall 2025

Assignment 2

**Answer the following questions.**

- Justify your answers.
- Explain your results.
- Provide the necessary calculations in a clear way.
- Provide the necessary supporting graphs and codes.
- Make sure the graphs are properly labeled and include the information (title and parameter values) necessary to understand your explanations.
- You may write your own code or adapt the template code provided in class.

1. Consider the following passive membrane equation

$$\tau \frac{dV}{dt} = -(V - E_L) + R I_{app}(t) \quad (1)$$

with  $E_L = -60$  mV,  $C = 1$   $\mu\text{F}/\text{cm}^2$ ,  $G_L = 0.1$   $\text{mS}/\text{cm}^2$ , Write a Matlab code (or use the template code) to solve eq. (1). Use  $V(0) = -60$  mV and the following units for  $V$ ,  $t$  and  $I_{app}$  respectively:  $[V] = \text{mV}$ ,  $[t] = \text{msec}$ ,  $[I_{app}] = \mu\text{A}/\text{cm}^2$ .

- (a) Calculate the time constant  $\tau$ .
- (b) Compute the steady-state solution for  $I_{app} = -0.5$  and  $I_{app} = 0.5$
- (c) For each these values of  $I_{app}$ , compare the numerical and analytical solutions to the passive membrane equation (1) by
  - i. Plotting superimposed graphs of these solutions
  - ii. plotting the error (absolute value of the difference between these two solutions).

Run your simulation for enough time so to allow  $V$  to reach a close vicinity of its steady state.

Use the initial condition  $V(0) = E_L$

(d) Consider the following oscillatory current

$$I_{app}(t) = I_0 \sin(2\pi f t/1000).$$

with  $I_0 = 0.5$ . The input frequency  $f$  is given in Hz (number of cycles per second) and the units of time are msec.

- i. Plot the numerical solution to the passive membrane equation (1) for  $f = 1$ ,  $f = 5$ ,  $f = 10$ ,  $f = 20$ , and  $f = 100$ .
- ii. Plot a graph relating the output frequency (y-axis) vs. the input frequency (x-axis)
- iii. Plot a graph relating the amplitude of the output oscillations (y-axis) vs. the input frequency (x-axis)
- iv. What is the effect of increasing and decreasing the time constant  $\tau$ ?

2. (Graduate level)

Repeat question 1d for a square-wave current  $I_{app,sqw}(t)$  with amplitude equal to 1.

Note: You can construct the square-wave current by making  $I_{app,sqw}(t) = 1$  if  $t \geq 0$  and  $I_{app,sqw}(t) = -1$  otherwise.

3. Find examples in the literature of both biological neuronal systems and models that behave like

- (Undergraduate level) low-pass filters
- (Graduate level) band-pass filters
- Speculate on the differences between them by sharing information between members of the two groups