Introduction to Computational Neuroscience

Biol698
Math635
Biol498
Math430

Bibliography:

"Mathematical Foundations of Neuroscience", by G. B. Ermentrout & D. H. Terman - Springer (2010), 1st edition. ISBN 978-0-387-87707-5

* "Foundations of Cellular Neurophysiology", by Daniel Johnston and Samuel M.-S. Wu. The MIT Press, 1995. ISBN 0-262-10053-3

* "Dynamical Systems in Neuroscience: The Geometry of Excitability and Bursting", by Eugene M. Izhikevich. The MIT Press, 2007. ISBN 0-262-09043-8

* "Biophysics of Computation - Information processing in single neurons", by Christof Koch. Oxford University Press, 1999. ISBN 0-19-510491-9

* "Theoretical Neuroscience: Computational and Mathematical Modeling of Neural Systems", by Peter Dayan and Larry F. Abbott. The MIT Press,2001. ISBN 0-262-04199-5

Figures were taken from some of these books

The passive membrane equation

- The passive membrane equation (review)
- Solutions to the passive membrane equation: constant currents
- Solutions to the passive membrane equation: square current pulses
- Solutions to the passive membrane equation: sinusoidal currents

Passive membrane equation

$$C\frac{dV}{dt} = -G_L(V - E_L) + I_{inj}(t)$$

$$C\frac{dV}{dt} = -G_L(V - E_L) + I_{app}(t)$$

$\tau = R\,C$ (time constant)

$$\tau\frac{dV}{dt} = -V + E_L + I_{inj}(t)R$$

Passive membrane equation

$$\tau \frac{dV}{dt} = -V + E_L + I_{inj}(t)R$$

τ = R C   (time constant)

Units:

- V = J / C
- F = C / V
- A = C / sec
- Ω = V / A
- Cal = 4.184 J

R: gas constant (1.98 cal/°K-mol)
F: Faraday's constant (96,480 C/mol))

Passive membrane equation

$$\tau \frac{dV}{dt} = -V + E_L + R\, I_{app}(t)$$

$\tau = R\,C$ (time constant)

$$V(t) = E_L + (V_0 - E_L)\, e^{-t/\tau} + \frac{e^{-t/\tau}}{\tau} R \int_0^t I_{app}(s)\, e^{s/\tau}\, ds$$

Passive membrane equation

- $I_{app} = I_0$ (const) $V_\infty = R\, I_0$

$$V(t) = \boxed{V_\infty + E_L} + (V_0 - E_L - V_\infty)\, e^{-t/\tau}$$

steady state

- $V_0 = E_L$

$$V(t) = \boxed{V_\infty + E_L} - V_\infty\, e^{-t/\tau}$$

steady state

Passive membrane equation

- $I_{app} = I_0$ (const) $V_\infty = R\, I_0$ (const)

$$V(t) = \boxed{V_\infty + E_L} + (V_0 - E_L - V_\infty)\, e^{-t/\tau}$$

steady state

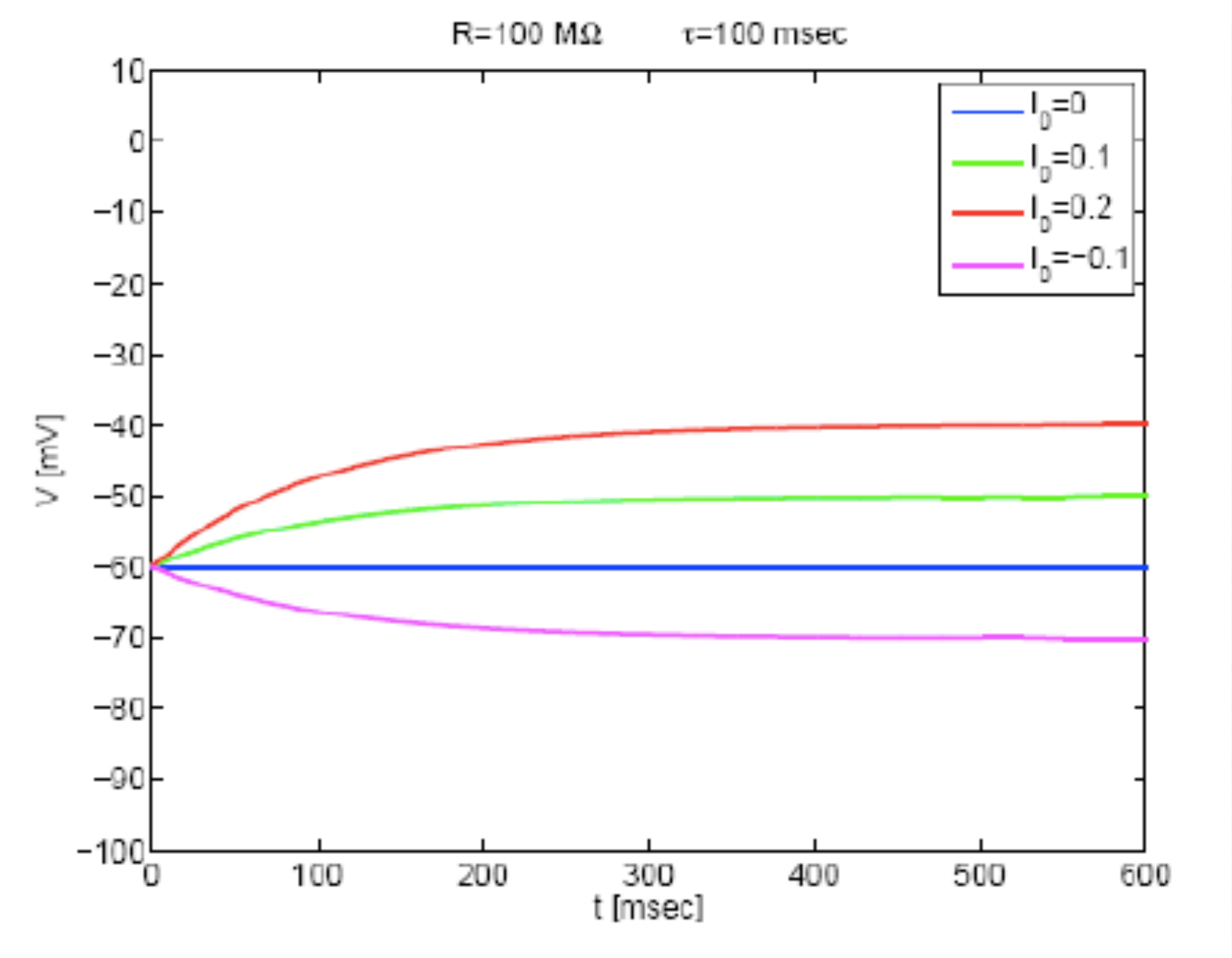
- $I_0 = \alpha$ nA
- $R = 100$ MΩ

$V_\infty = \alpha * 10^2$ mV

We will use the following units: mV, nA, msec

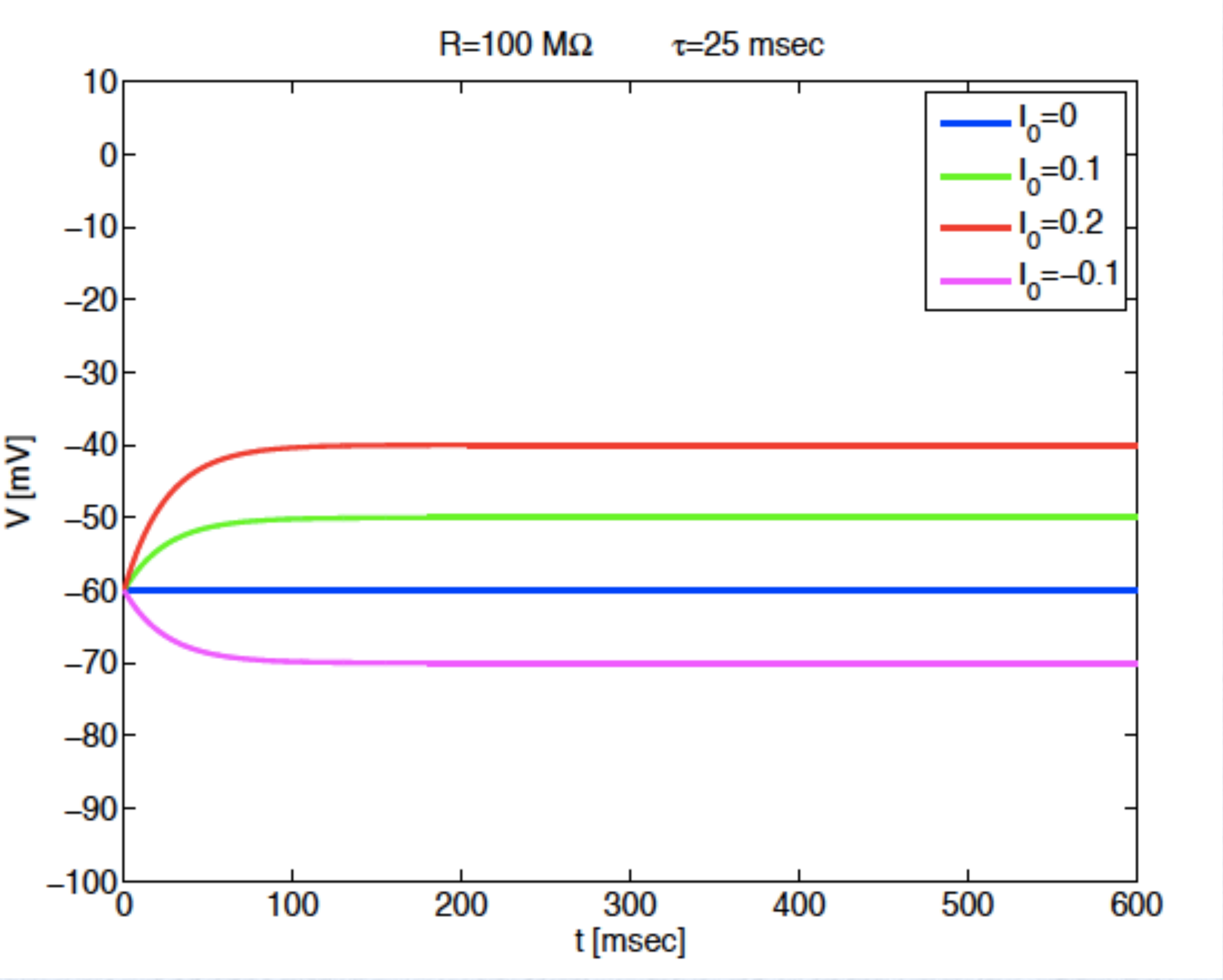
Passive membrane equation

$$V(t) = \boxed{V_\infty + E_L} - V_\infty\, e^{-t/\tau}$$



Passive membrane equation

$$V(t) = \boxed{V_\infty + E_L} - V_\infty \; e^{-t/\tau}$$



Passive membrane equation

- $I_{app}(t) = I_0 \, H(t - t_i) \;\; H(t_f - t)$

  H(t): Heaviside function

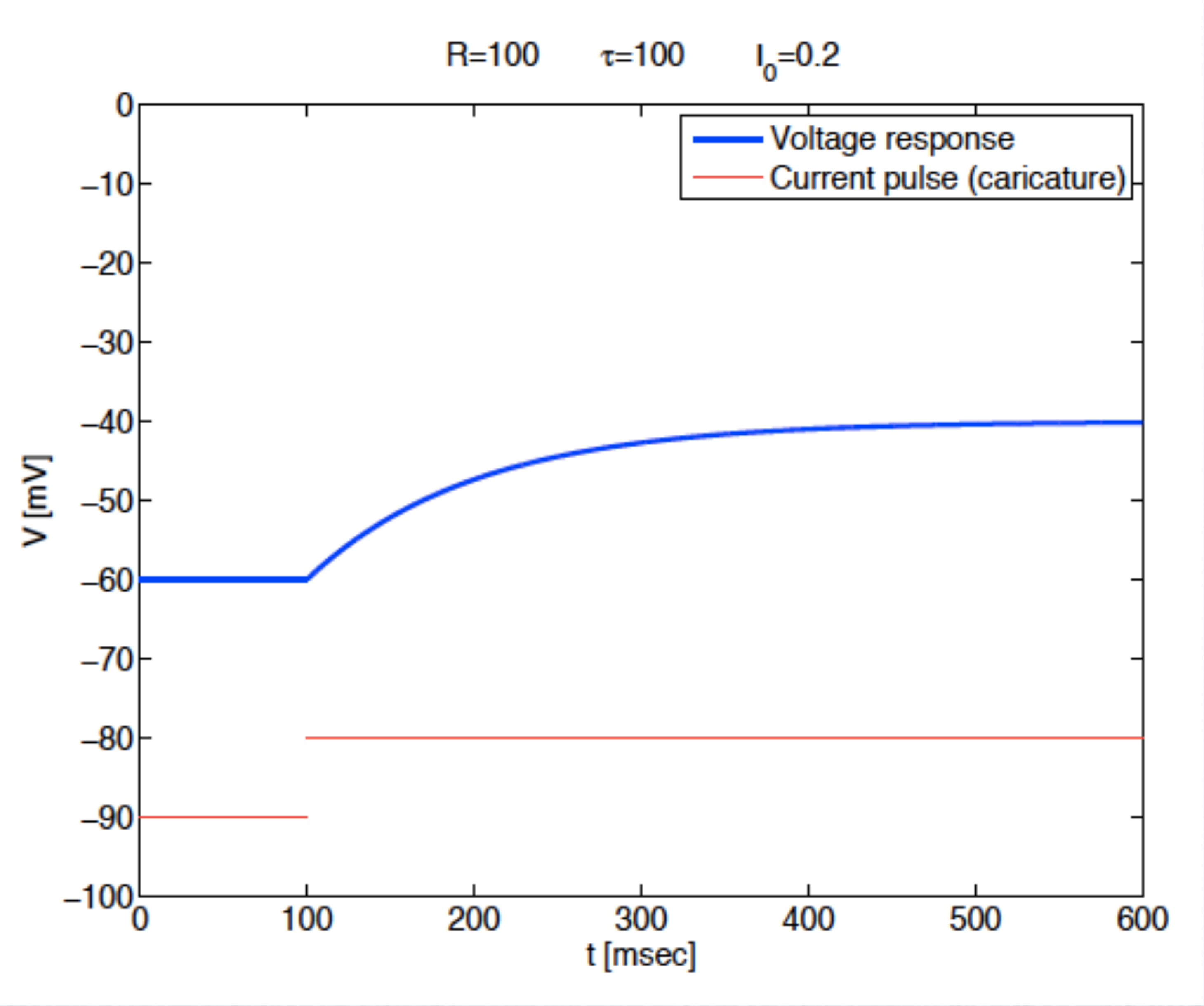
  $I_{app}(t)$: square pulse of current starting at $t = t_i$ and ending at $t = t_f$

✤ $V_0 = E_L$
✤ $V_\infty = R \, I_0$ (const)

$$V(t) - E_L = V_\infty \left[H(t - t_i)\left(1 - e^{-\frac{t - t_i}{\tau}}\right) - H(t - t_f)\left(1 - e^{-\frac{t - t_f}{\tau}}\right) \right]$$

Passive membrane equation

$$V(t) - E_L = V_\infty \left[H(t - t_i)\left(1 - e^{-\frac{t - t_i}{\tau}}\right) - H(t - t_f)\left(1 - e^{-\frac{t - t_f}{\tau}}\right) \right]$$



Passive membrane equation

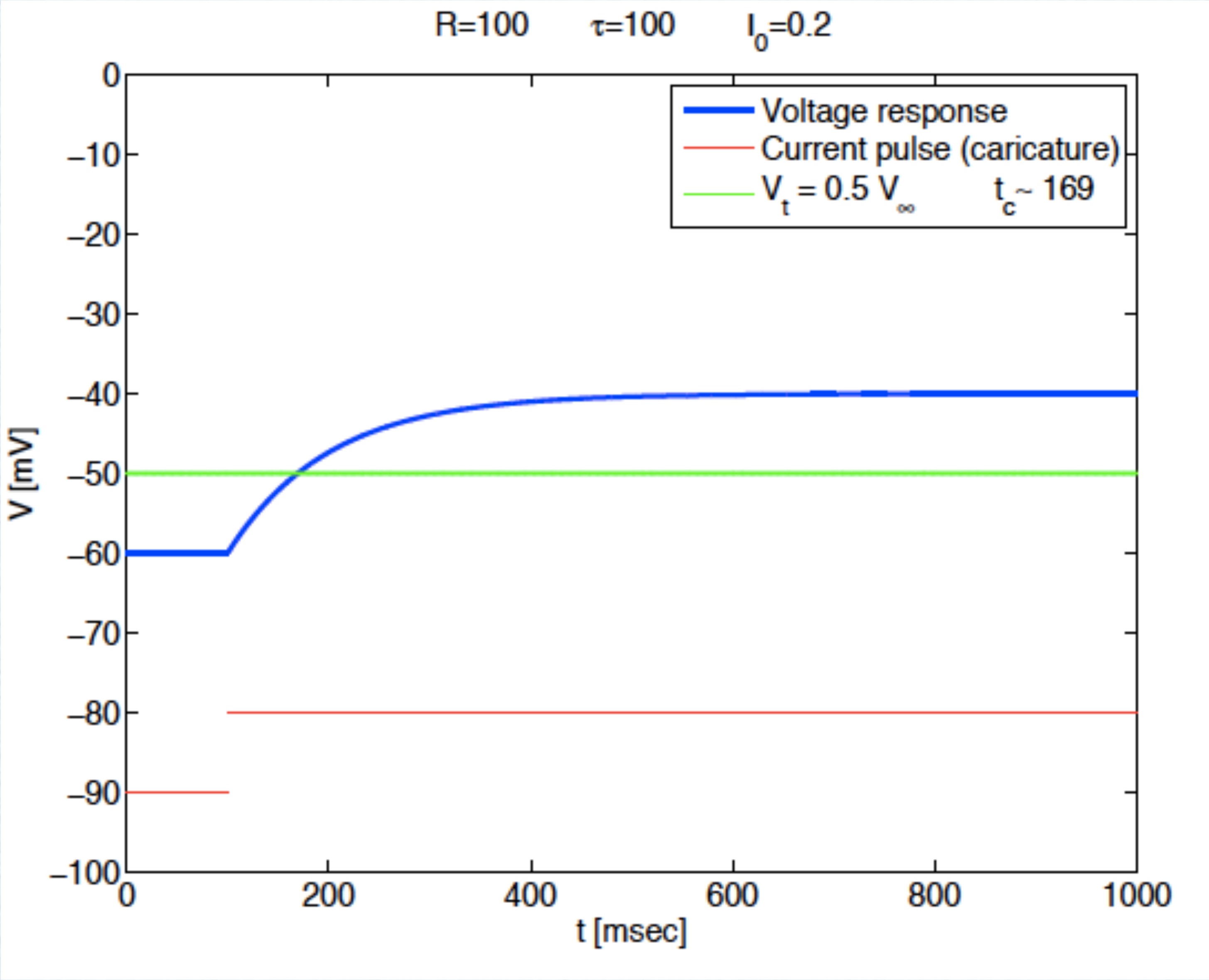
- Calculation of the time t_c that takes V to reach the value V_t

$$V(t) - E_L = V_\infty H(t - t_i)\left(1 - e^{-\frac{t - t_i}{\tau}}\right)$$

$$V_t = \eta V_\infty \quad \rightarrow \quad t_c = t_i - \tau \ln(1 - \eta)$$

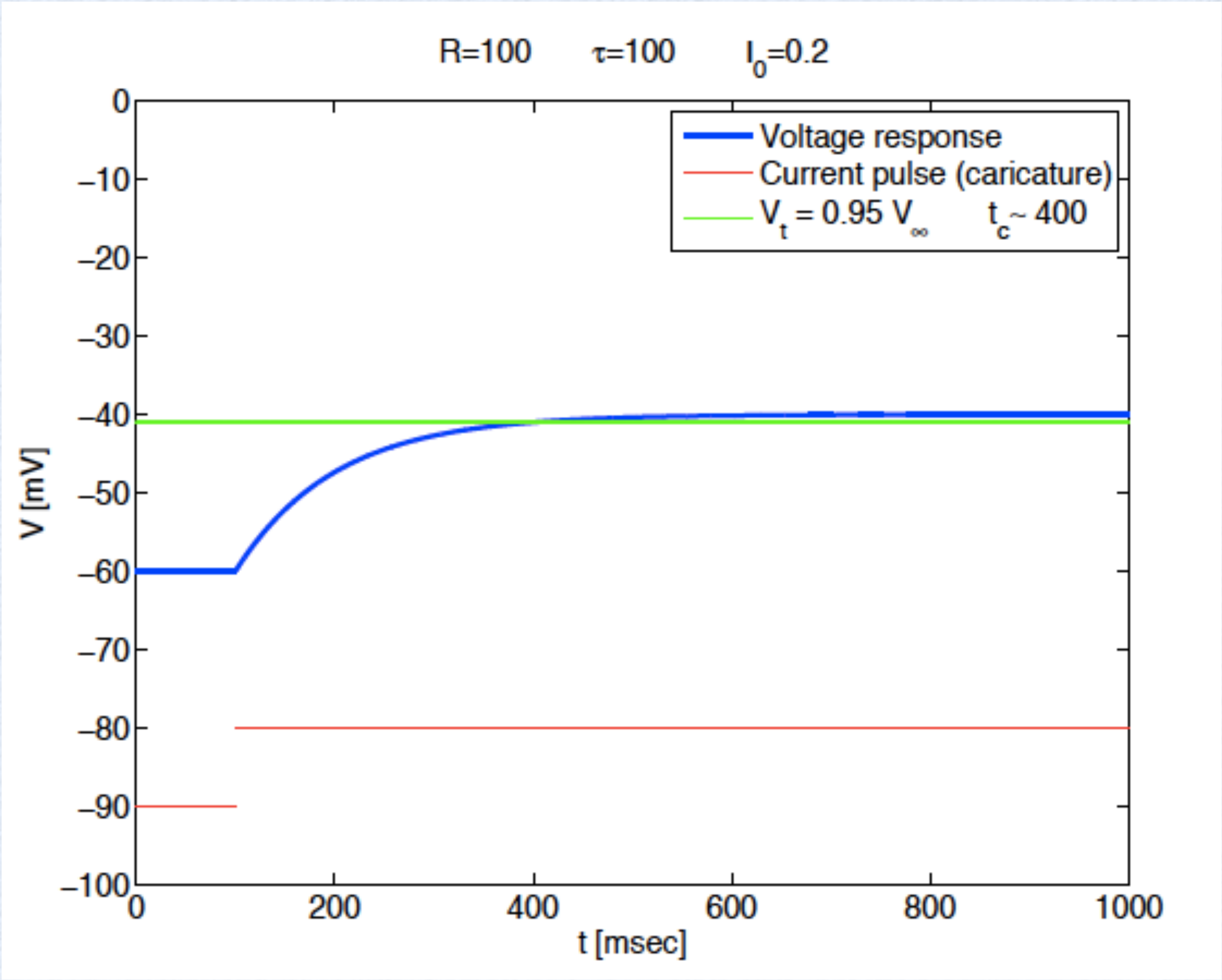
Passive membrane equation

$$V(t) - E_L = V_\infty \left[H(t - t_i)\left(1 - e^{-\frac{t - t_i}{\tau}}\right) - H(t - t_f)\left(1 - e^{-\frac{t - t_f}{\tau}}\right)\right]$$



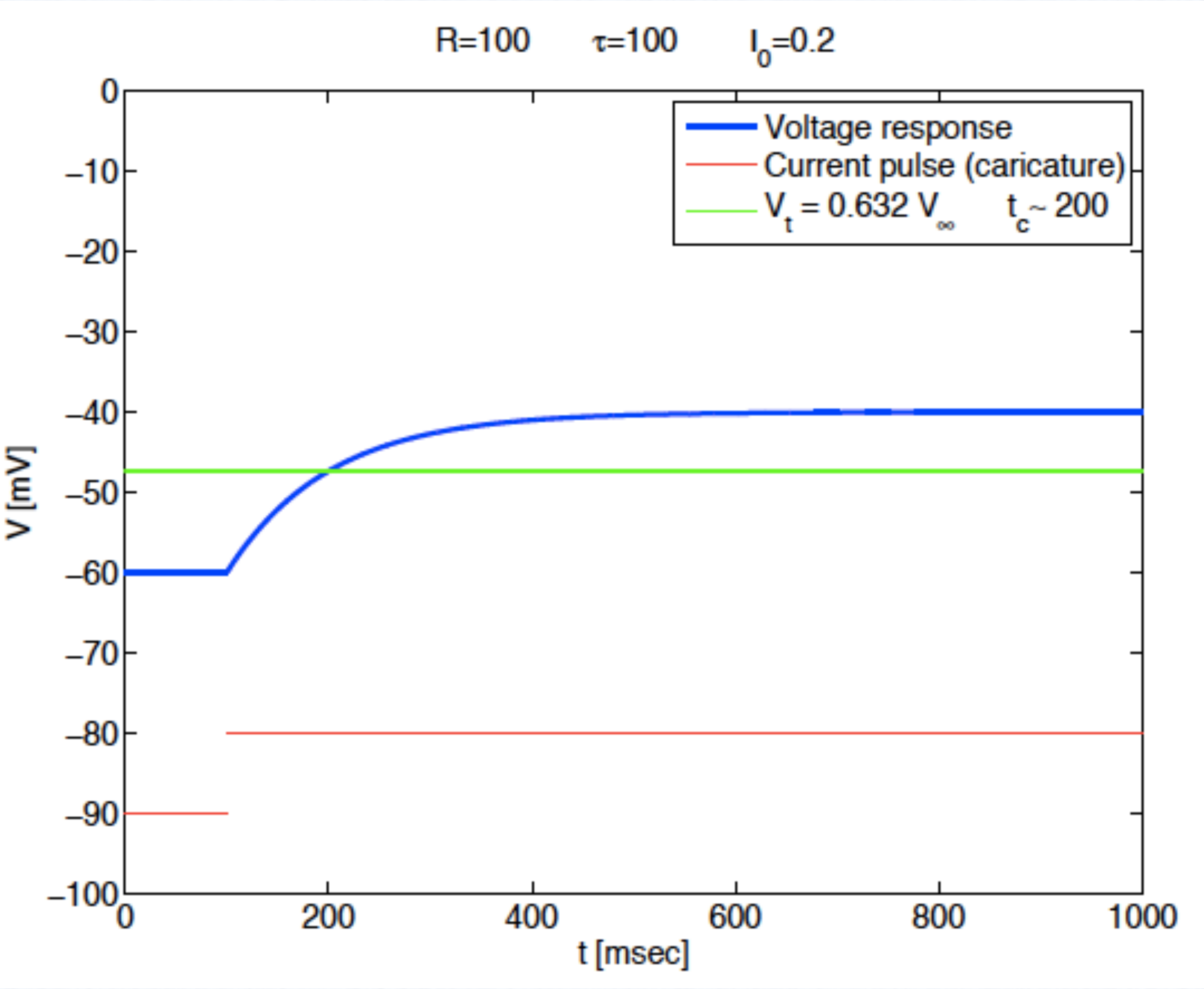
Passive membrane equation

$$V(t) - E_L = V_\infty \left[H(t - t_i)\left(1 - e^{-\frac{t - t_i}{\tau}}\right) - H(t - t_f)\left(1 - e^{-\frac{t - t_f}{\tau}}\right)\right]$$



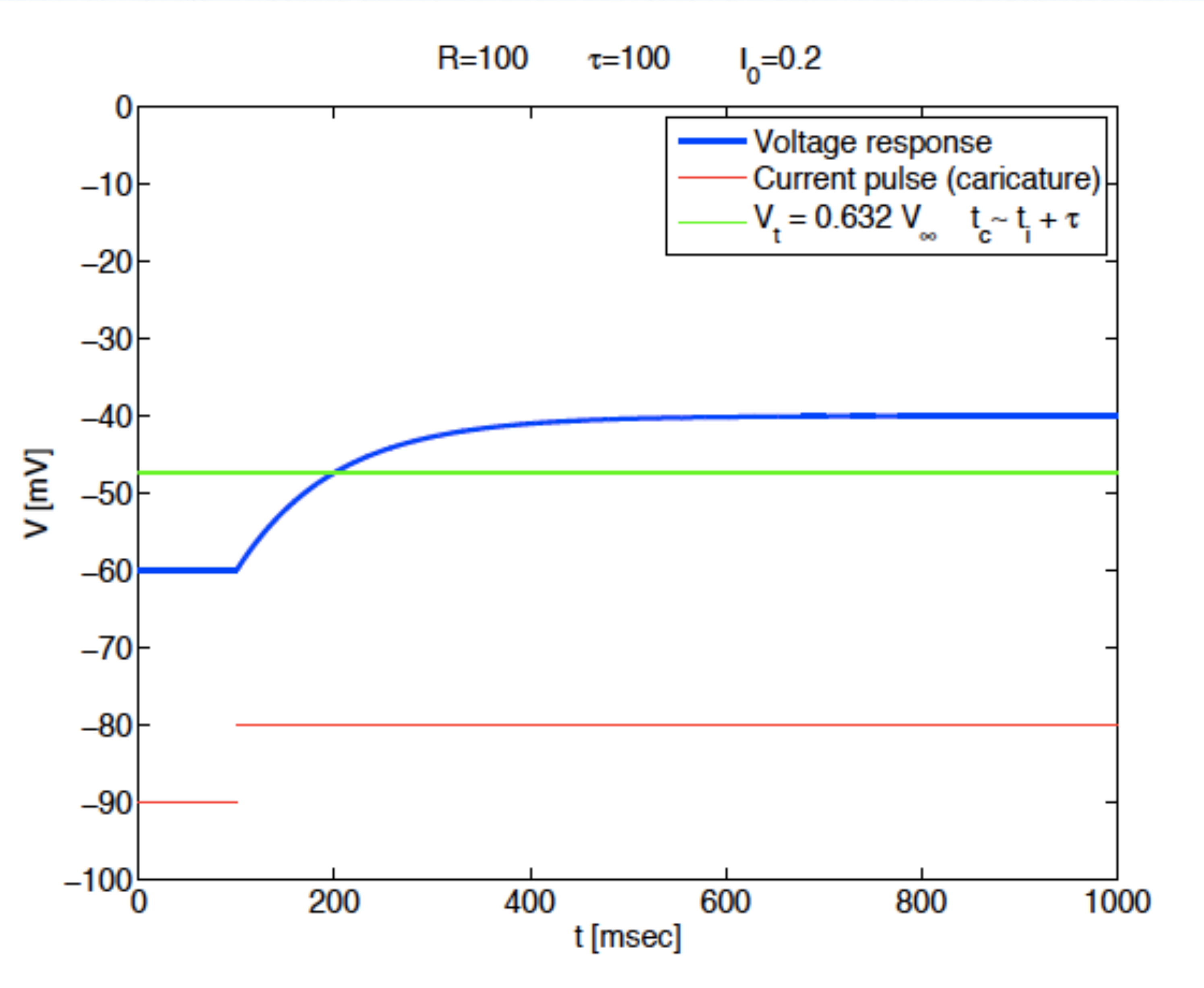
Passive membrane equation

$$V(t) - E_L = V_\infty \left[H(t - t_i)\left(1 - e^{-\frac{t - t_i}{\tau}}\right) - H(t - t_f)\left(1 - e^{-\frac{t - t_f}{\tau}}\right) \right]$$



Passive membrane equation

$$V(t) - E_L = V_\infty \left[H(t - t_i)\left(1 - e^{-\frac{t - t_i}{\tau}}\right) - H(t - t_f)\left(1 - e^{-\frac{t - t_f}{\tau}}\right) \right]$$



Passive membrane equation

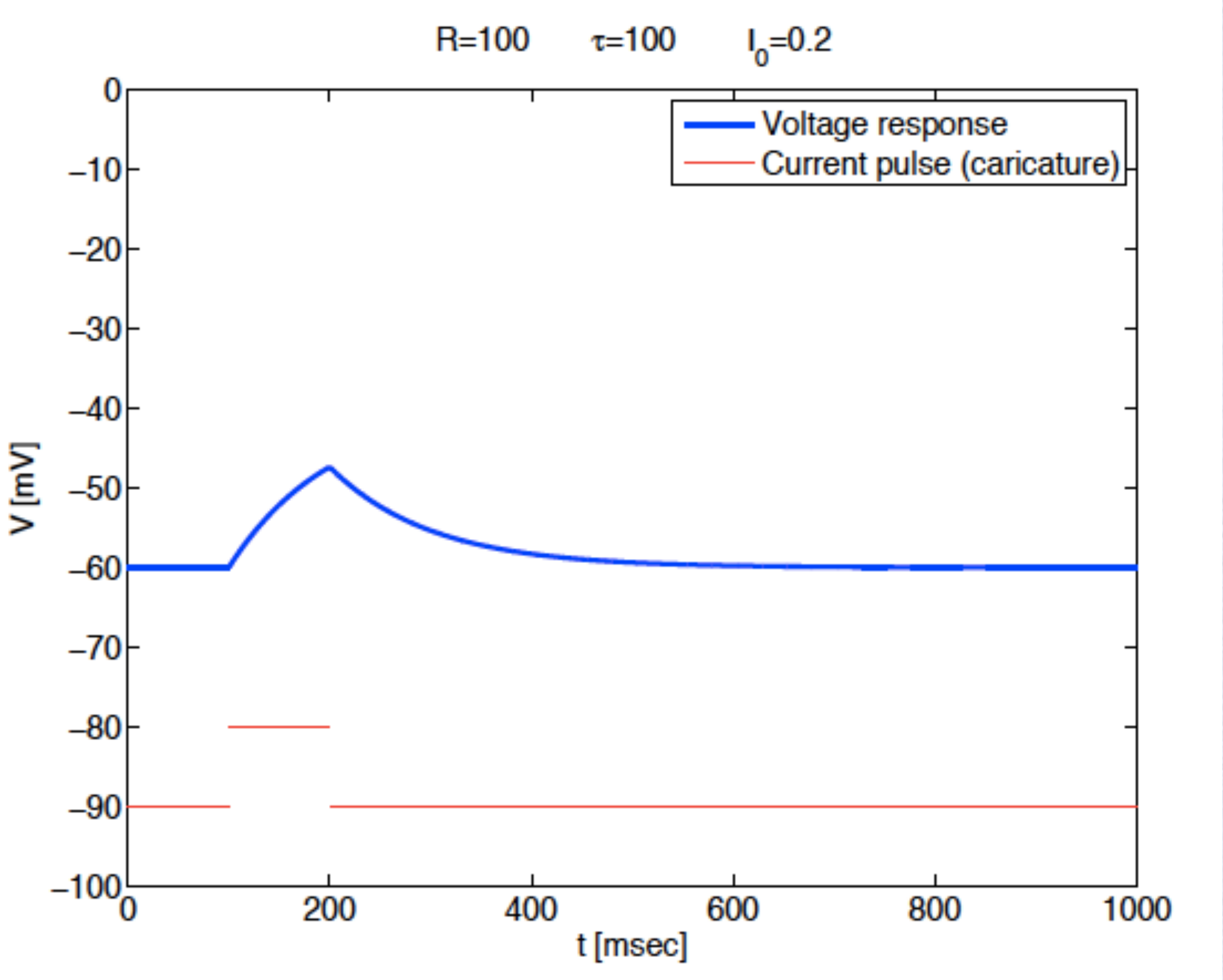
- The time it takes $V(t)$ to rise to 63% of its final value is the time constant $\tau = RC$

- Given experimental data we can calculate

  - E_L Resting potential in the absence of I_{app}
  - R From steady state V_∞ (constant I_{app})
  - C From $\tau = RC$

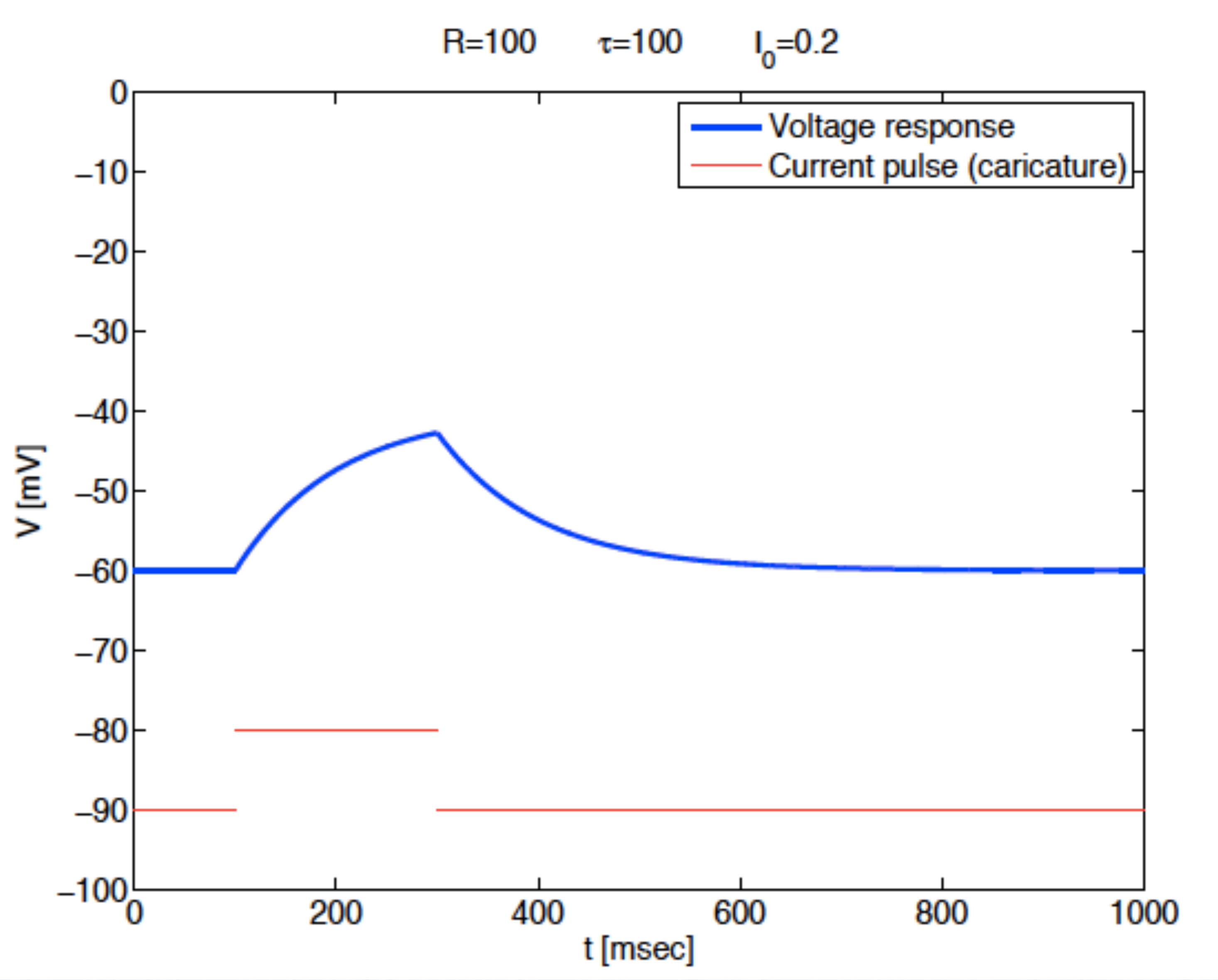
Passive membrane equation

$$V(t) - E_L = V_\infty \left[H(t - t_i)\left(1 - e^{-\frac{t - t_i}{\tau}}\right) - H(t - t_f)\left(1 - e^{-\frac{t - t_f}{\tau}}\right)\right]$$



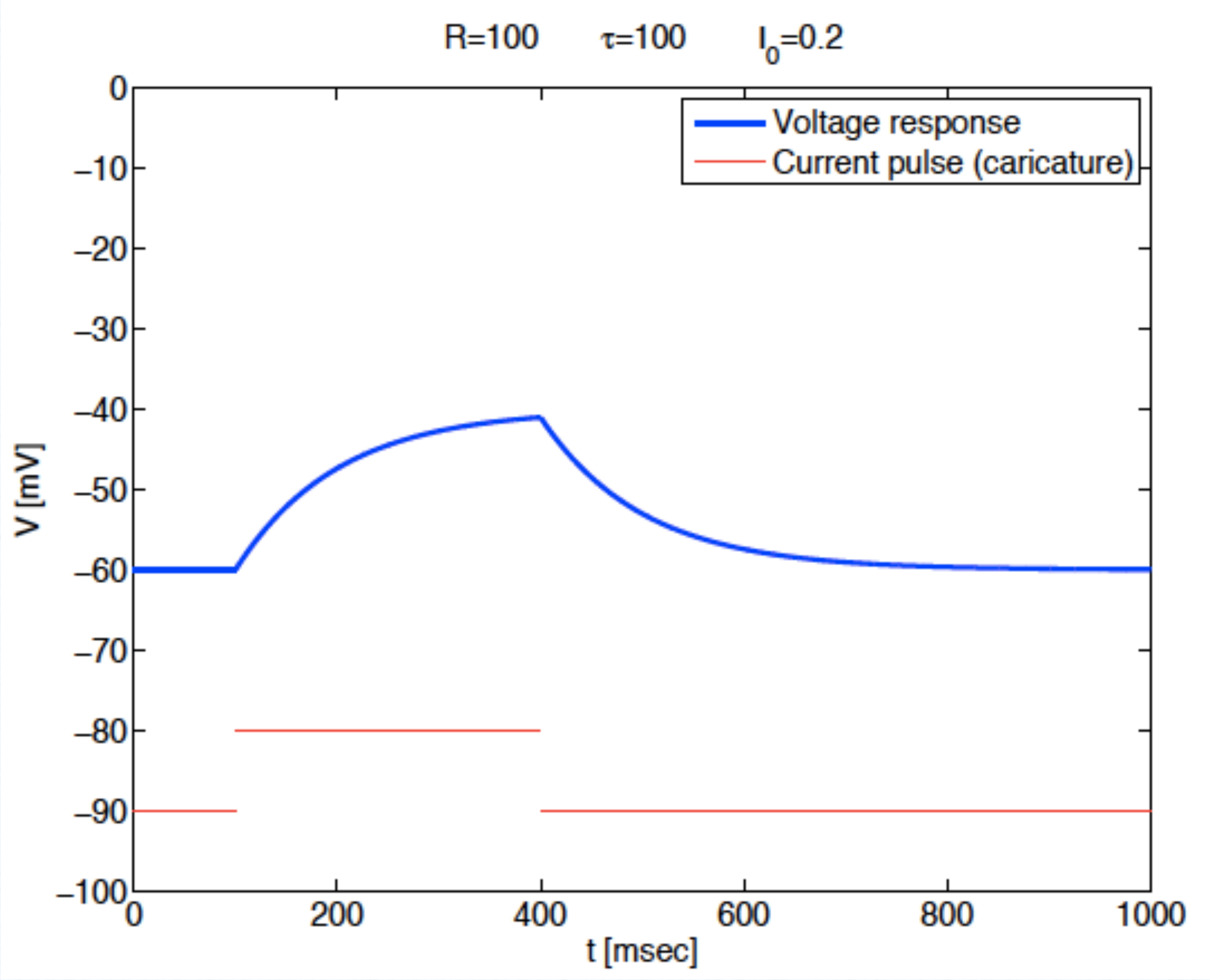
Passive membrane equation

$$V(t) - E_L = V_\infty \left[H(t - t_i)\left(1 - e^{-\frac{t - t_i}{\tau}}\right) - H(t - t_f)\left(1 - e^{-\frac{t - t_f}{\tau}}\right) \right]$$



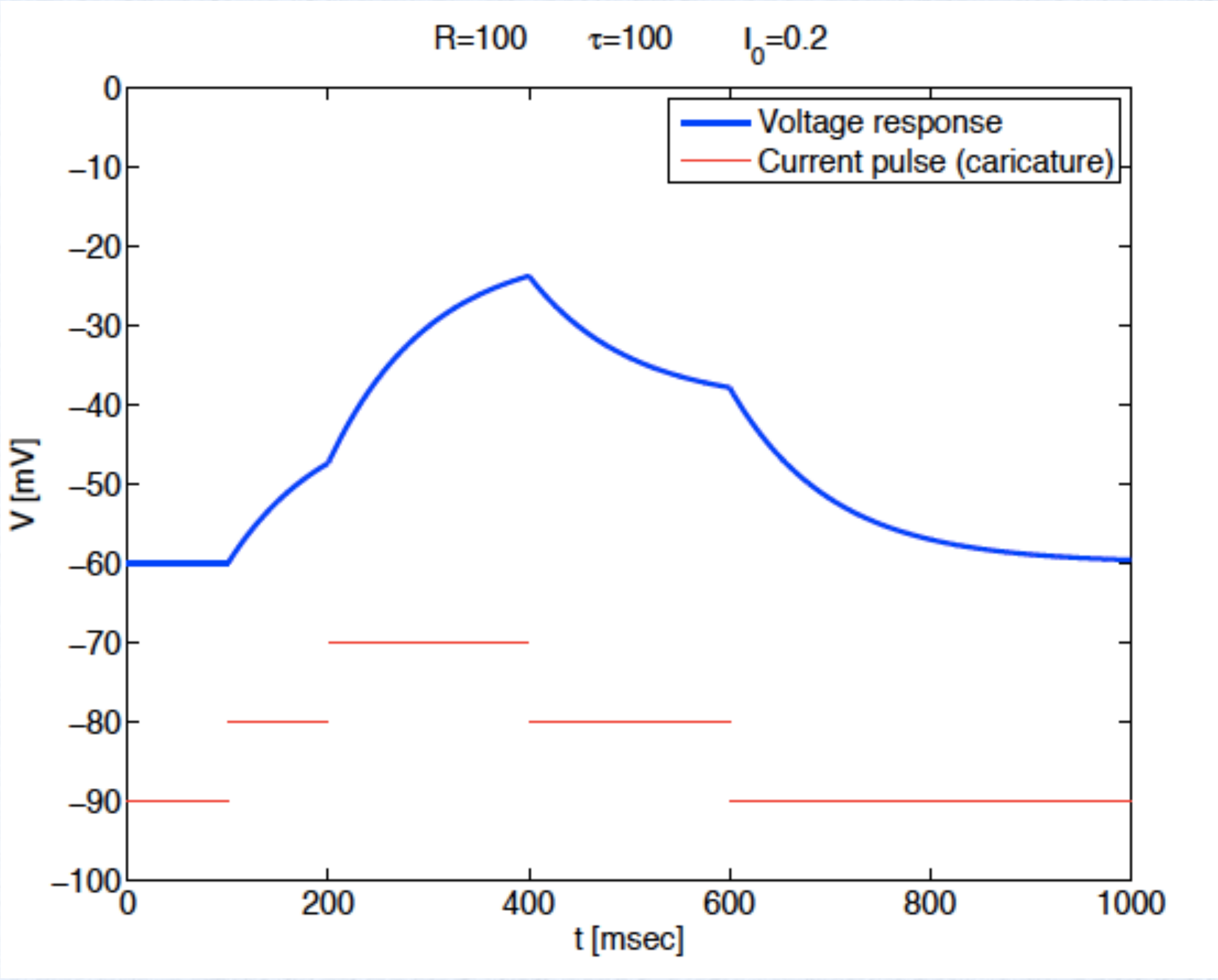
Passive membrane equation

$$V(t) - E_L = V_\infty \left[H(t - t_i)\left(1 - e^{-\frac{t - t_i}{\tau}}\right) - H(t - t_f)\left(1 - e^{-\frac{t - t_f}{\tau}}\right) \right]$$



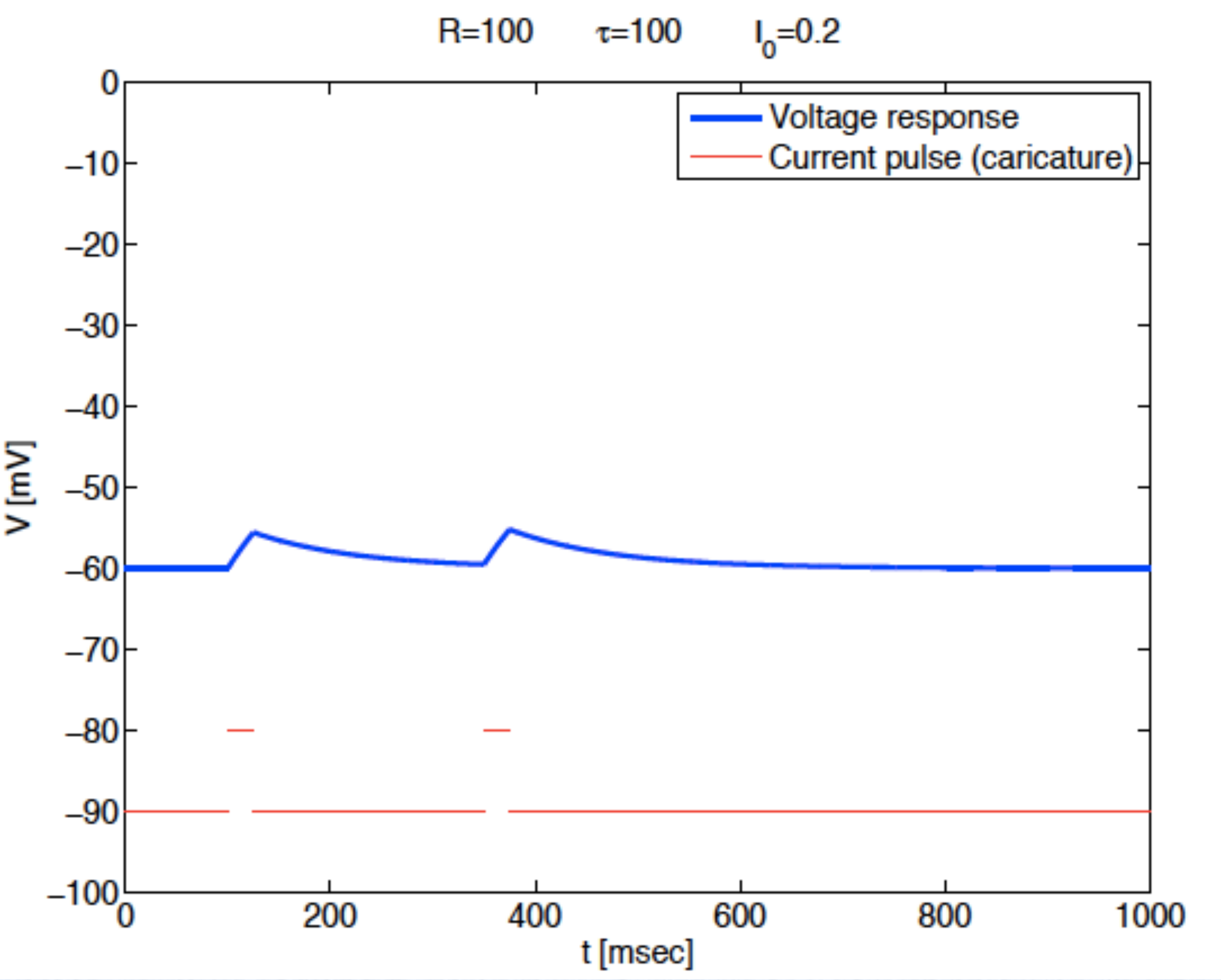
Passive membrane equation

$$V(t) - E_L = V_\infty \left[H(t-t_i)\left(1-e^{-\frac{t-t_i}{\tau}}\right) - H(t-t_f)\left(1-e^{-\frac{t-t_f}{\tau}}\right) \right]$$



Passive membrane equation

$$V(t) - E_L = V_\infty \left[H(t - t_i)\left(1 - e^{-\frac{t - t_i}{\tau}}\right) - H(t - t_f)\left(1 - e^{-\frac{t - t_f}{\tau}}\right) \right]$$



Passive membrane equation

- $I_{app}(t) = A_\omega \sin(\omega t)$ sinusoidal input current

$$V(t) = E_L + (V_0 - E_L)\, e^{-t/\tau} + \frac{R\,A\,\omega\,\tau}{1+\omega^2\tau^2} e^{-t/\tau} +$$

$$+ \frac{R\,A}{1+\omega^2\tau^2}\left[\sin(\omega t) - \omega\tau\cos(\omega t)\right]$$

$A = A_\omega$

Passive membrane equation

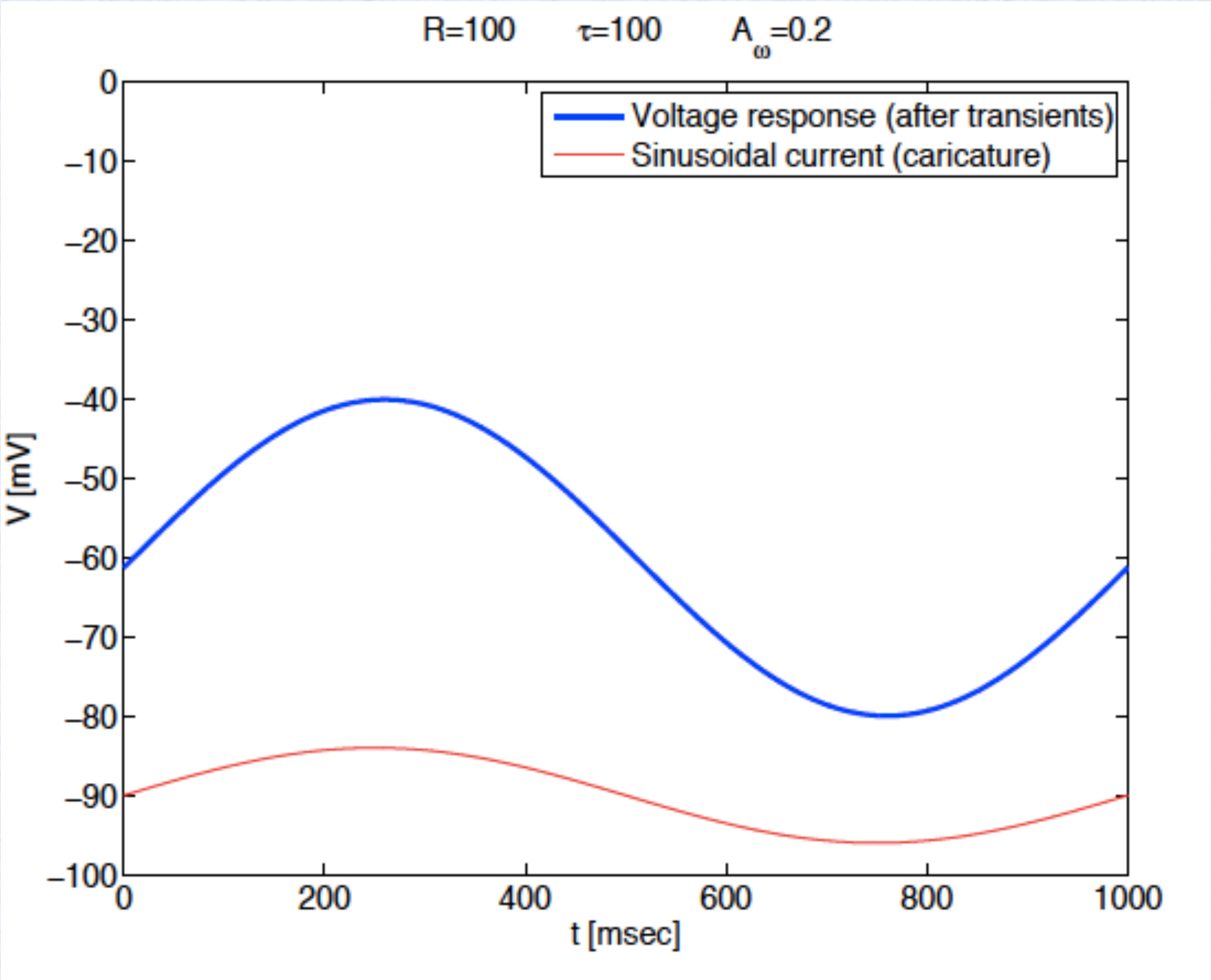
- $I_{app}(t) = A_\omega \sin(\omega t)$ sinusoidal input current

  - $t \rightarrow \infty$ & $V_0 = E_L$

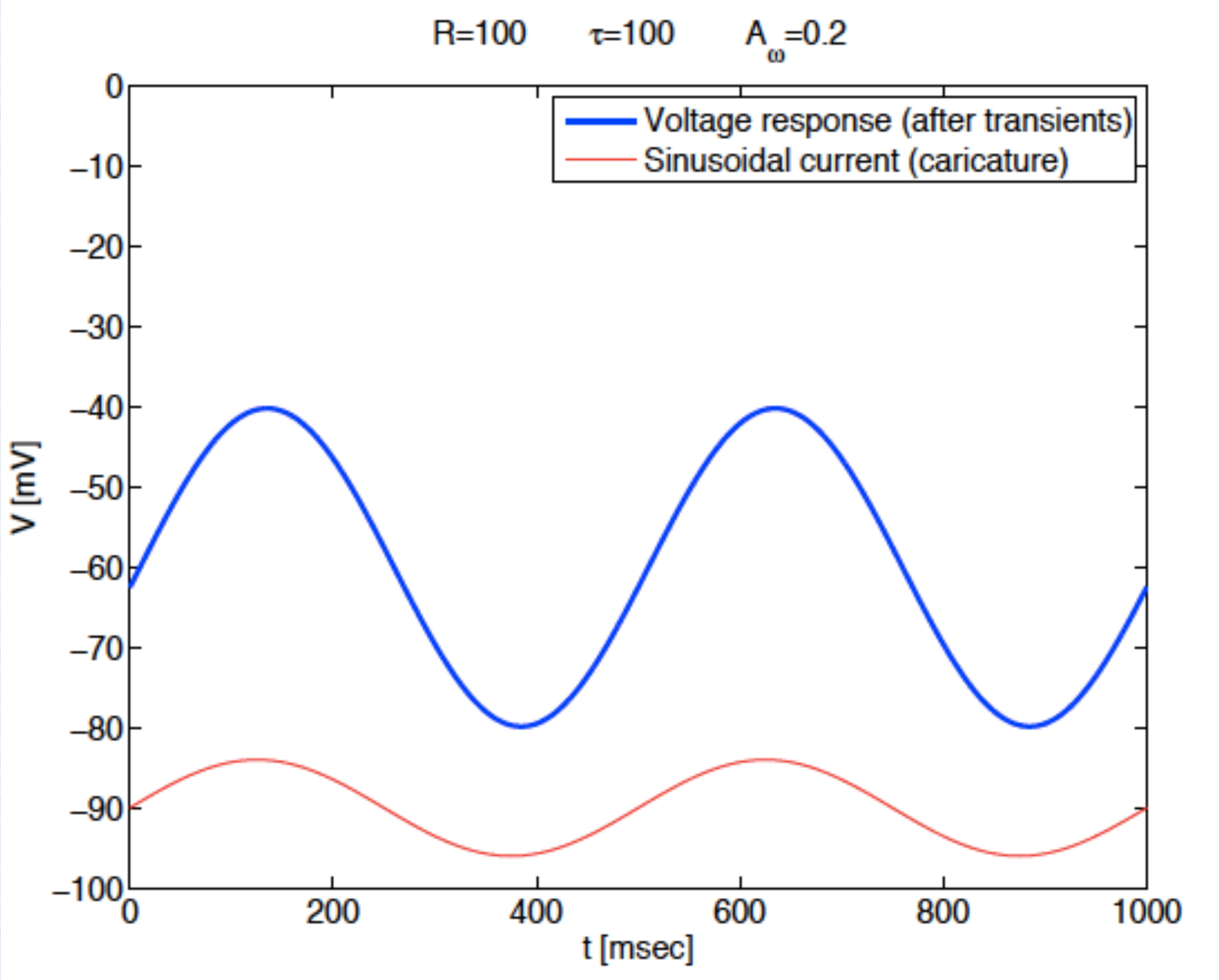
$$V(t) - E_L = \frac{RA}{1+\omega^2\tau^2}\left[\sin(\omega t) - \omega\tau\cos(\omega t)\right]$$

$A = A_\omega$

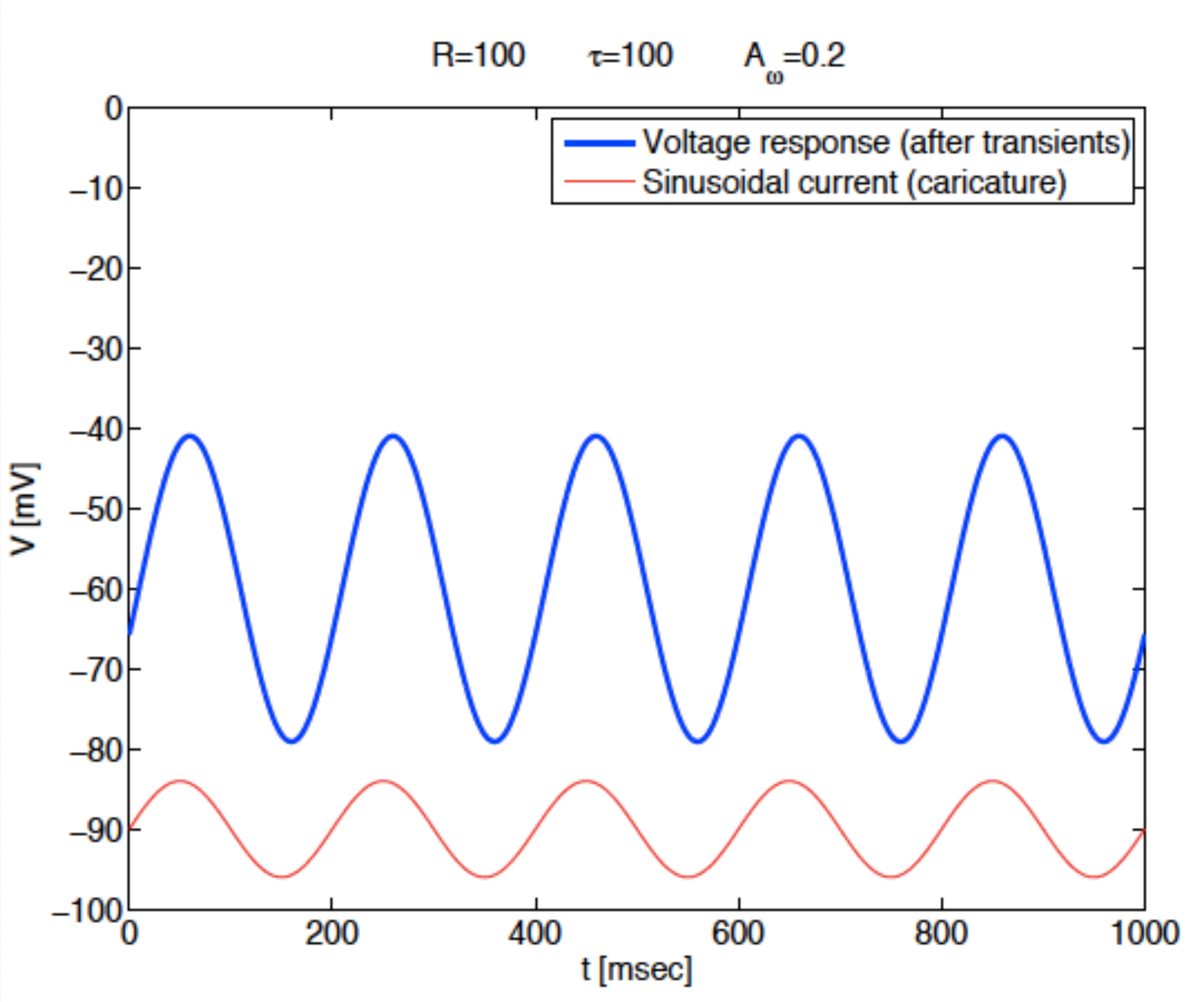
Passive membrane equation



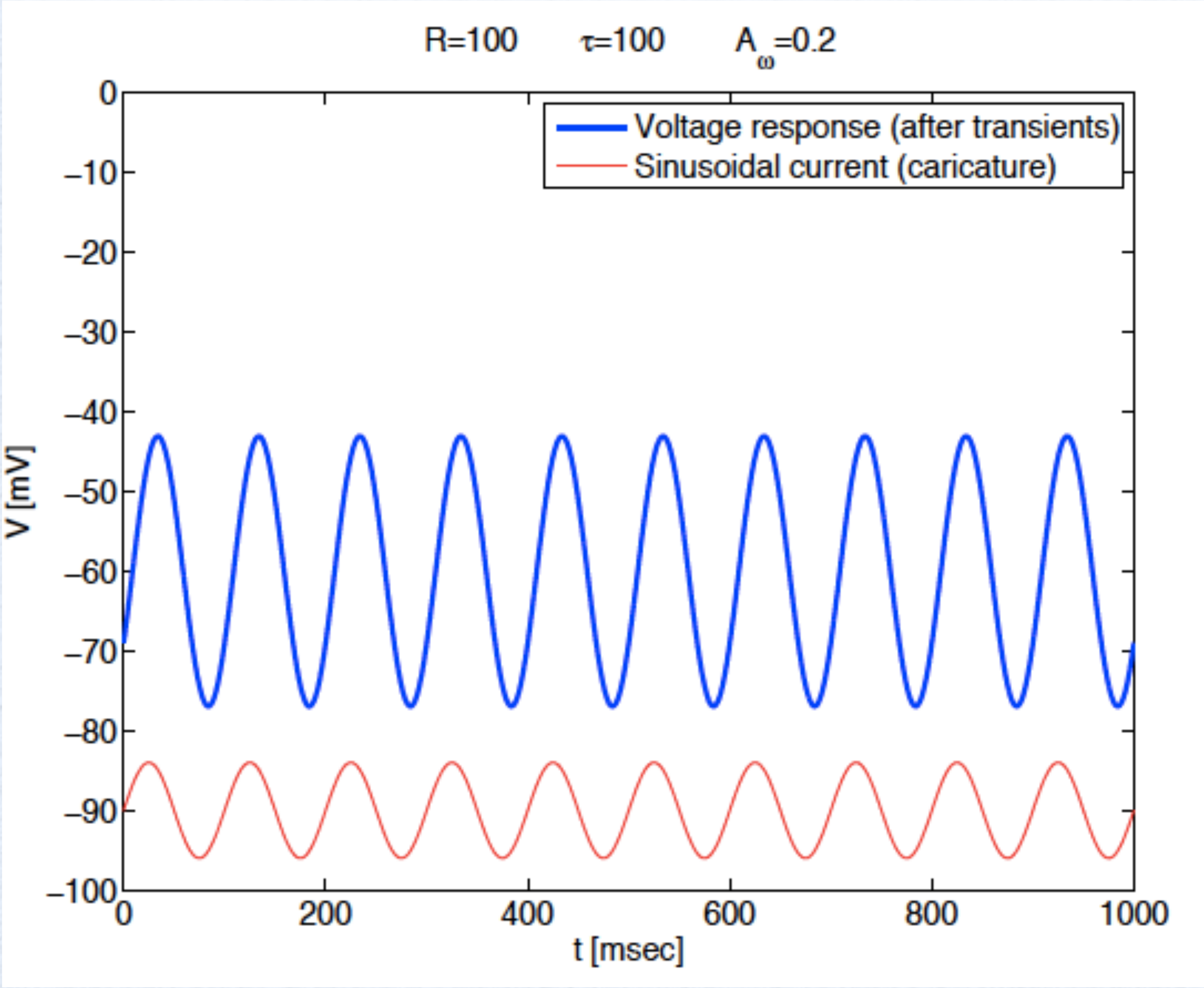
Passive membrane equation



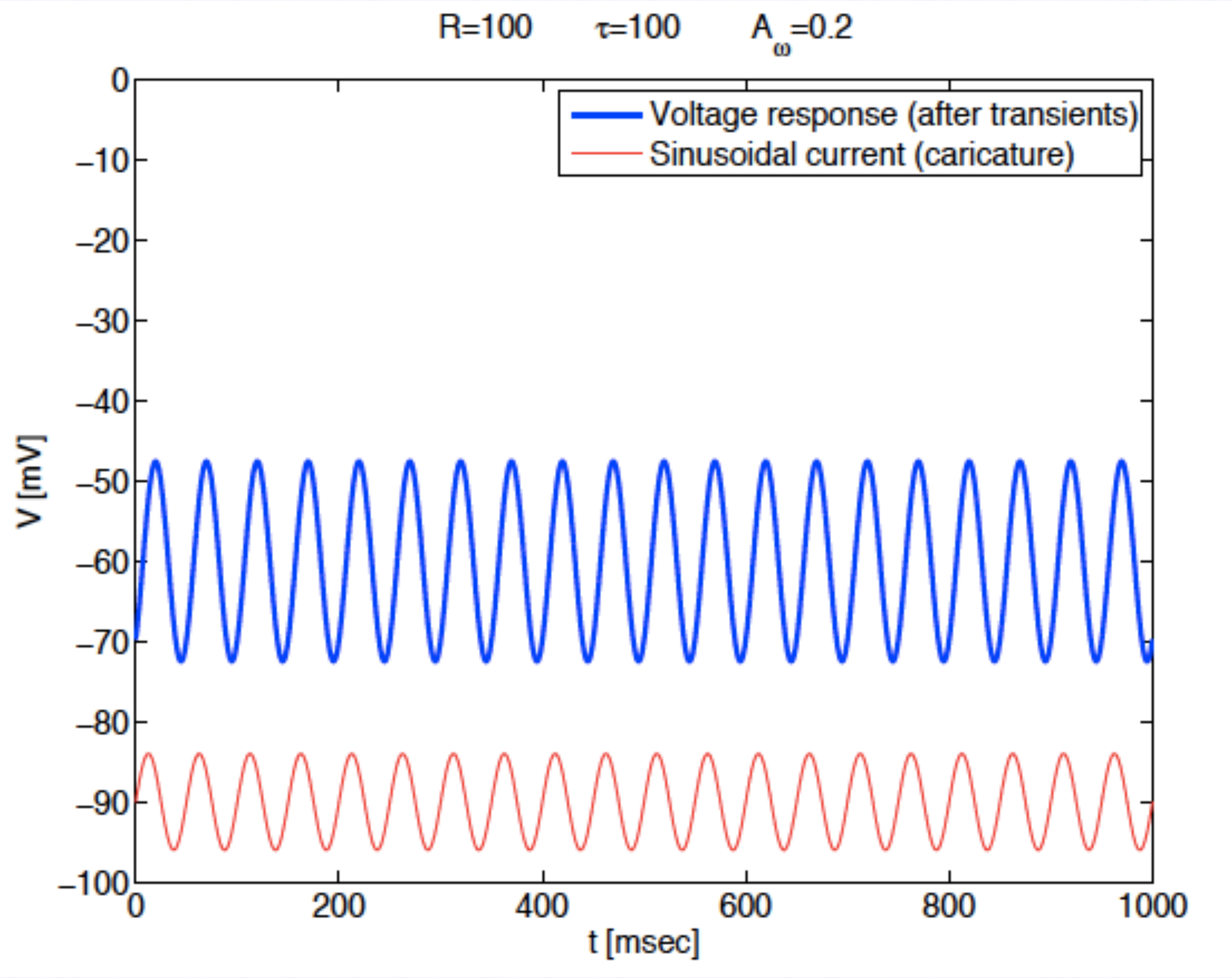
Passive membrane equation



Passive membrane equation

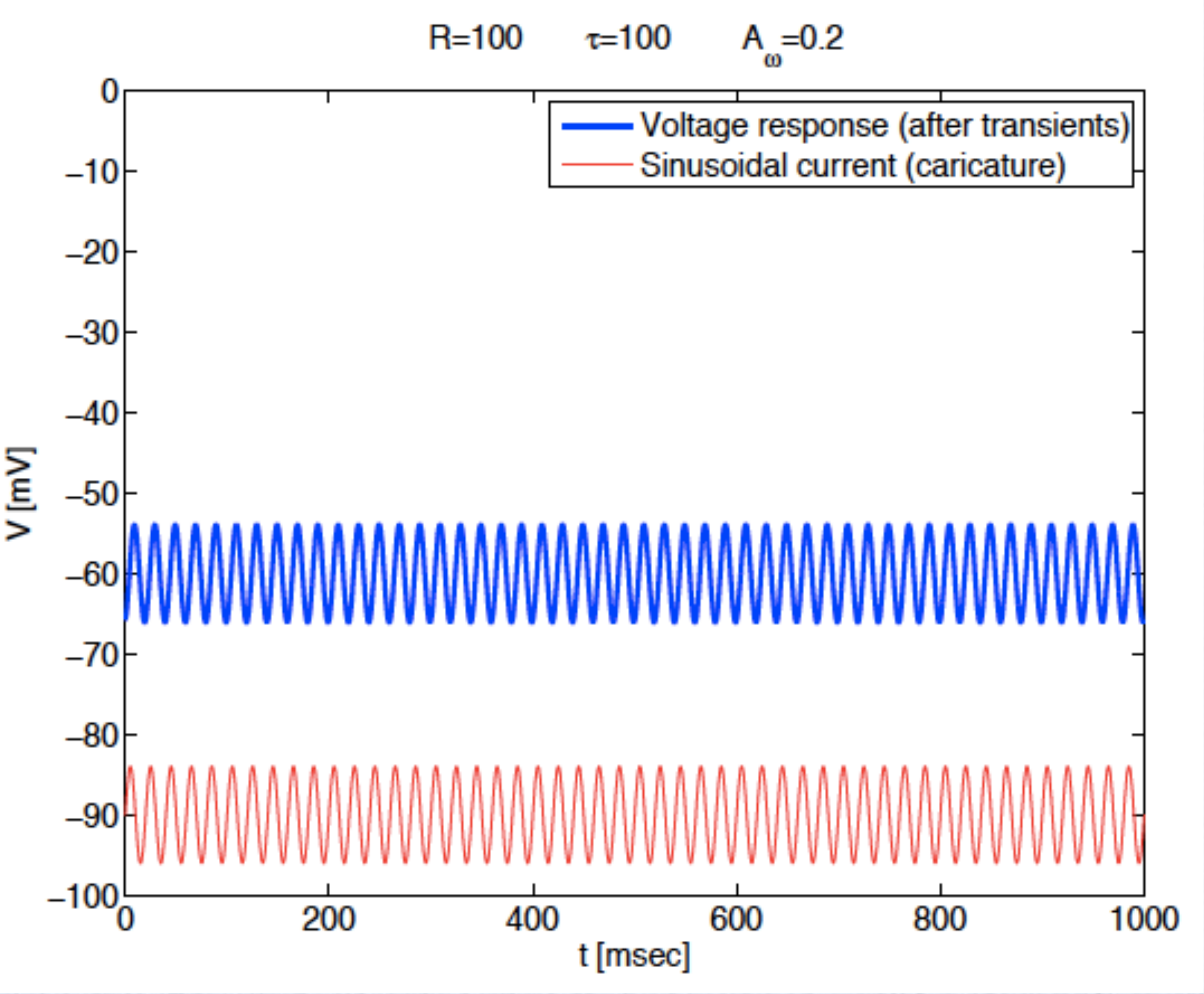


Passive membrane equation

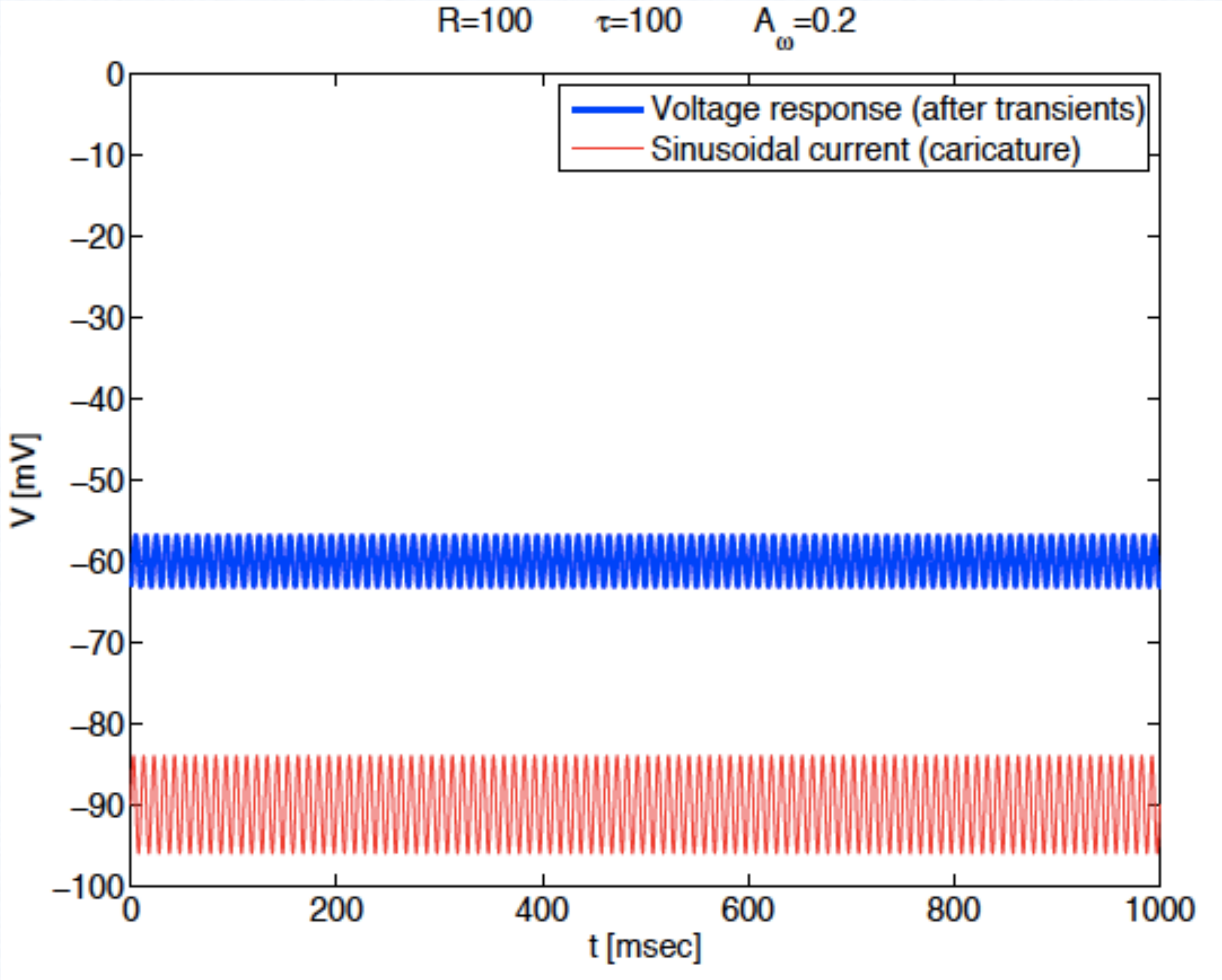


Passive membrane equation

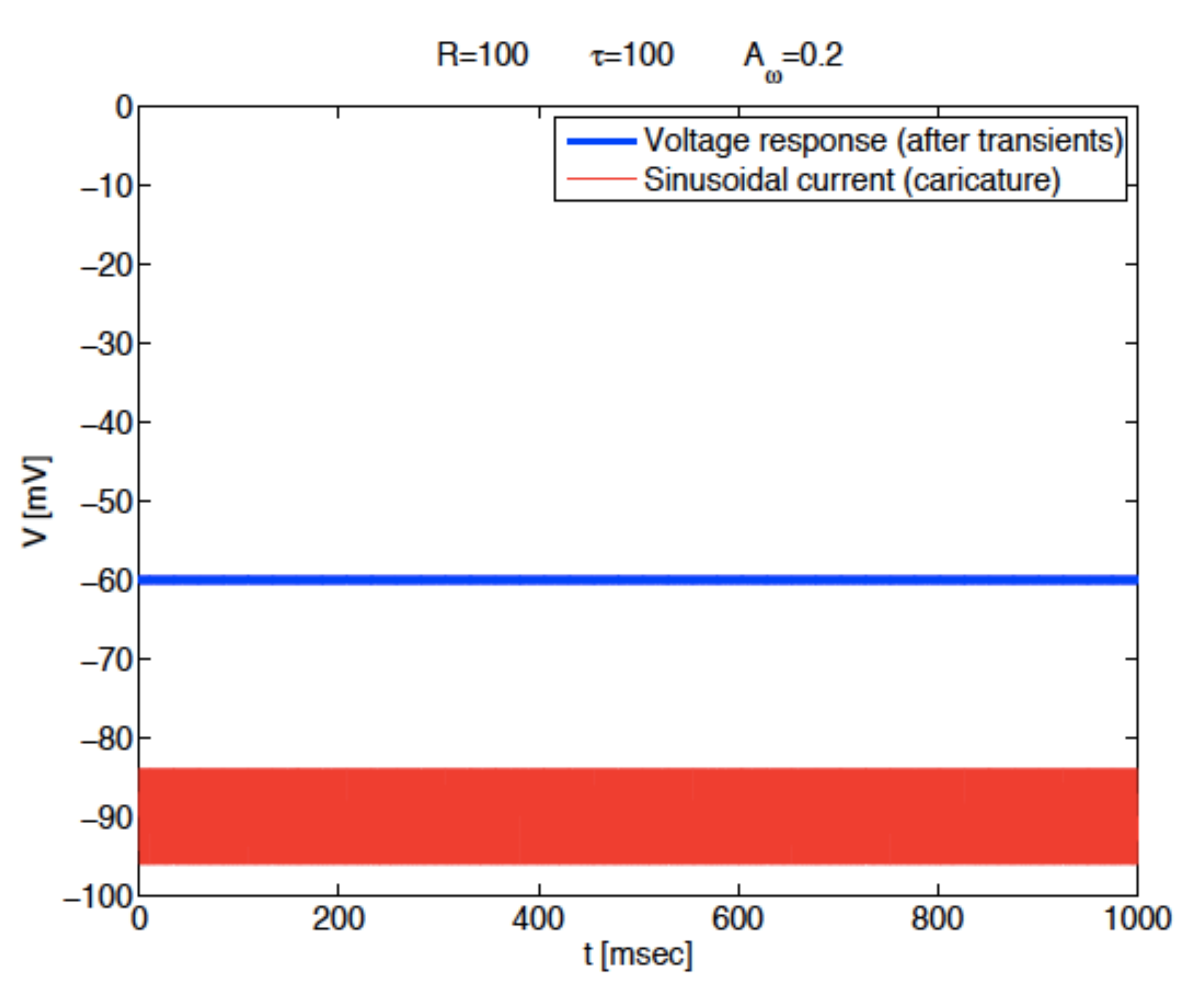
Passive membrane equation



Passive membrane equation



Passive membrane equation



Passive membrane equation

- $I_{app}(t) = A_\omega \sin(\omega t)$ sinusoidal input current

  - $t \to \infty$ & $V_0 = E_L$

$$V(t) - E_L = \frac{RA}{1+\omega^2\tau^2}\left[\sin(\omega t) - \omega\tau\cos(\omega t)\right]$$

$$V(t) - E_L = \boxed{\frac{RA}{\sqrt{1+\omega^2\tau^2}}} \sin(\omega t + \phi)$$

$A = A_\omega$

$$\phi = \arctan(\omega\tau)$$

Passive membrane equation

- $I_{app}(t) = A_\omega \sin(\omega t)$ sinusoidal input current

  - Impedance:

$$\left|Z(\omega)\right| = \frac{R}{\sqrt{1+\omega^2\tau^2}}$$

$$Z(\omega) = \frac{F[V(t)](\omega)}{F[I_{app}(t)](\omega)}$$