Analytical Computational Neuroscience

Math 635

Syllabus

Bibliography:

"Mathematical Foundations of Neuroscience", by G. B. Ermentrout & D. H. Terman - Springer (2010), 1st edition. ISBN 978-0-387-87707-5

* "Foundations of Cellular Neurophysiology", by Daniel Johnston and Samuel M.-S. Wu. The MIT Press, 1995. ISBN 0-262-10053-3

* "Dynamical Systems in Neuroscience: The Geometry of Excitability and Bursting", by Eugene M. Izhikevich. The MIT Press, 2007. ISBN 0-262-09043-8

* "Biophysics of Computation - Information processing in single neurons", by Christof Koch. Oxford University Press, 1999. ISBN 0-19-510491-9

* "Theoretical Neuroscience: Computational and Mathematical Modeling of Neural Systems", by Peter Dayan and Larry F. Abbott. The MIT Press,2001. ISBN 0-262-04199-5

Figures were taken from some of these books

Introduction

Neuroscience is the study of the brain

Brain function underlies all behavior

- Motor activity (walking, smiling, etc.)
- Higher order cognitive behavior (thinking, learning, memory, decision making)

The brain consists of neurons and glial cells.

Introduction

Neurons are the basic signaling units of the nervous system.

Glial cells are important in maintaining the health of neurons and how they behave and interact

Neurons communicate with each other at synapses

Introduction

Human Brain:

- 10^{11} neurons
- 10^{15} synapses

Neurons typically receive inputs from a large number of other cells

A typical 1 mm3 cortical tissue contains 10^5 neurons

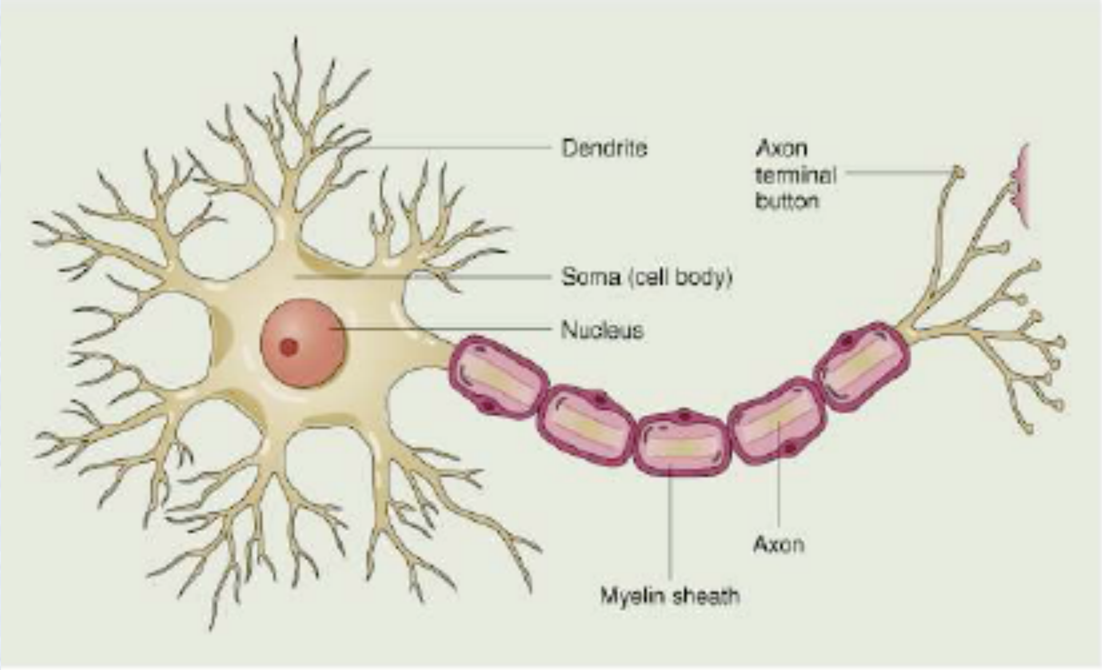
Neuron size varies from the micrometer to the meter scales

The estimated number of different morphological classes of neurons in the vertebrate brain is 10,000

Introduction

Neurons:

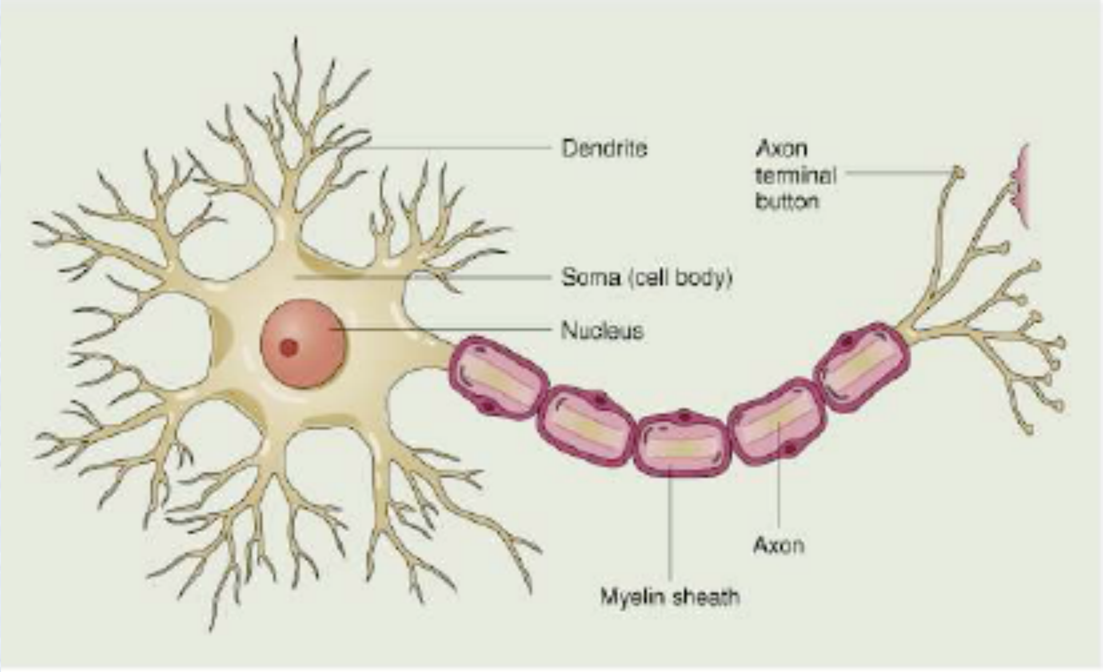
- Soma or cell body
- Dendrites
- Axon



Introduction

Soma:

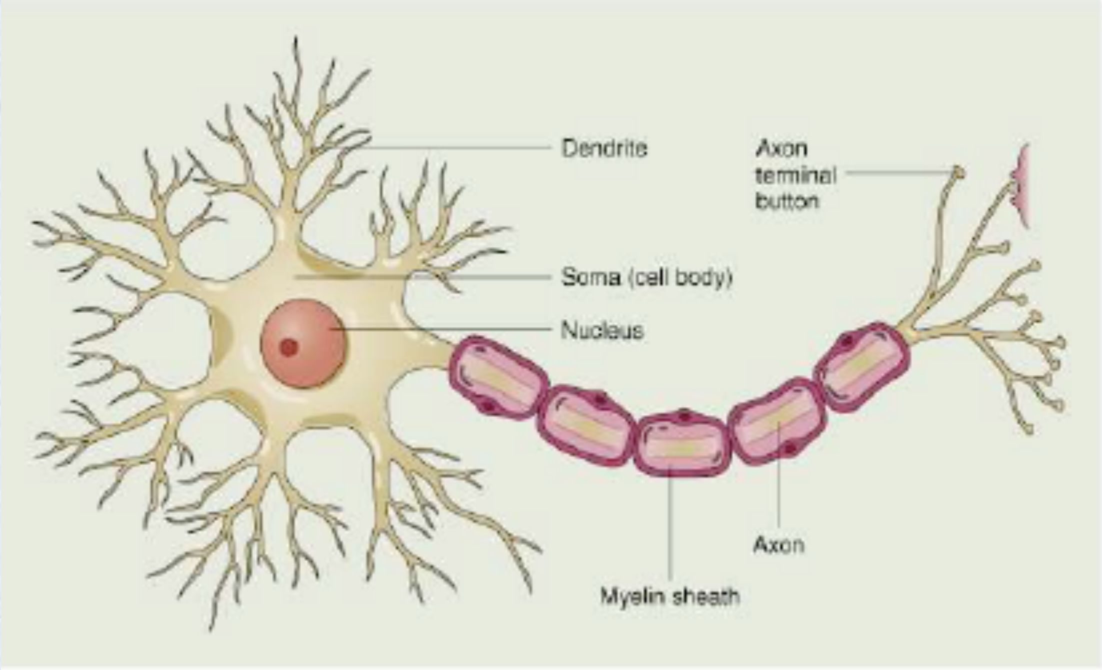
- Site of the nucleus and all the normal cellular machinery
- Functionally the soma plays the role of integrating all of the inputs of the cell to produce some output



Introduction

Dendrites:

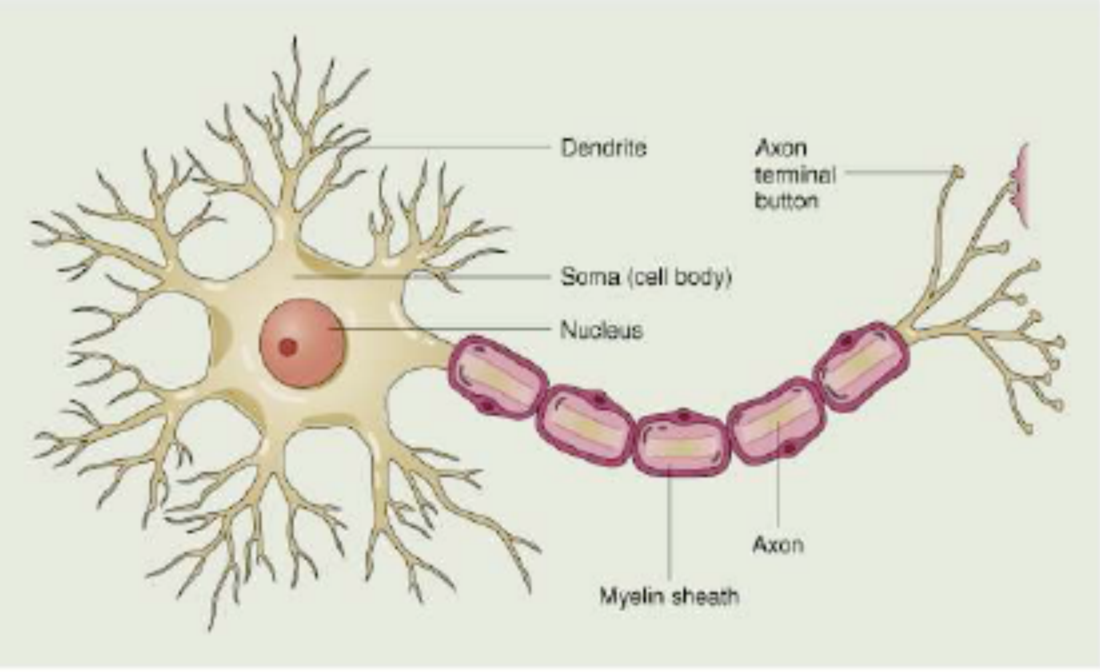
- “Input line” to the neuron
- Dendrites that lie at the top of the neuron are called apical
- Dendrites that lie at the base of the neuron are called basal
- In many cases, the majority of the surface area of the cell is taken up by the dendrites



Introduction

Axon:

- Major conducting unit of the neuron ("output line")
- The diameter of an axon may range from .2 to 20 μm and may extend for up to one meter
- It can convey information great distances by propagating a transient electrical signal called the action potential
- Large axons are surrounded by a fatty insulating sheath called myelin
- The sheath is interrupted at regular intervals by nodes of Ranvier.



Introduction

Neurons, like all other cells, maintain a potential difference of about 65 mV across their external membrane

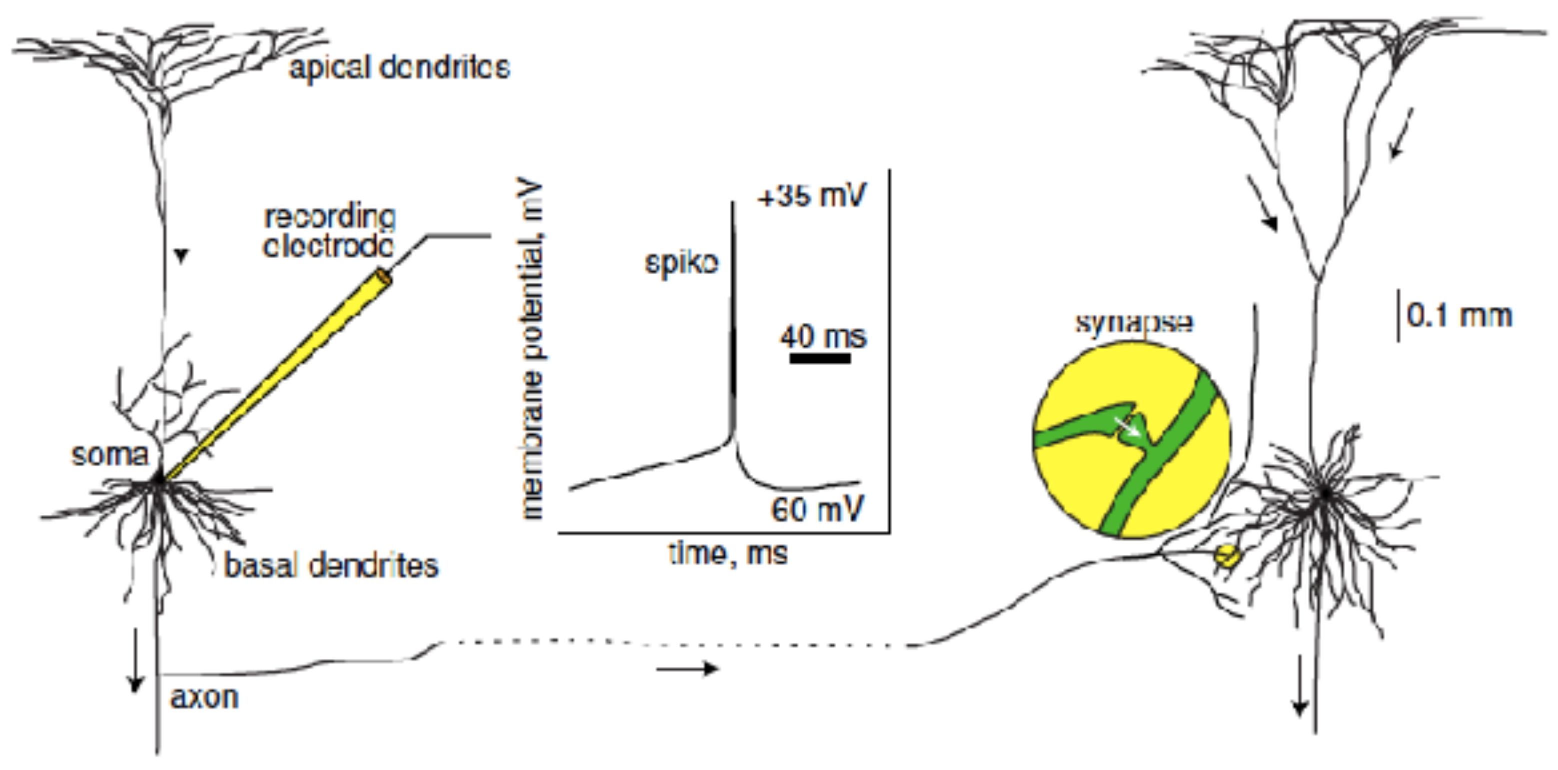


Figure 1.1: Two interconnected cortical pyramidal neurons (hand drawing) and *in vitro* recorded spike.

Introduction

What distinguishes neurons and other excitable cells from most cells in the body is that this resting membrane potential can be altered and can, therefore, serve as a signaling mechanism

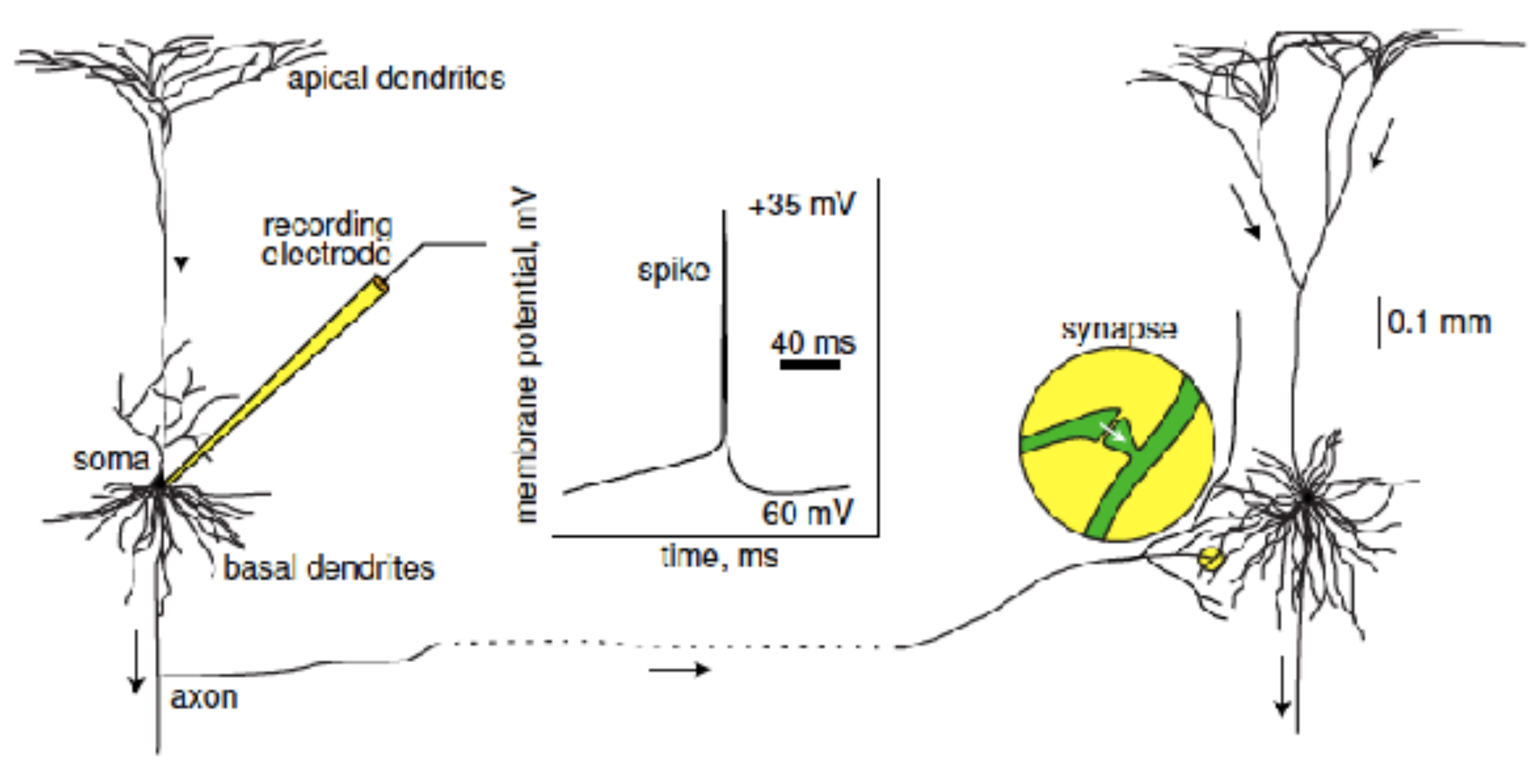


Figure 1.1: Two interconnected cortical pyramidal neurons (hand drawing) and *in vitro* recorded spike.

Introduction

Action potentials:

- Fast change in the neuron's membrane potential
- It is initiated at a specialized region of the soma called the axon hillock
- It is initiated when the membrane potential reaches some threshold value
- It propagates along the axon
- May last as short as 1 msec
- May travel at rates that vary between about 1 and 100 meters per second
- The shape and speed of each action potential does not depend on the stimulus

Refractory period:

- After each action potential, there is a period during which a second impulse cannot be initiated

Introduction

Firing frequency:

- Stronger stimuli often produce higher frequencies of impulse firing
- Since the shape and speed of each action potential does not depend on the stimulus, information about the stimulus is often carried in the frequency of firing
- Limited by the refractory period
- Some neurons are capable of continuously generating action potentials, even without inputs
- The temporal pattern of firing may be quite complicated.

Introduction

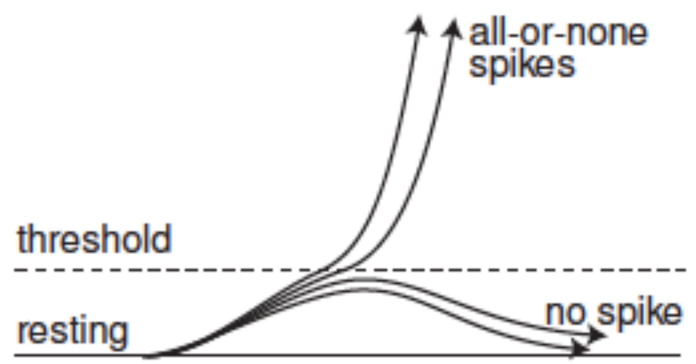


Figure 1.3: The concept of a firing threshold.

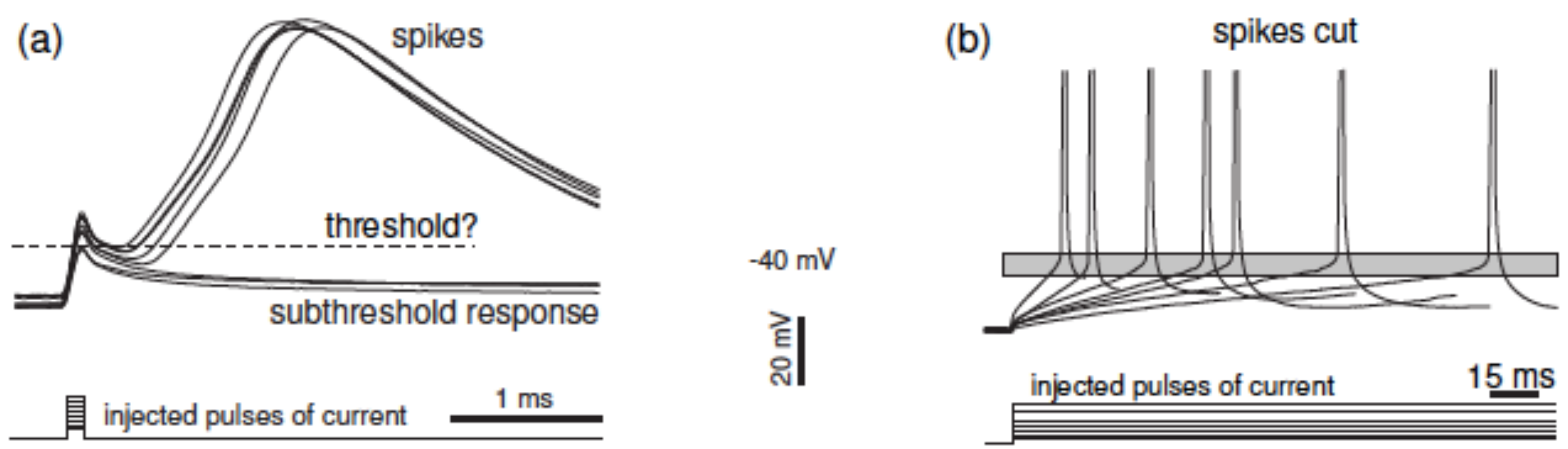


Figure 1.4: Where is the firing threshold? Shown are *in vitro* recordings of two layer 5 pyramidal neurons of rat. Notice the difference of voltage and time scales.

Introduction

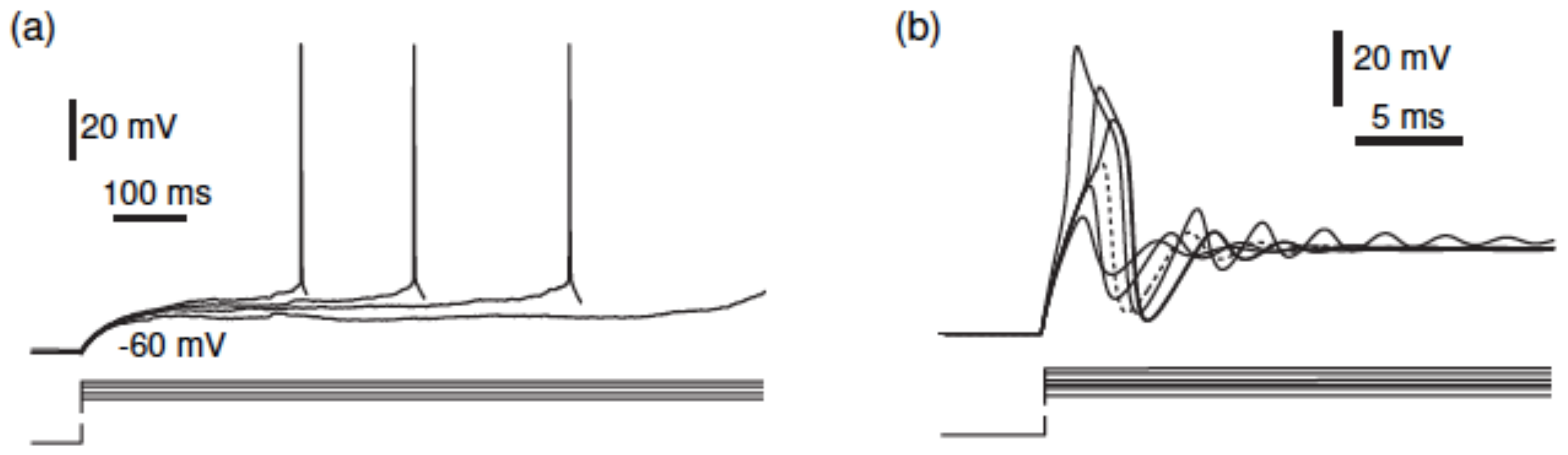


Figure 1.5: Where is the rheobase, i.e., the minimal current that fires the cell? (a) *in vitro* recordings of pyramidal neuron of layer 2/3 of rat's visual cortex show increasing latencies as the amplitude of the injected current decreases. (b) Simulation of the $I_{\mathrm{Na,p}}+I_{\mathrm{K}}$-model shows spikes of graded amplitude.

Introduction

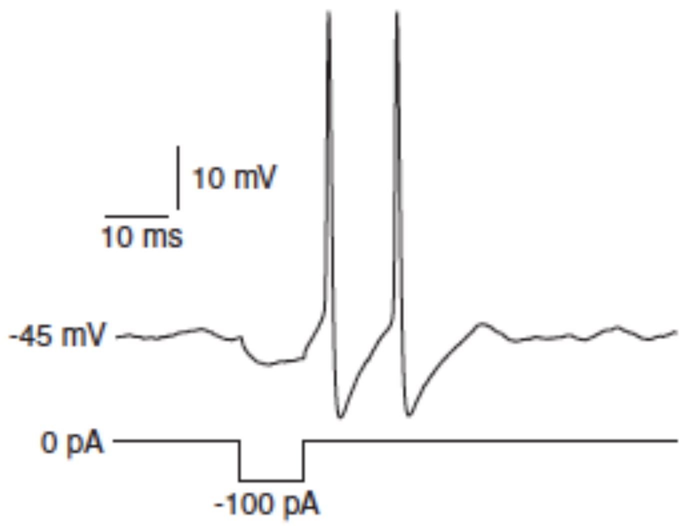


Figure 1.6: *In vitro* recording of rebound spikes of rats brainstem mesV neuron in response to a brief hyperpolarizing pulse of current.

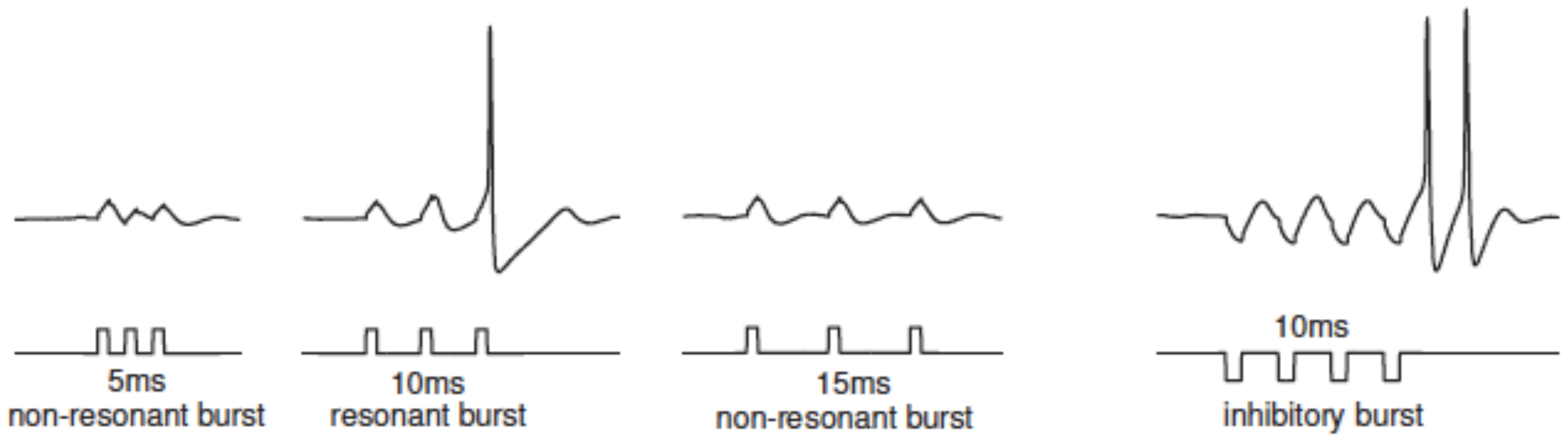


Figure 1.7: Resonant response of the mesencephalic V neuron of rat brainstem to pulses of injected current having 10 ms period (*in vitro*).

Introduction

Dynamical systems tools:

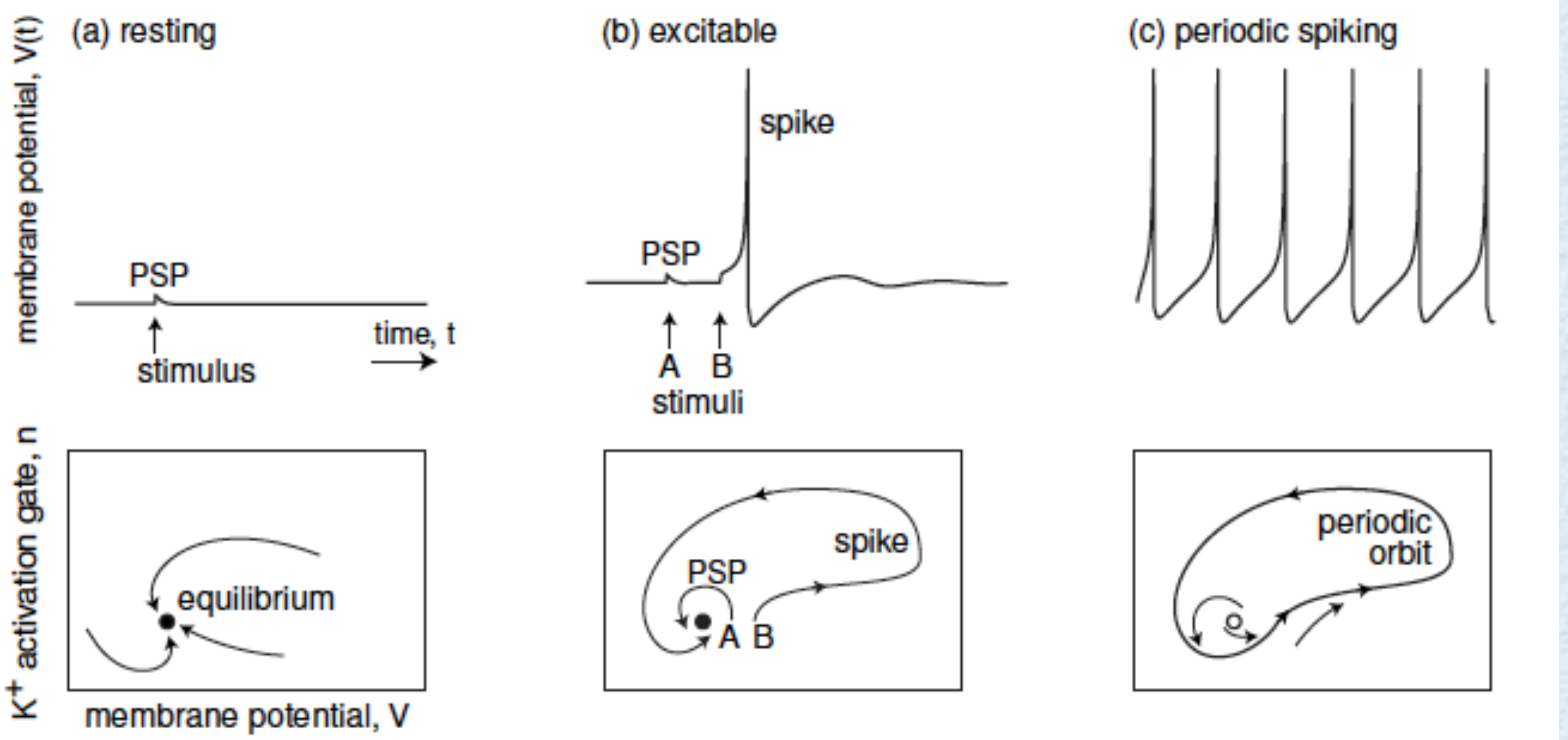


Figure 1.9: Resting, excitable, and periodic spiking activity correspond to a stable equilibrium (a and b) or limit cycle (c), respectively.

Introduction

Bursting

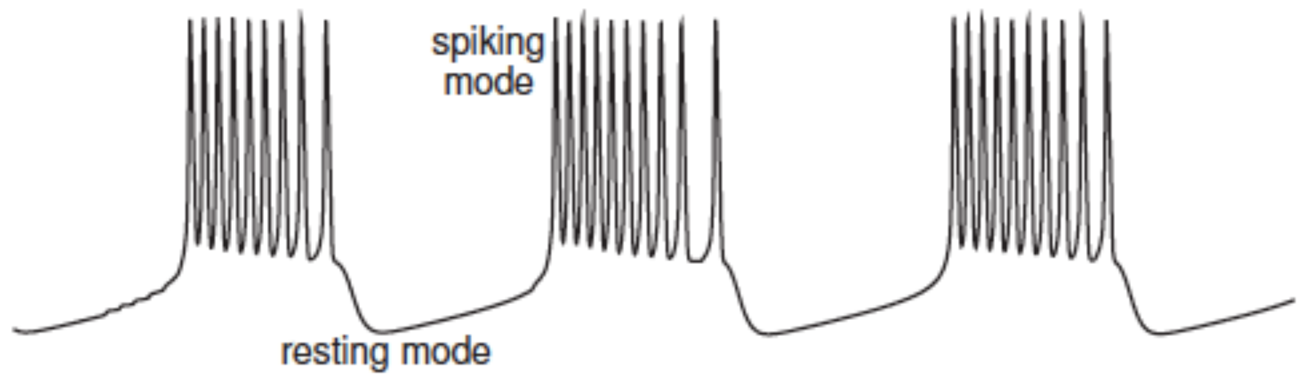


Figure 1.10: Rhythmic transitions between resting and spiking modes result in bursting behavior.

Introduction

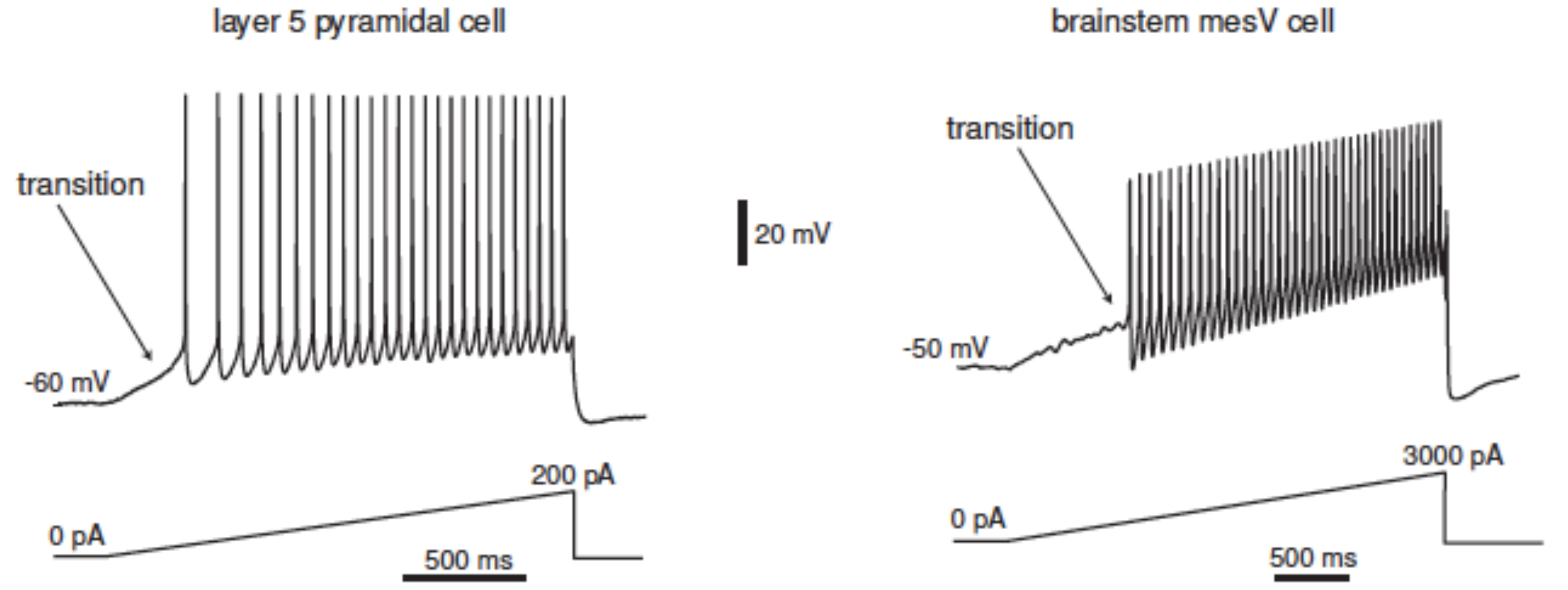
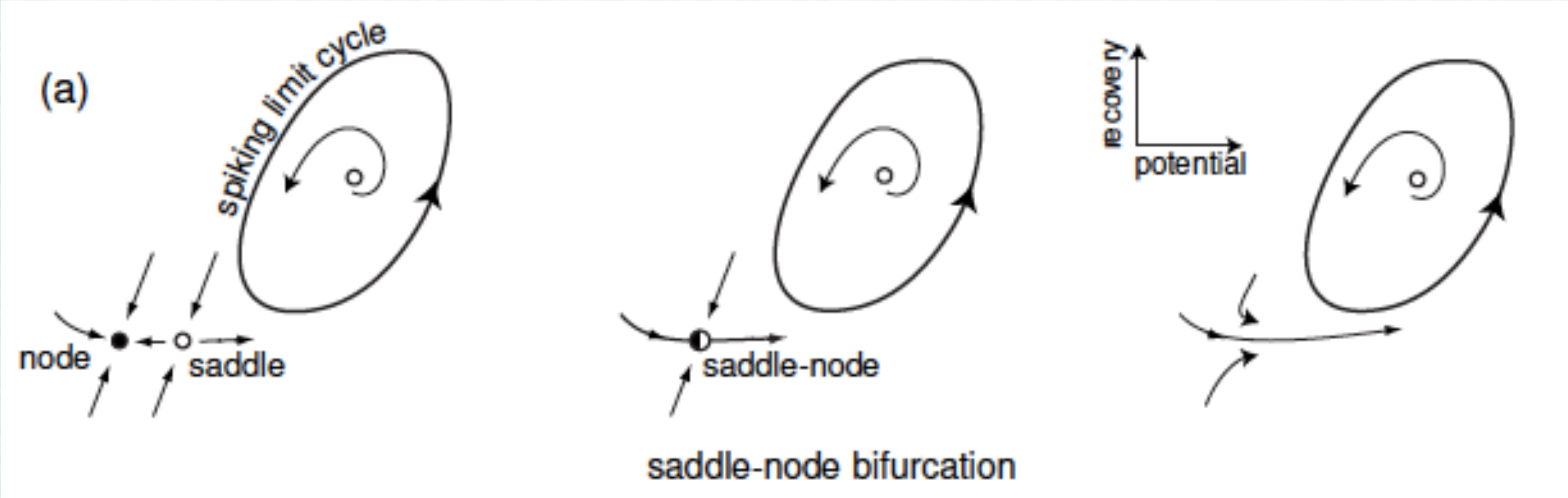


Figure 1.11: As the magnitude of the injected current slowly increases, the neurons bifurcate from resting (equilibrium) to tonic spiking (limit cycle) modes.

Introduction

Bifurcations:



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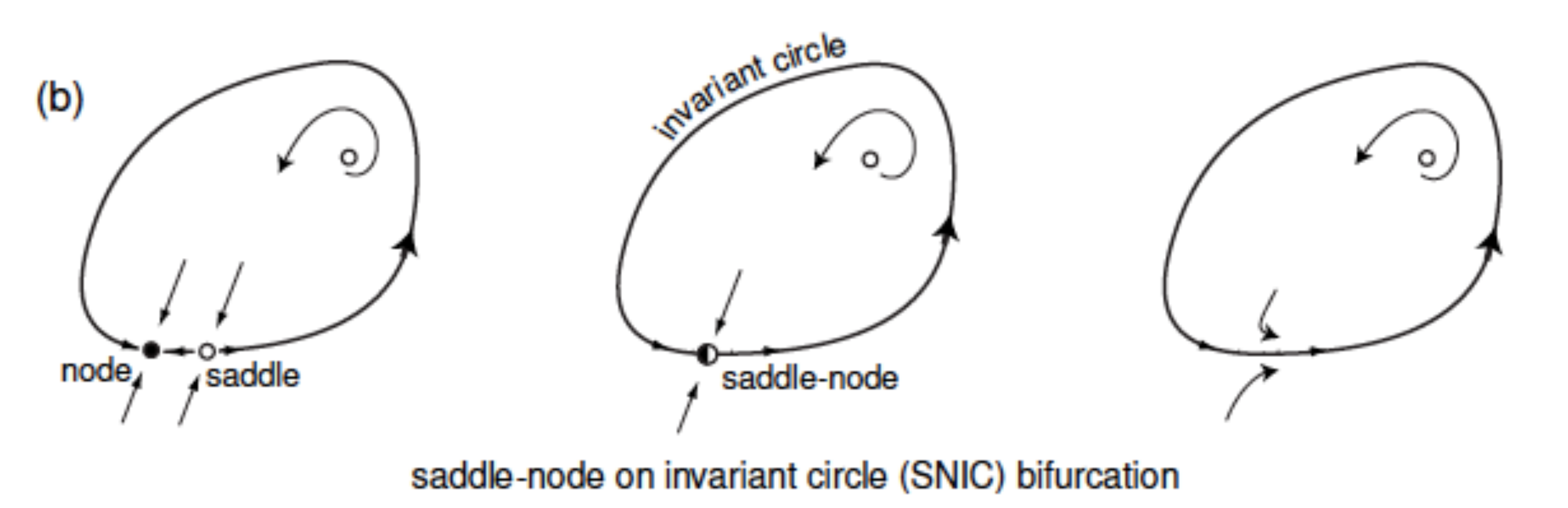

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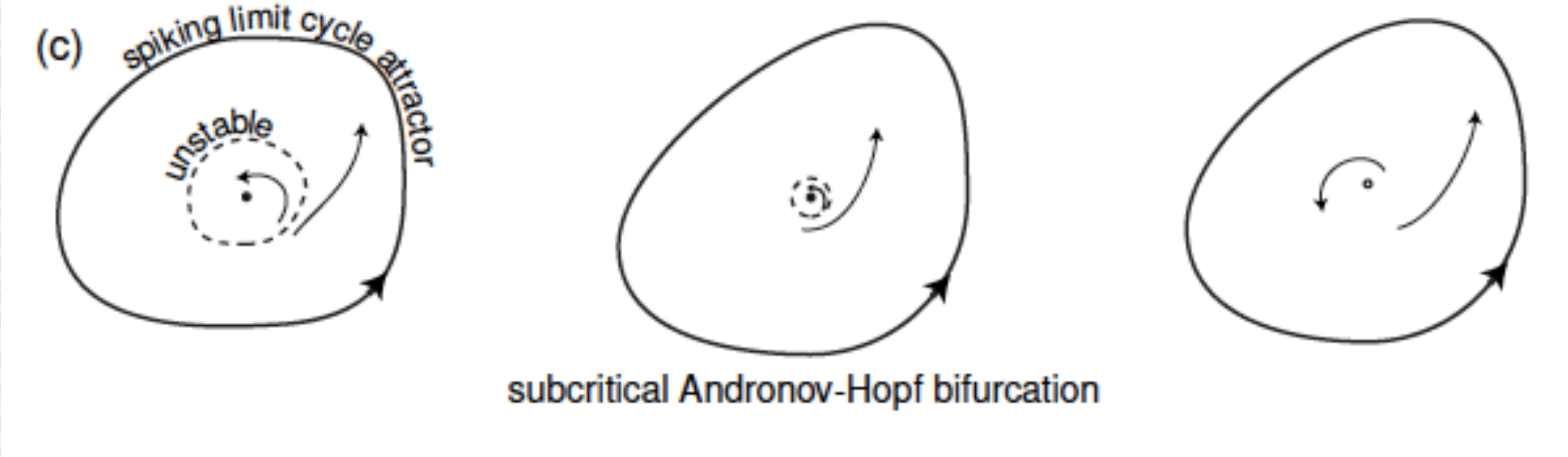

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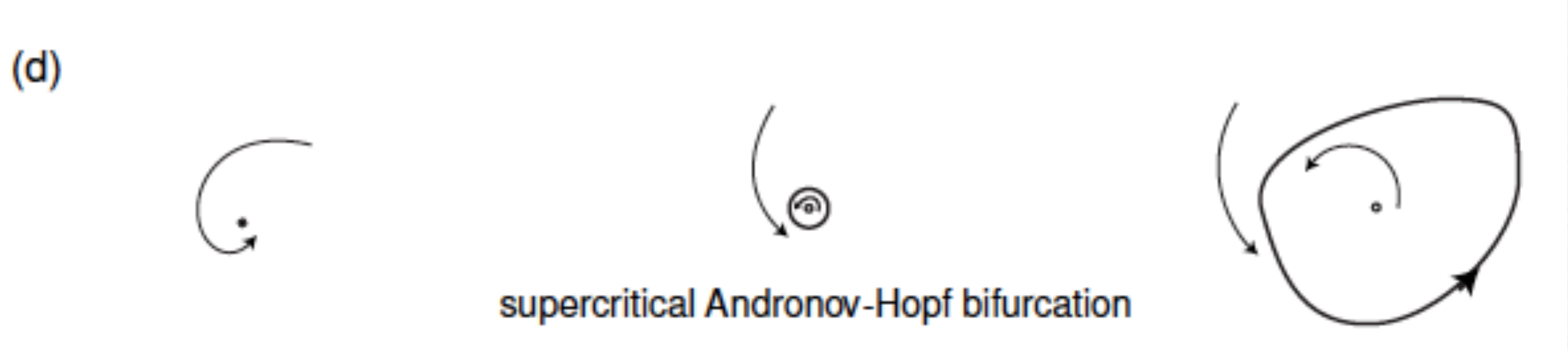

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Bifurcations:

co-existence of resting and spiking states

subthreshold oscillations	YES (bistable)	NO (monostable)
NO (integrator)	saddle-node	saddle-node on invariant circle
YES (resonator)	subcritical Andronov-Hopf	supercritical Andronov-Hopf

Figure 1.13: Classification of neurons into monostable/bistable integrators/resonators according to the bifurcation of the resting state in Fig. 1.12.

Introduction

Bifurcations:

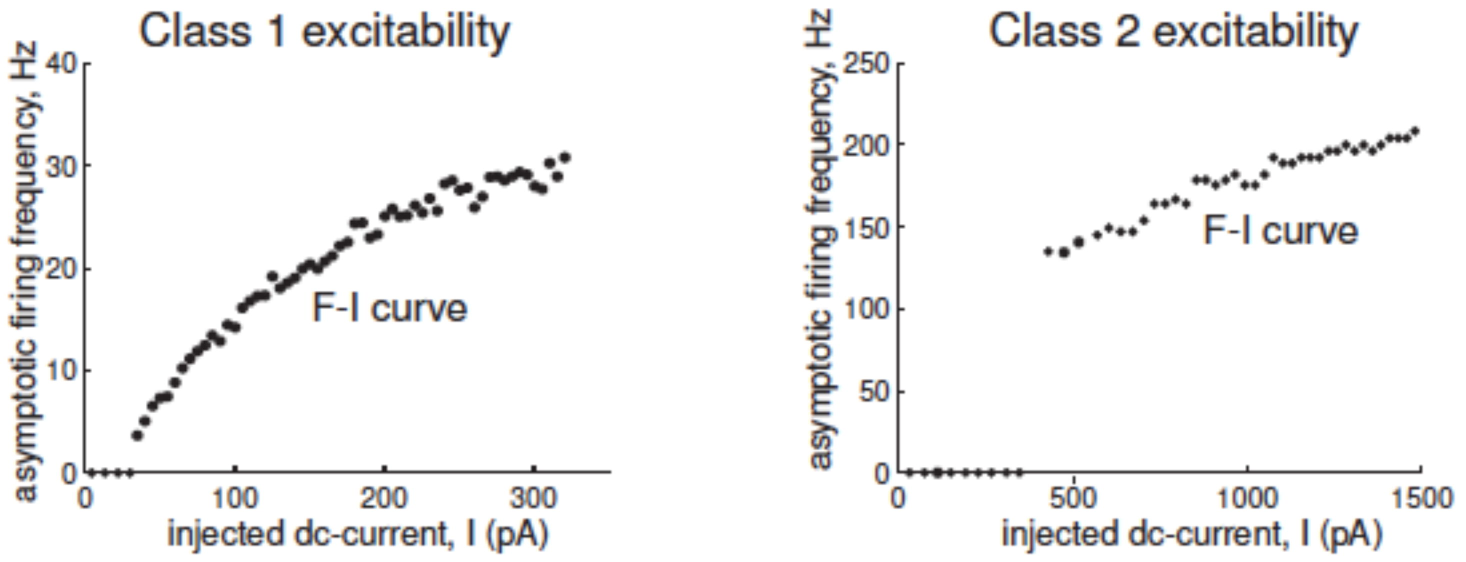

Figure 1.14: Frequency-current (F-I) curves of cortical pyramidal neuron and brainstem mesV neuron from Fig. 7.3. These are the same neurons used in the ramp experiment in Fig. 1.11.

Introduction

Hodgkin-Huxley classification of neural excitability:

- Class 1: Action potentials can be generated with arbitrarily low frequency, depending on the strength of the applied current
- Class 2: Action potentials are generated in a certain frequency band that is relatively insensitive to changes in the strength of the applied current.
- Class 3: A single action potential is generated in response to a pulse of current. Repetitive (tonic) spiking can be generated only for extremely strong injected currents or not at all.

Introduction

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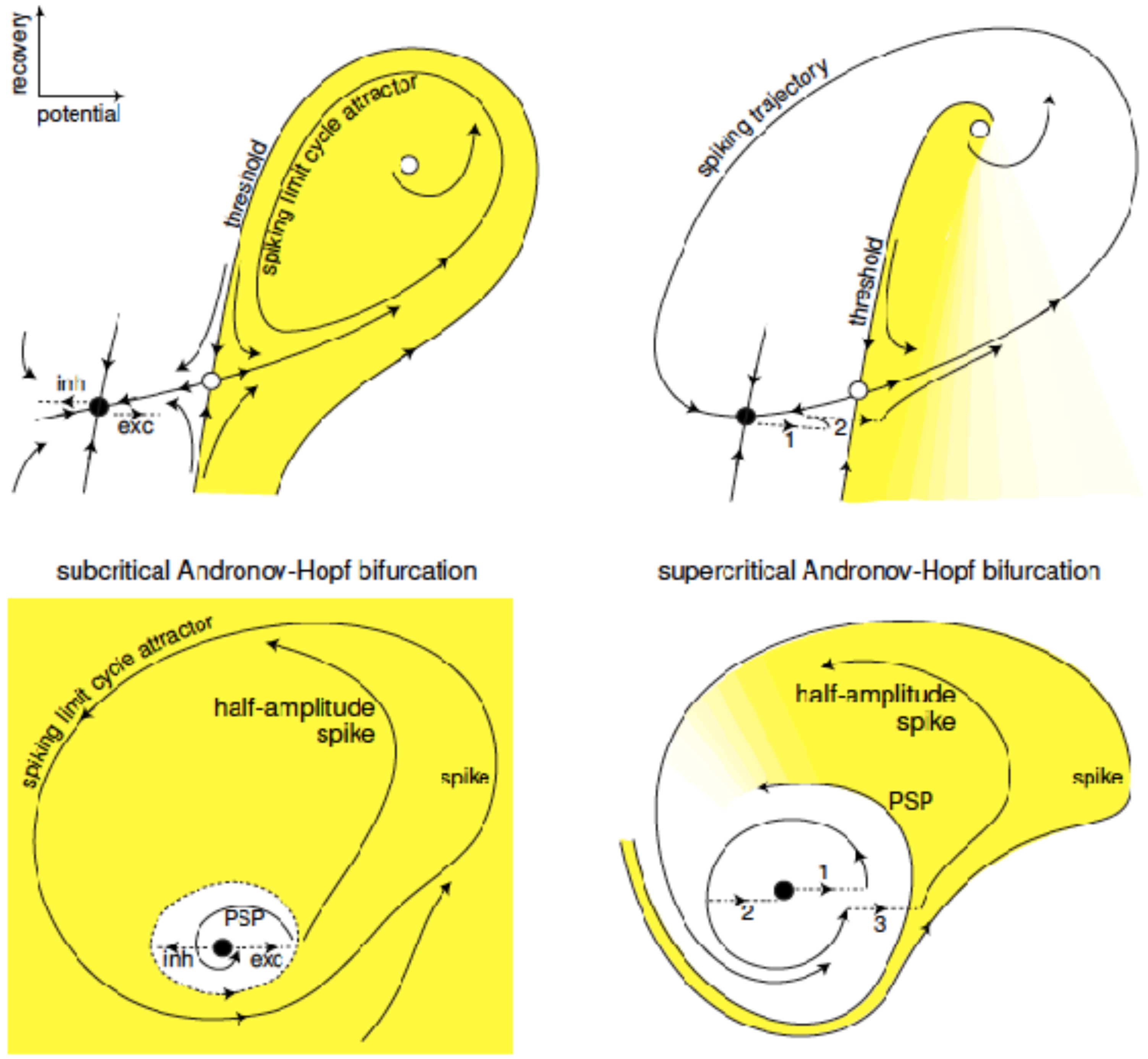

Figure 1.15: The geometry of phase portraits of excitable systems near 4 bifurcations can explain many neuro-computational properties (see Sect. 1.2.4 for detail).