Introduction to Computational Neuroscience

Biol 698
Math 635
Biol 498
Math 430

Bibliography:

"Mathematical Foundations of Neuroscience", by G. B. Ermentrout & D. H. Terman - Springer (2010), 1st edition. ISBN 978-0-387-87707-5

* "Foundations of Cellular Neurophysiology", by Daniel Johnston and Samuel M.-S. Wu. The MIT Press, 1995. ISBN 0-262-10053-3

* "Dynamical Systems in Neuroscience: The Geometry of Excitability and Bursting", by Eugene M. Izhikevich. The MIT Press, 2007. ISBN 0-262-09043-8

* "Biophysics of Computation - Information processing in single neurons", by Christof Koch. Oxford University Press, 1999. ISBN 0-19-510491-9

* "Theoretical Neuroscience: Computational and Mathematical Modeling of Neural Systems", by Peter Dayan and Larry F. Abbott. The MIT Press,2001. ISBN 0-262-04199-5

Figures were taken from some of these books

Integrate-and-fire neuron models

- The passive membrane equation (review)
- Response of V to changes in the equilibrium potential
- Integrate-and-fire-model
- Spike-rate adaptation

Passive membrane equation (review)

$$\tau \frac{dV}{dt} = -V + E_L + R\, I_{app}(t)$$

$\tau = R\,C$ (time constant)

$$V(t) = E_L + (V_0 - E_L)\, e^{-t/\tau} + \frac{e^{-t/\tau}}{\tau} R \int_0^t I_{app}(s)\, e^{s/\tau}\, ds$$

Passive membrane equation (review)

- $I_{app} = I_0$ (const) $V_\infty = R\, I_0$

$$V(t) = \boxed{V_\infty + E_L} + (V_0 - E_L - V_\infty)\, e^{-t/\tau}$$

steady state

- $V_0 = E_L$

$$V(t) = \boxed{V_\infty + E_L} - V_\infty\, e^{-t/\tau}$$

steady state

Passive membrane equation (review)

- $I_{app} = I_0$ (const) $V_\infty = R\, I_0$ (const)

$$V(t) = \boxed{V_\infty + E_L} + (V_0 - E_L - V_\infty)\, e^{-t/\tau}$$

steady state

- ✤ $I_0 = \alpha$ nA
- ✤ $R = 100$ MΩ

$V_\infty = \alpha * 10^2$ mV

We will use the following units: mV, nA, msec

Passive membrane equation (review)

- $I_{app}(t) = I_0\, H(t - t_i)\ \ H(t_f - t)$

  H(t): Heaviside function

  $I_{app}(t)$: square pulse of current starting at $t = t_i$ and ending at $t = t_f$

✤ $V_0 = E_L$
✤ $V_\infty = R\, I_0$ (const)

$$V(t) - E_L = V_\infty \left[H(t - t_i)\left(1 - e^{-\frac{t - t_i}{\tau}}\right) - H(t - t_f)\left(1 - e^{-\frac{t - t_f}{\tau}}\right)\right]$$

Passive membrane equation (review)

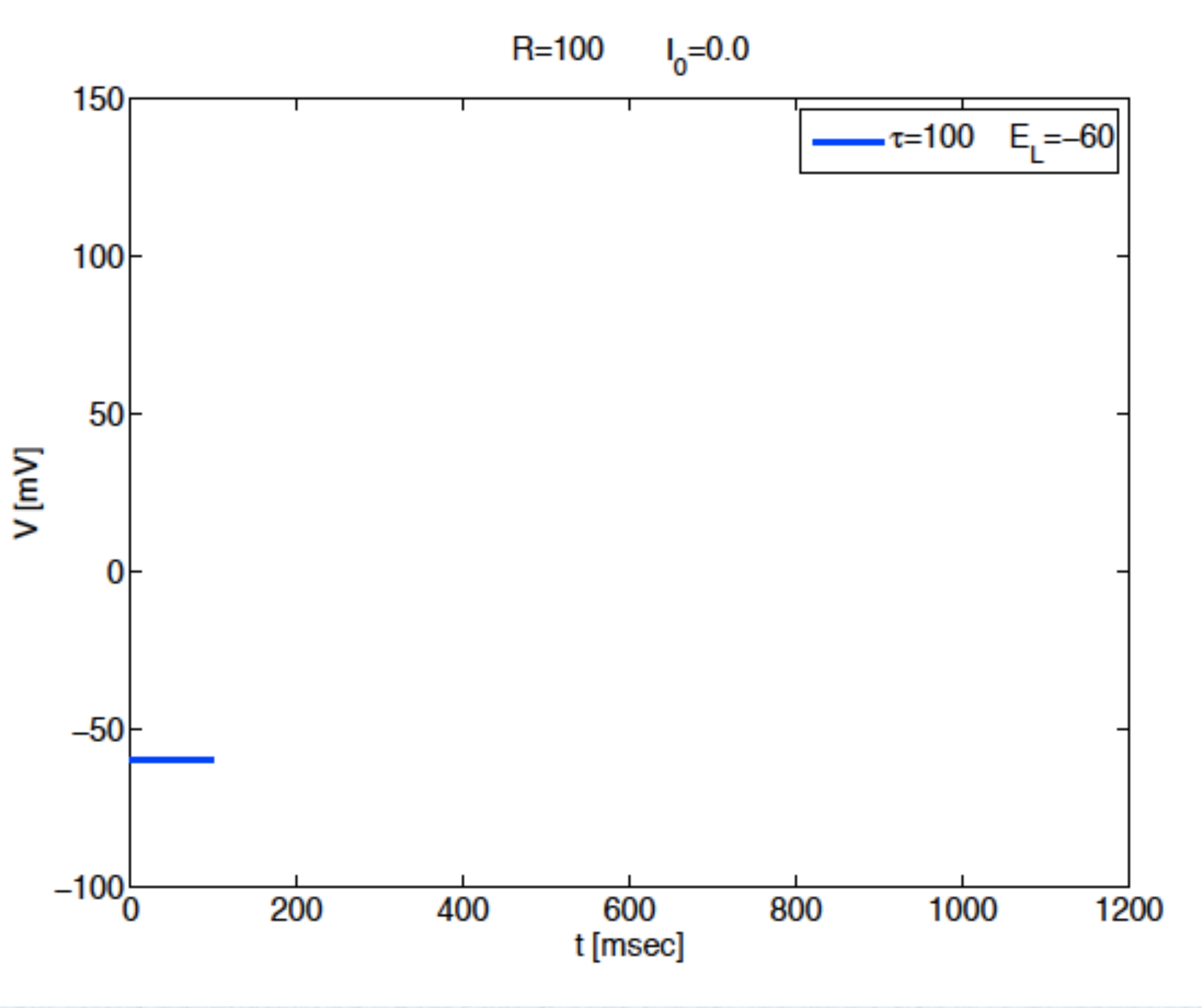
- Calculation of the time t_c that takes V to reach the value V_t

$$V(t) - E_L = V_\infty H(t - t_i)\left(1 - e^{-\frac{t - t_i}{\tau}}\right)$$

$$V_t = \eta V_\infty \quad \rightarrow \quad t_c = t_i - \tau \ln(1 - \eta)$$

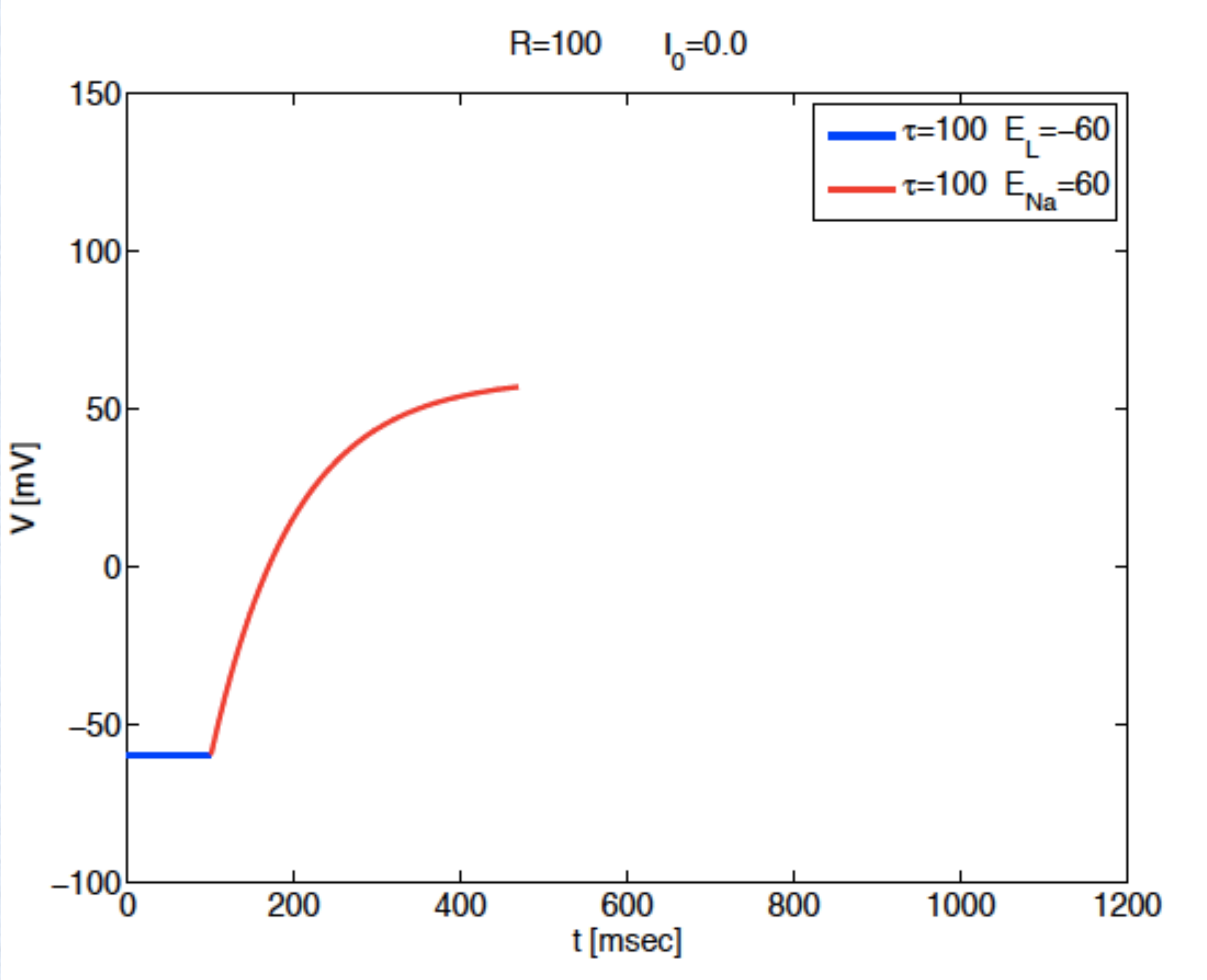
Response of V to changes in E_x

$$\tau \frac{dV}{dt} = -V + E_L \qquad V(0) = E_L$$



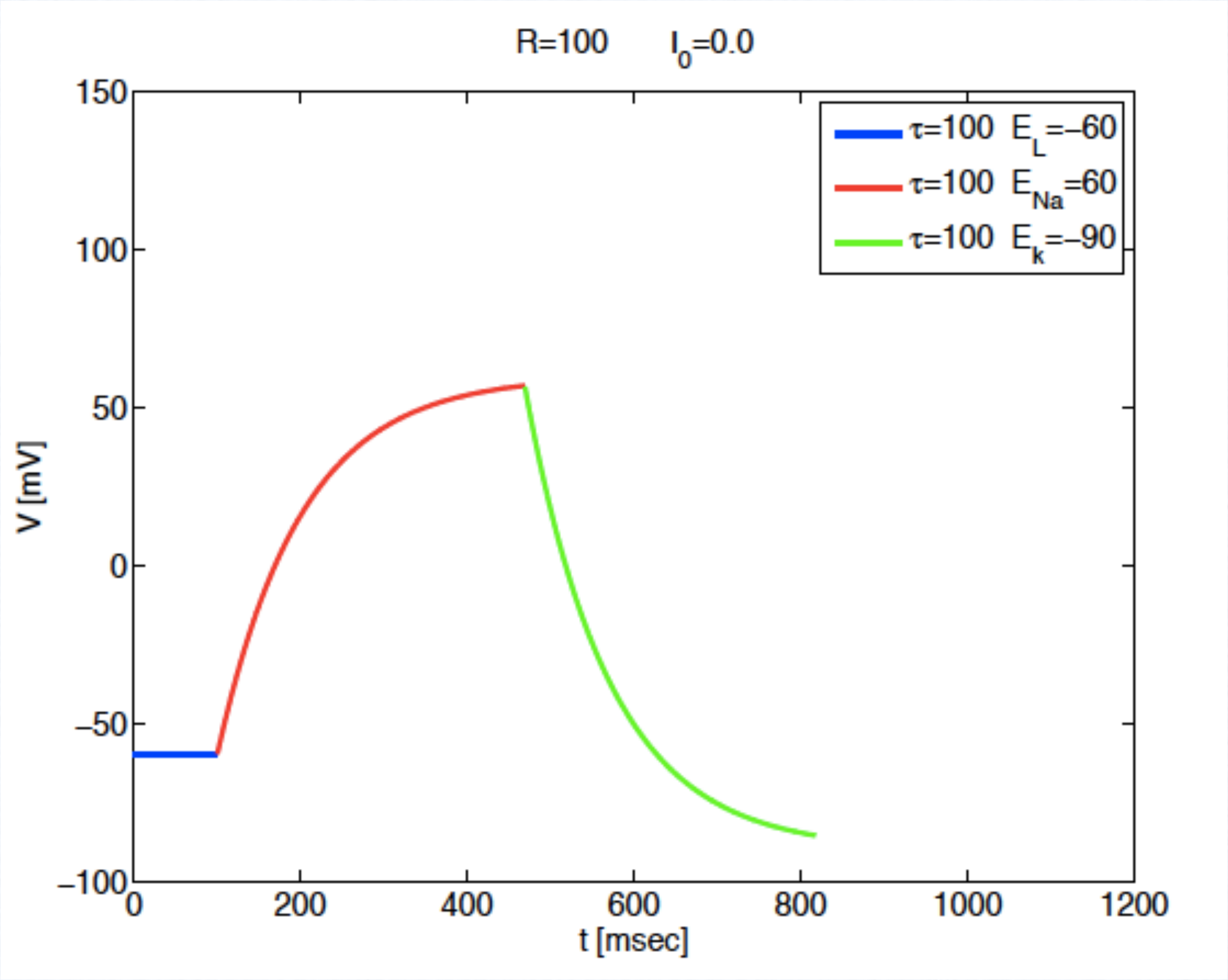
Response of V to changes in E_x

$$\tau \frac{dV}{dt} = -V + E_{Na} \qquad V(t_1) = E_L$$



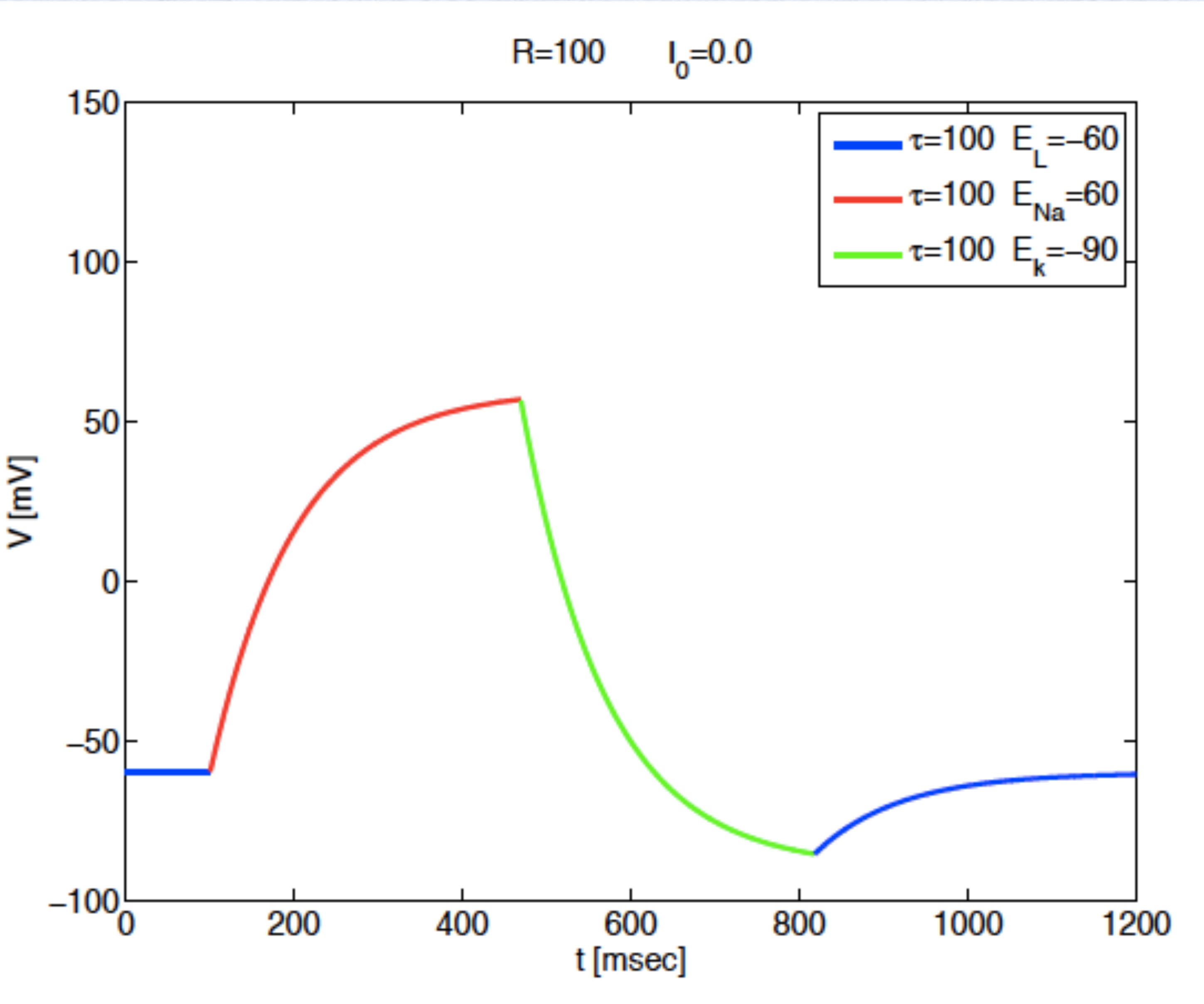
Response of V to changes in E_x

$$\tau \frac{dV}{dt} = -V + E_K \qquad V(t_2) = E_{Na}$$



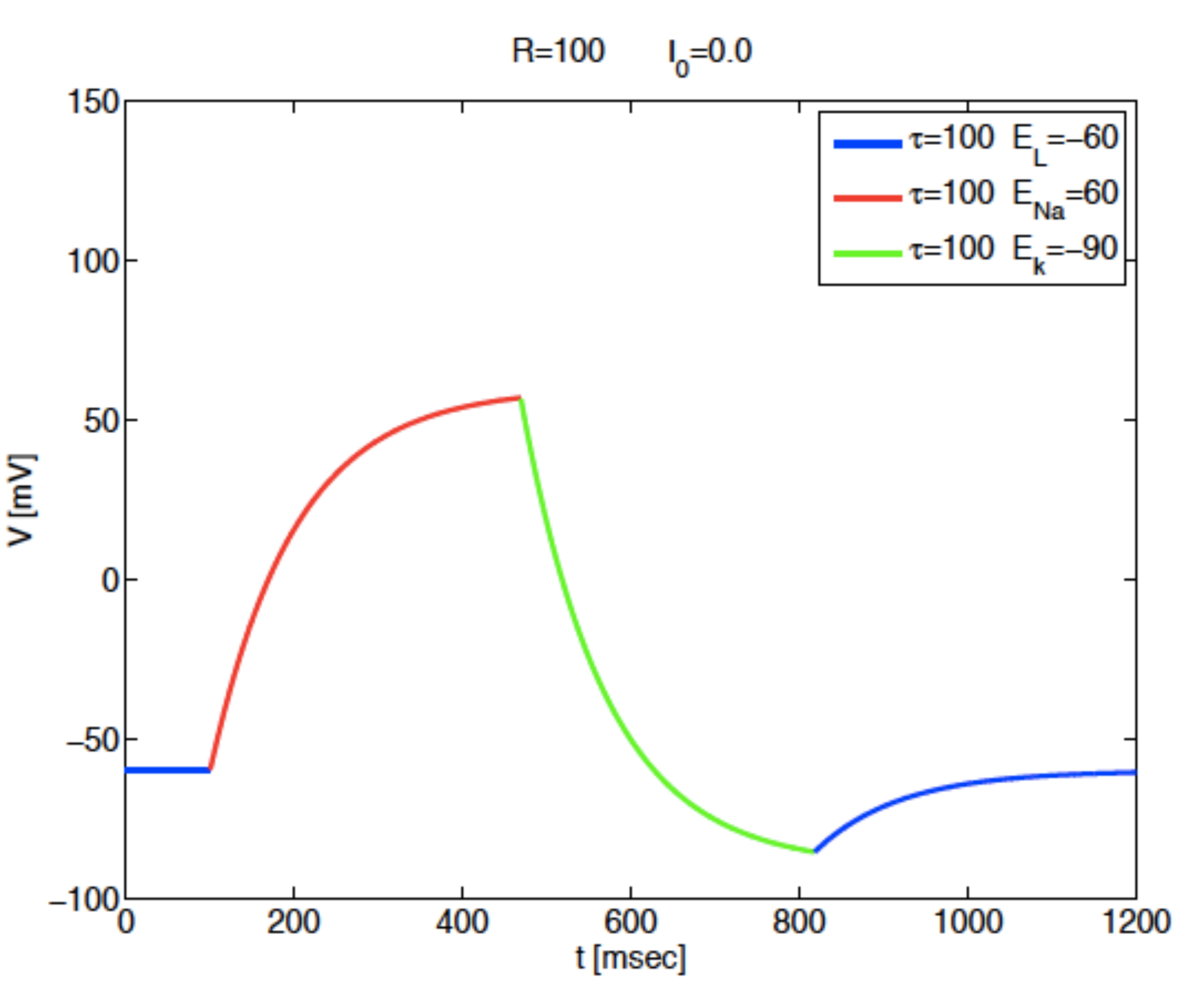
Response of V to changes in E_x

$$\tau \frac{dV}{dt} = -V + E_L \qquad V(t_3) = E_K$$



Response of V to changes in E_x

- V tracks the equilibrium potential E_x (E_L, E_{Na}, E_K)



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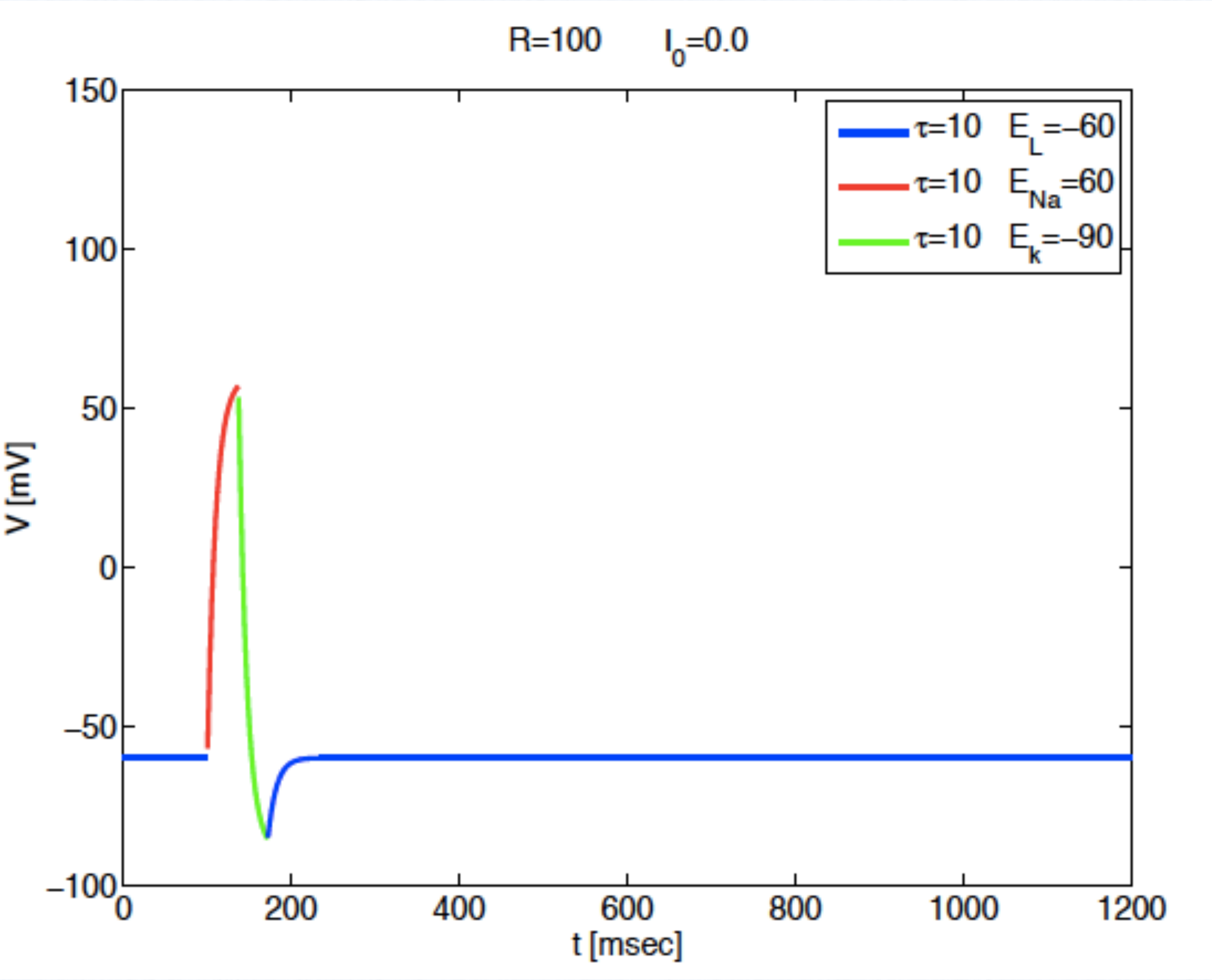

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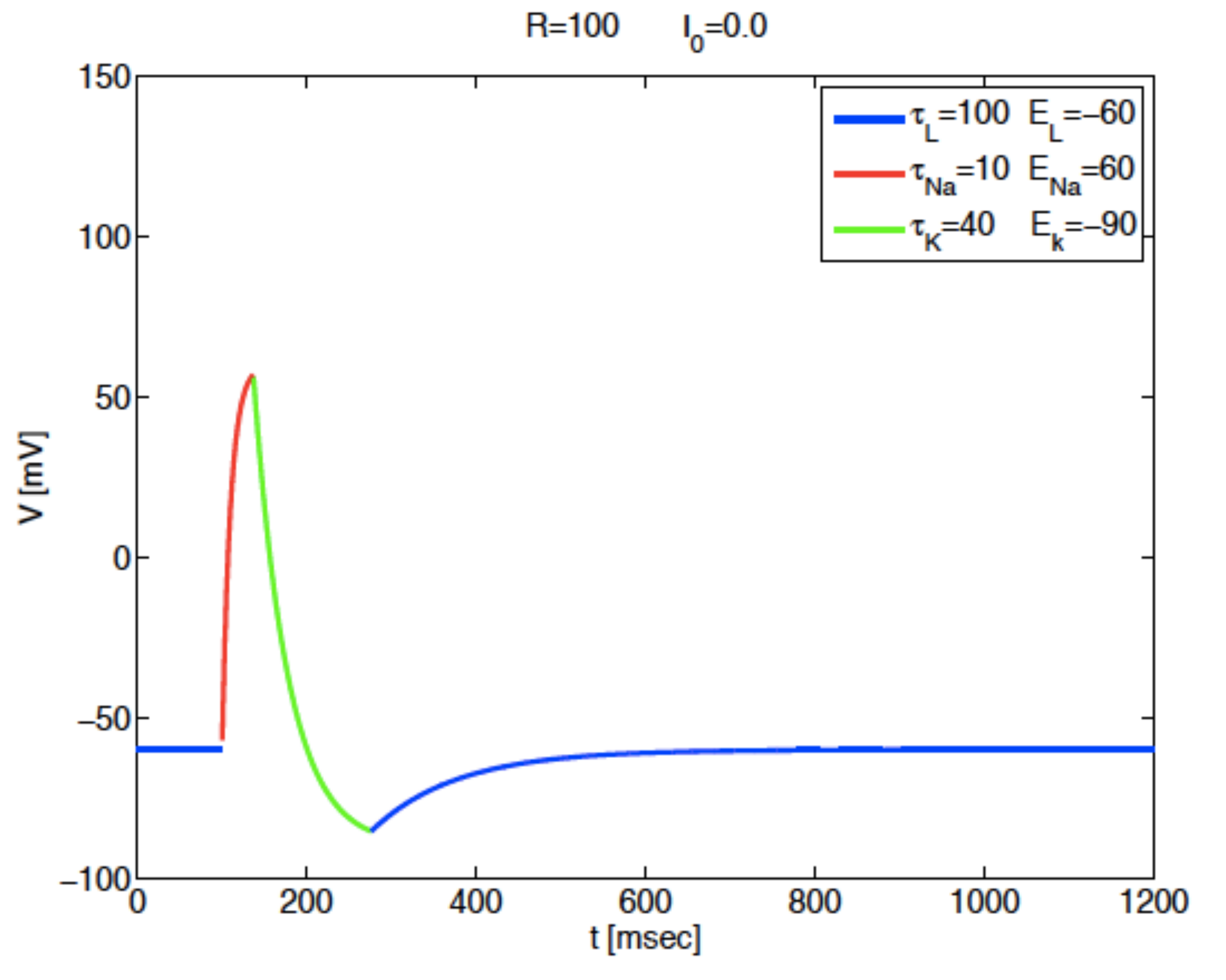


Integrate-and-fire neuron models

- A neuron will typically fire and action potential (or spike) when its membrane potential reaches a threshold value of about -55 to -50 mV

- Action potentials:

  - V increases rapidly and then decreases to a value below threshold

  - Stereotypical trajectory

  - The mechanism involves the activation of Na^+ and K^+ conductances

Integrate-and-fire neuron models

- Passive membrane equation

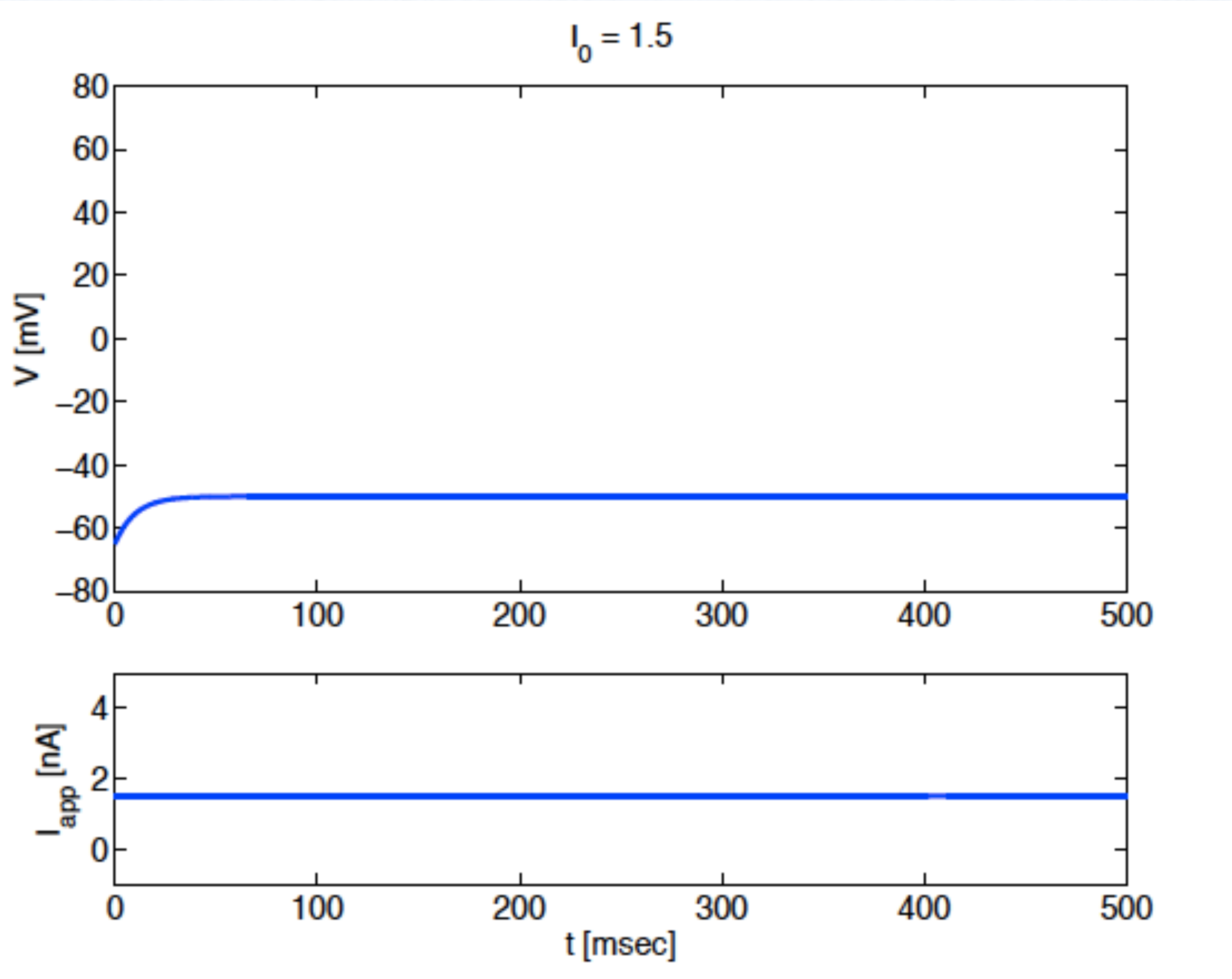
$$\tau \frac{dV}{dt} = -V + E_L + R\, I_{app}(t) \qquad V(0) = V_0$$

- An action potential occurs whenever $V = V_{th}$ (voltage threshold for spike generation)
- After the action potential, V is reset to a value $V = V_{rst}$ (voltage reset value)

Integrate-and-fire neuron models

- $E_L = -65$ $V_{th} = -50$ $V_{rst} = -65$ $\tau = 10$ $R = 10$

(Units: mV, nA, msec, MΩ)



Integrate-and-fire neuron models

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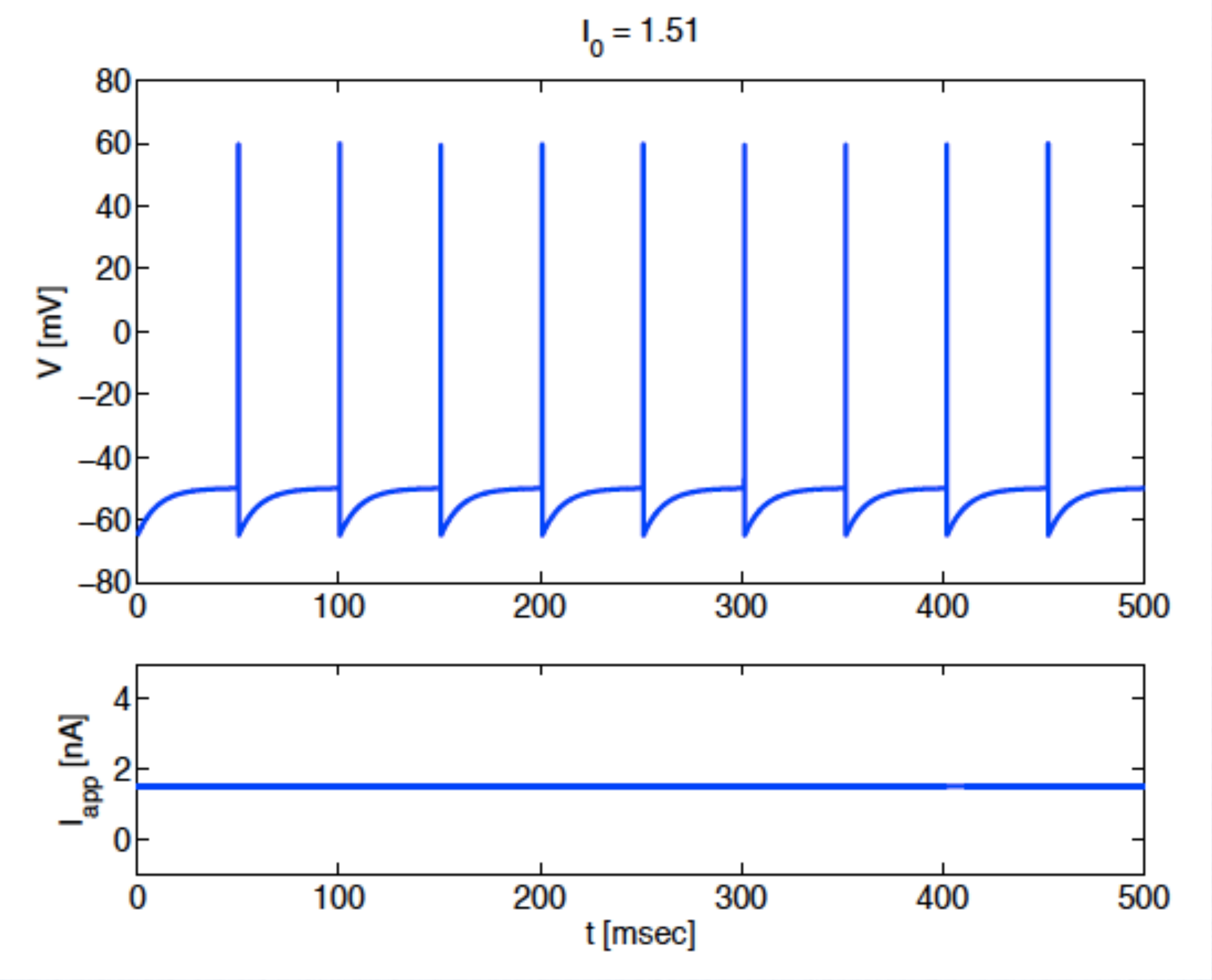


Integrate-and-fire neuron models

- Calculation of the firing frequency (r_{isi}) and inter-spike interval (ISI)

$$V(t) = \boxed{V_\infty + E_L} + (V_0 - E_L - V_\infty)\, e^{-t/\tau}$$

$$t_{isi} = \tau \ln \frac{V_{rst} - V_\infty - E_L}{V_{th} - V_\infty - E_L}$$

$$r_{isi} = \frac{1}{t_{isi}} = \frac{1}{\tau}\left[\ln \frac{V_{rst} - V_\infty - E_L}{V_{th} - V_\infty - E_L}\right]^{-1}$$

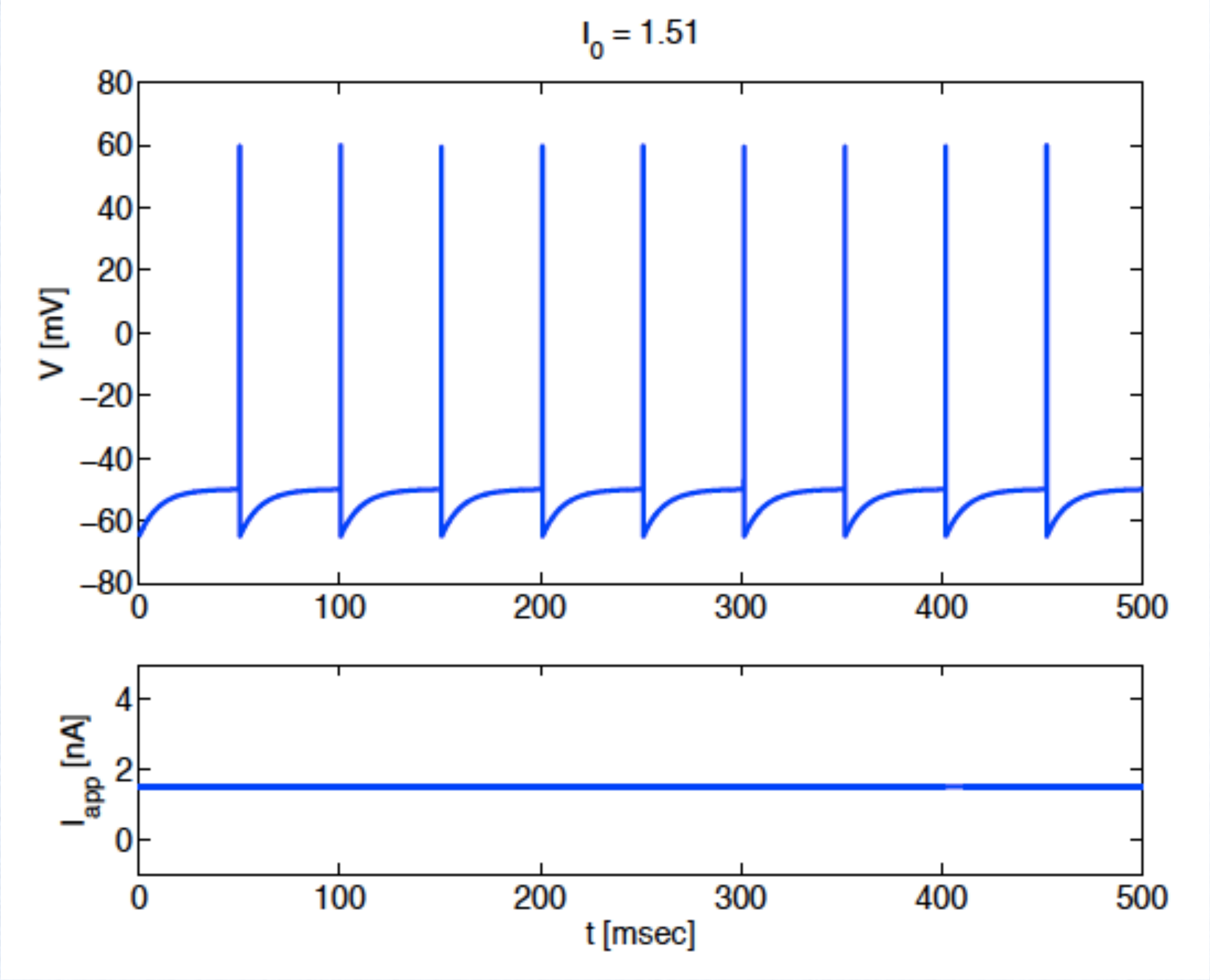
Integrate-and-fire neuron models

- E_L= -65 V_{th}= -50 V_{rst} = -65   τ = 10   R = 10

(Units: mV, nA, msec, MΩ)

t_{isi} ~ 50 msec

r_{isi} ~ 20 spk /sec



Integrate-and-fire neuron models

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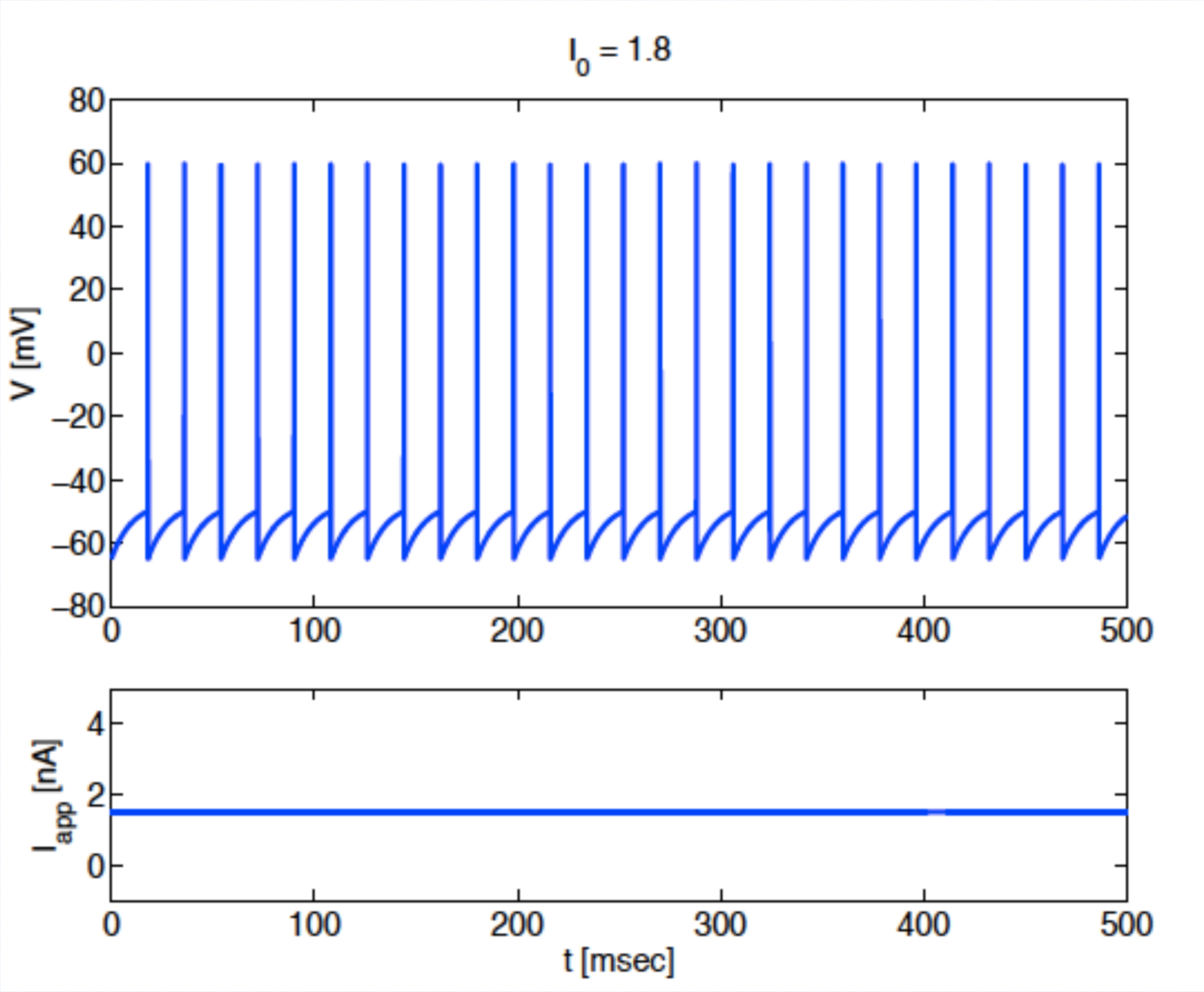

Integrate-and-fire neuron models

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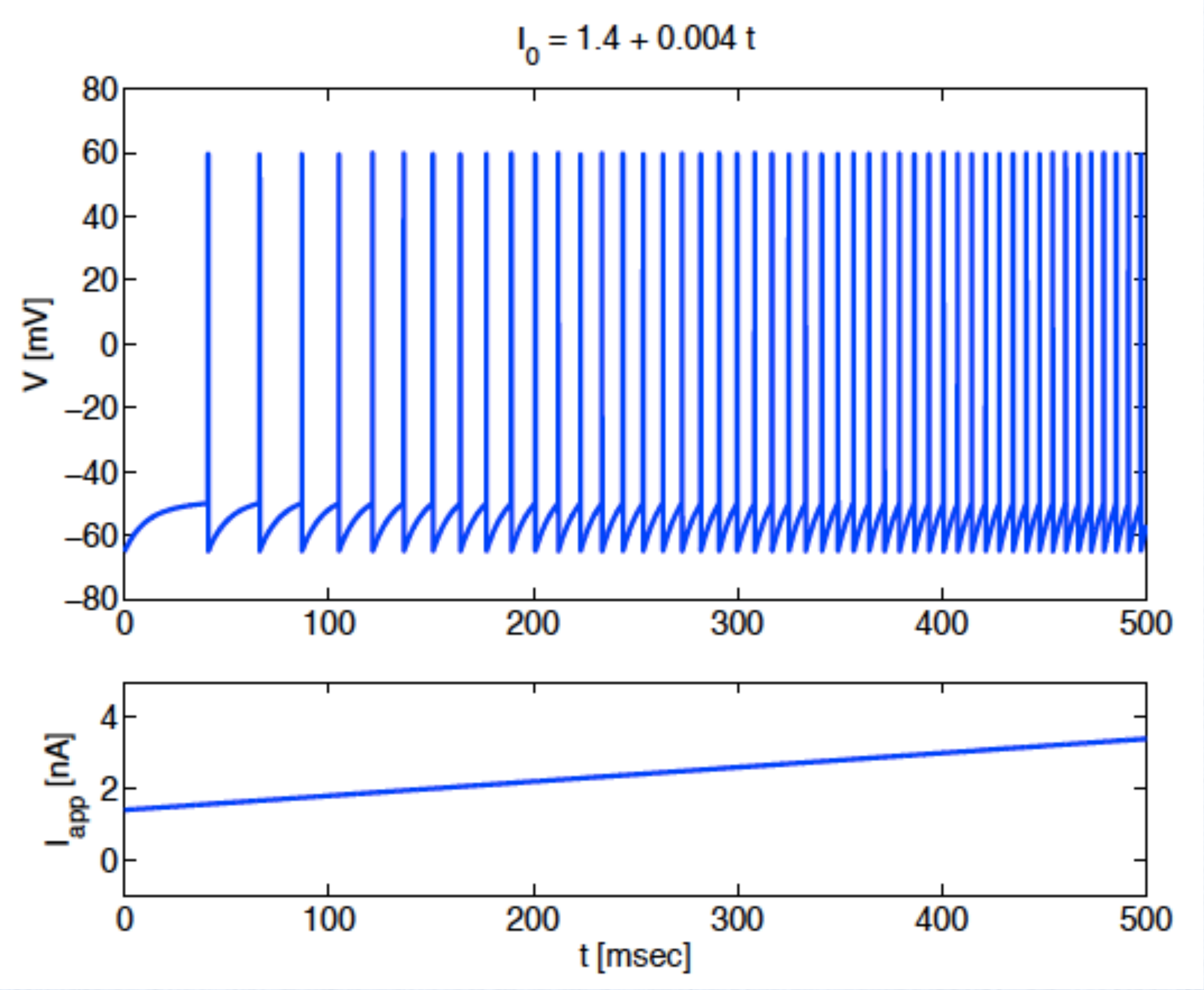

Integrate-and-fire neuron models

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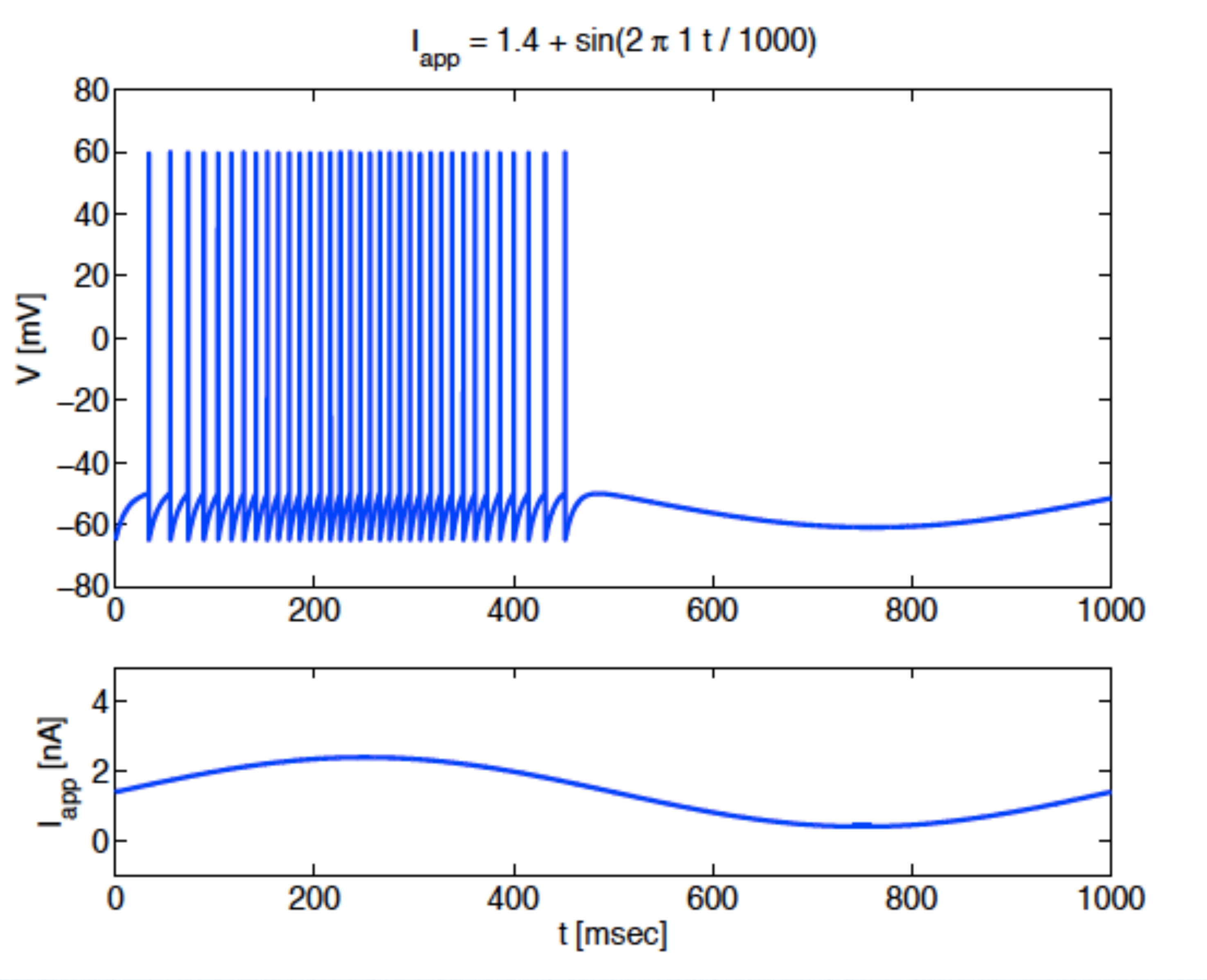
(Units: mV, nA, msec, MΩ)



Integrate-and-fire neuron models

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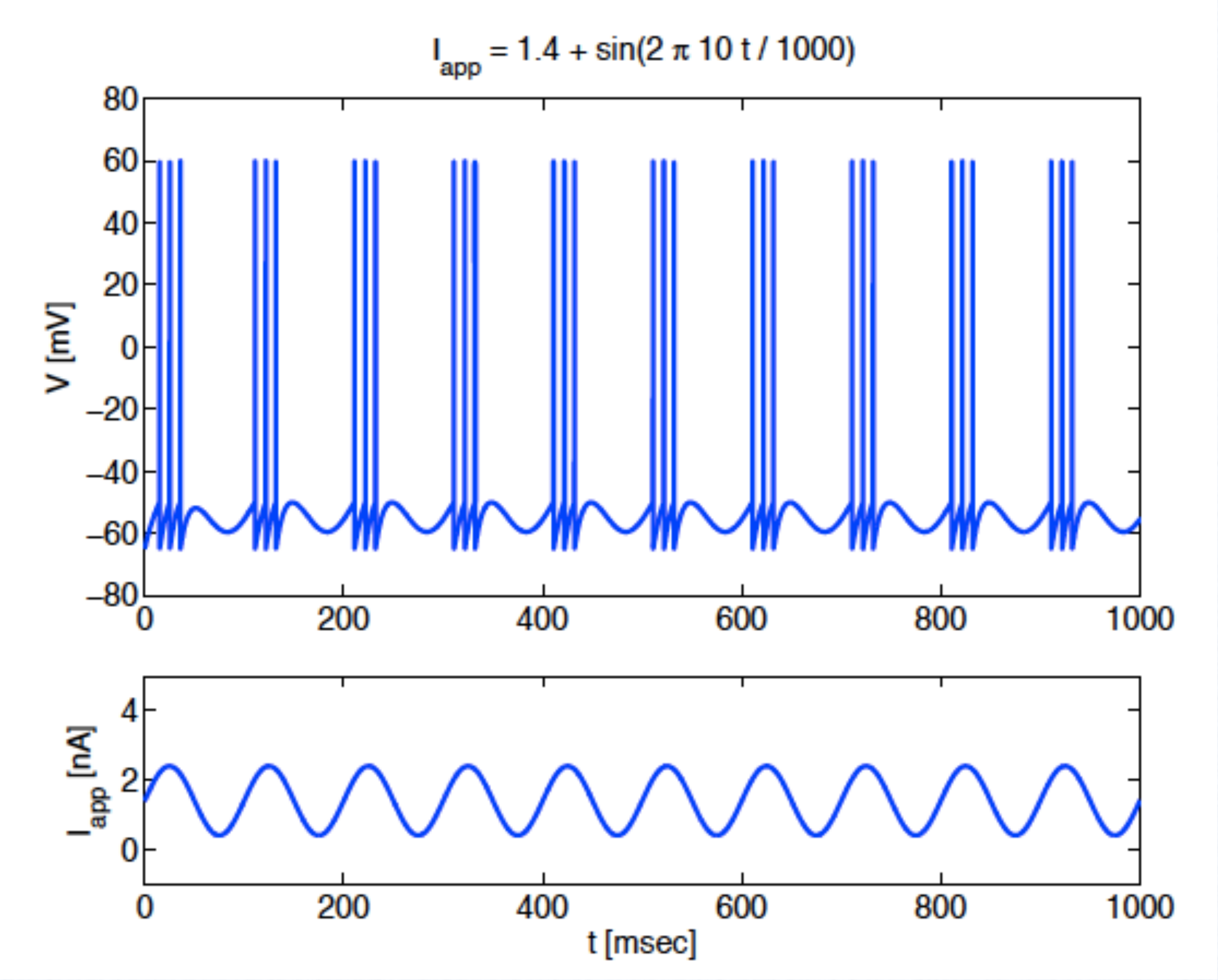
(Units: mV, nA, msec, MΩ)



Integrate-and-fire neuron models

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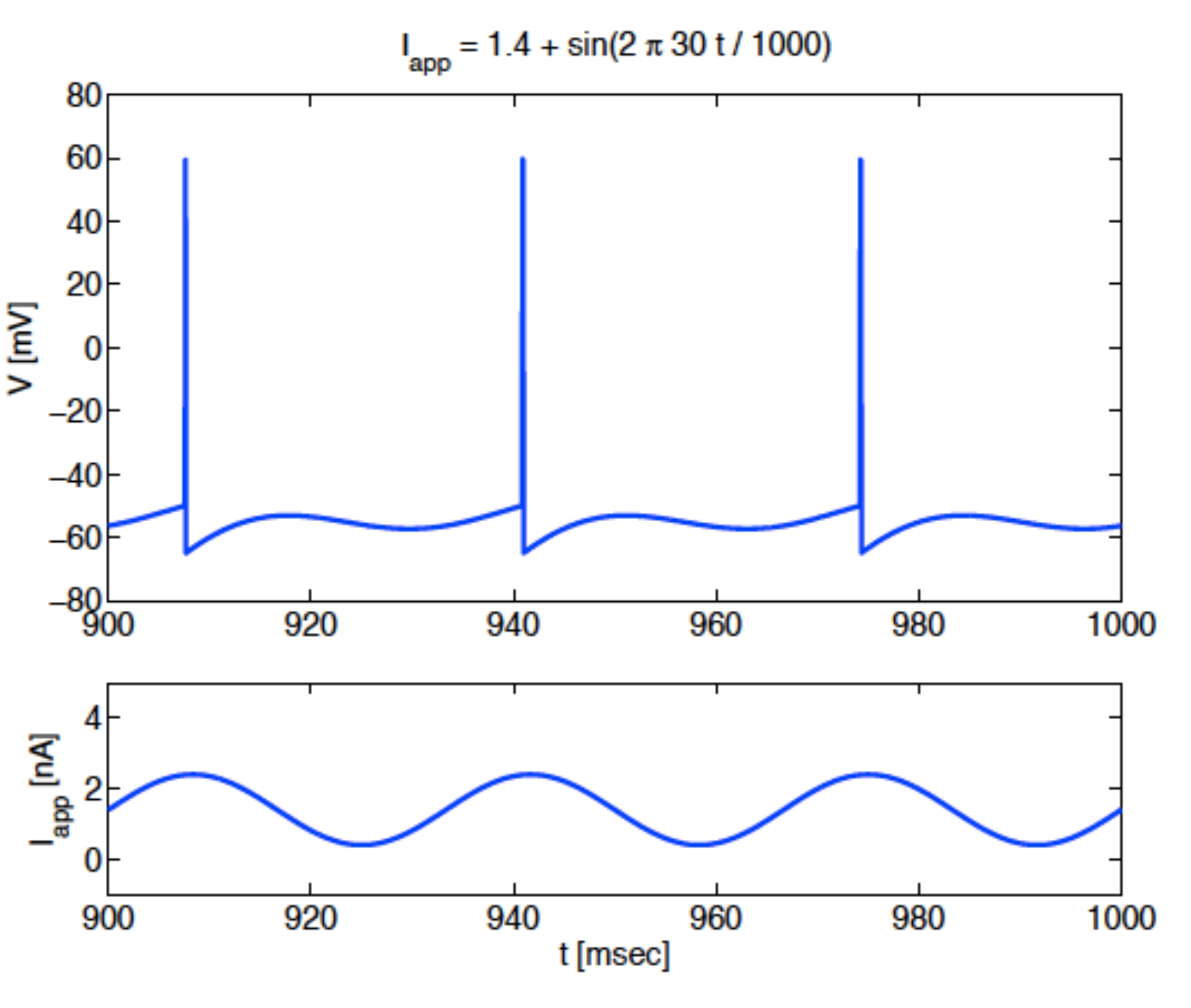
(Units: mV, nA, msec, MΩ)



Integrate-and-fire neuron models

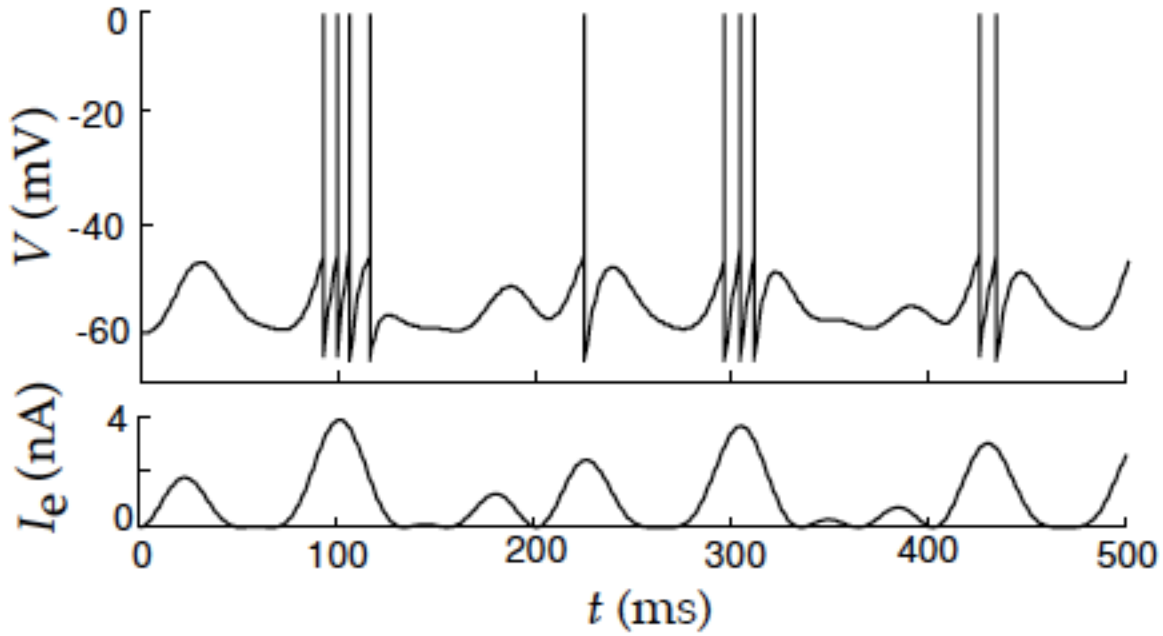


Figure 5.5 A passive integrate-and-fire model driven by a time-varying electrode current. The upper trace is the membrane potential, and the bottom trace the driving current. The action potentials in this figure are simply pasted onto the membrane potential trajectory whenever it reaches the threshold value. The parameters of the model are $E_L = V_{reset} = -65$ mV, $V_{th} = -50$ mV, $\tau_m = 10$ ms, and $R_m = 10$ MΩ.

Dayan & Abbott

Integrate-and-fire neuron models

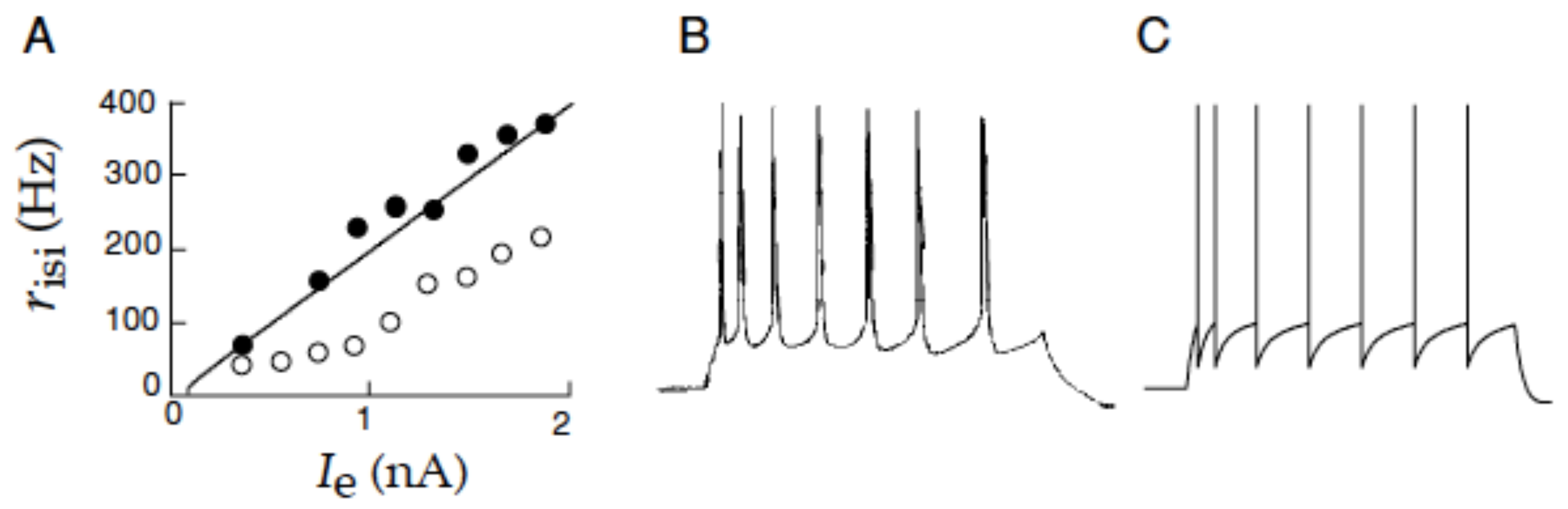


Figure 5.6 (A) Comparison of interspike-interval firing rates as a function of injected current for an integrate-and-fire model and a cortical neuron measure in vivo. The line gives r_{isi} for a model neuron with $\tau_m = 30$ ms, $E_L = V_{reset} = -65$ mV, $V_{th} = -50$ mV, and $R_m = 90$ MΩ. The data points are from a pyramidal cell in the primary visual cortex of a cat. The filled circles show the inverse of the interspike interval for the first two spikes fired, and the open circles show the steady-state interspike-interval firing rate after spike-rate adaptation. (B) A recording of the firing of a cortical neuron under constant current injection, showing spike-rate adaptation. (C) Membrane voltage trajectory and spikes for an integrate-and-fire model with an added current, with $r_m \Delta g_{sra} = 0.06$, $\tau_{sra} = 100$ ms, and $E_K = -70$ mV (see equations 5.13 and 5.14). (Data in A from Ahmed et al., 1998; B from McCormick, 1990.)

Dayan & Abbott

Integrate-and-fire neuron models

- E_L= -65 V_{th}= -50 V_{rst} = -65 τ = 10 R = 10

(Units: mV, nA, msec, MΩ)

.ode code:

```
dV/dt = (F(V) + Rc * Iapp(t)) / tau

F(V)= - (V - El)

Iapp(t) = heav(t-ti) * heav(tf-t) * (Io + B*t) + A * sin(2*pi*freq*t/1000)

global 1 V-Vthr {V=60}

global -1 V-59 {V=Vrst}

aux stim=Rc * Iapp(t)
```

Integrate-and-fire neuron models

- E_L= -65 V_{th}= -50 V_{rst} = -65 τ = 10 R = 10

(Units: mV, nA, msec, MΩ)

.ode code:

```
par R=10, Rc=10, tau=10, EL=-65, Io=0
par A=0, freq=1, par B=0
par ti=0, tf=10000
par Vthr=-50, Vrst=-65

V(0)=-65

@ total=1000
@ xlo=0,xhi=1000,ylo=-100,yhi=100,maxstor=1000000
@ back=black, bounds=1e6x
@ yp1=V, nplot=1
```

Spike-rate adaptation

- Integrate-and-fire model with an additional current in the membrane equation

$$\tau \frac{dV}{dt} = -V + E_L - R\, g_{sra}\,(V - E_k) + R\, I_{app}(t)$$

$$\tau_{sra} \frac{dg_{sra}}{dt} = -\, g_{sra}$$

- If $V = V_{th}$ at $t = t^*$, then at $t = t^*$

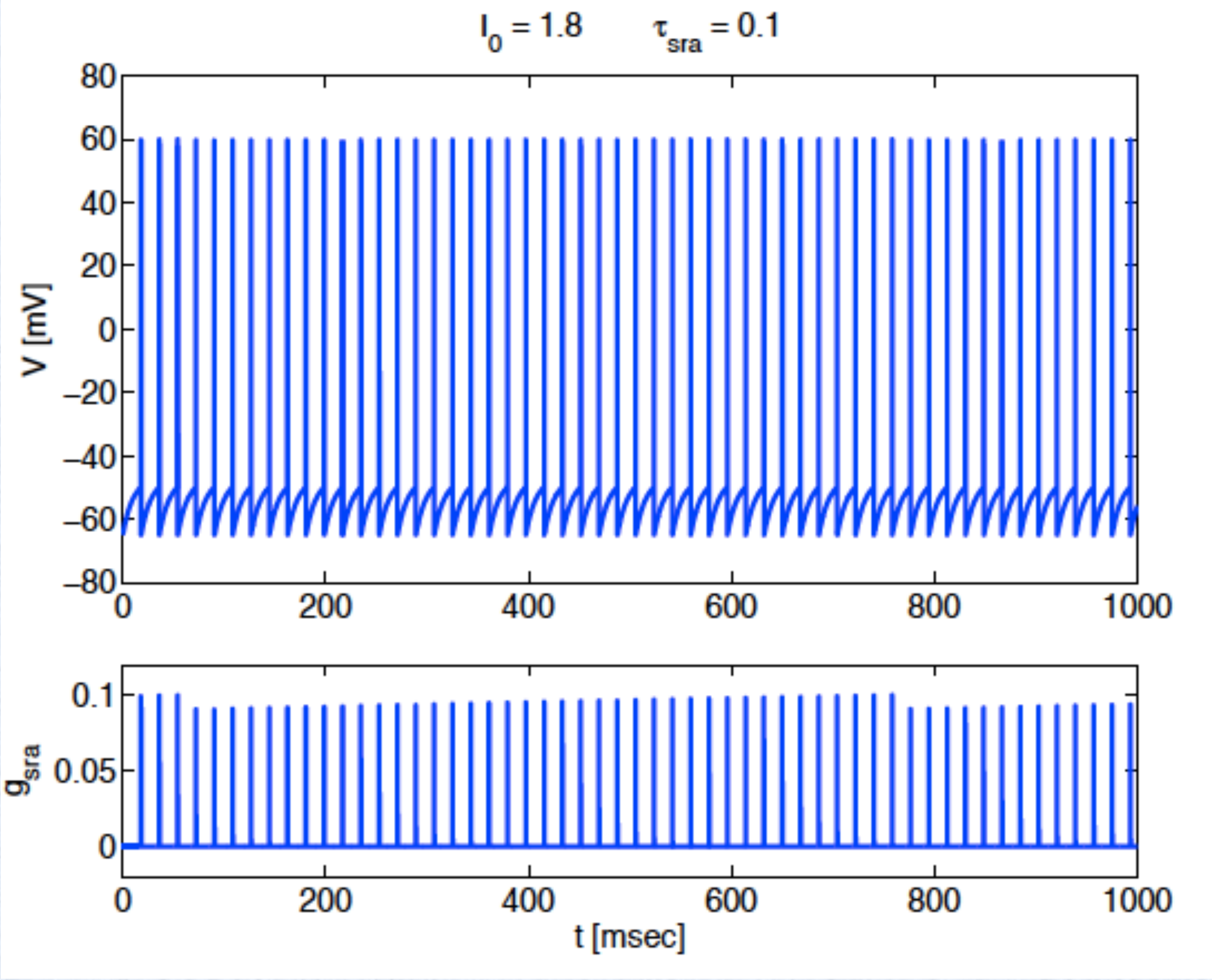
(1) A spike is generated

(2) $g_{sra} \rightarrow g_{sra} + \Delta\, g_{sra}$

Integrate-and-fire neuron models

- E_L= -65 V_{th}= -50 V_{rst} = -65 τ = 10 R = 10
- E_K= -85 Δ g_{sra}= 0.1

(Units: mV, nA, msec, MΩ)



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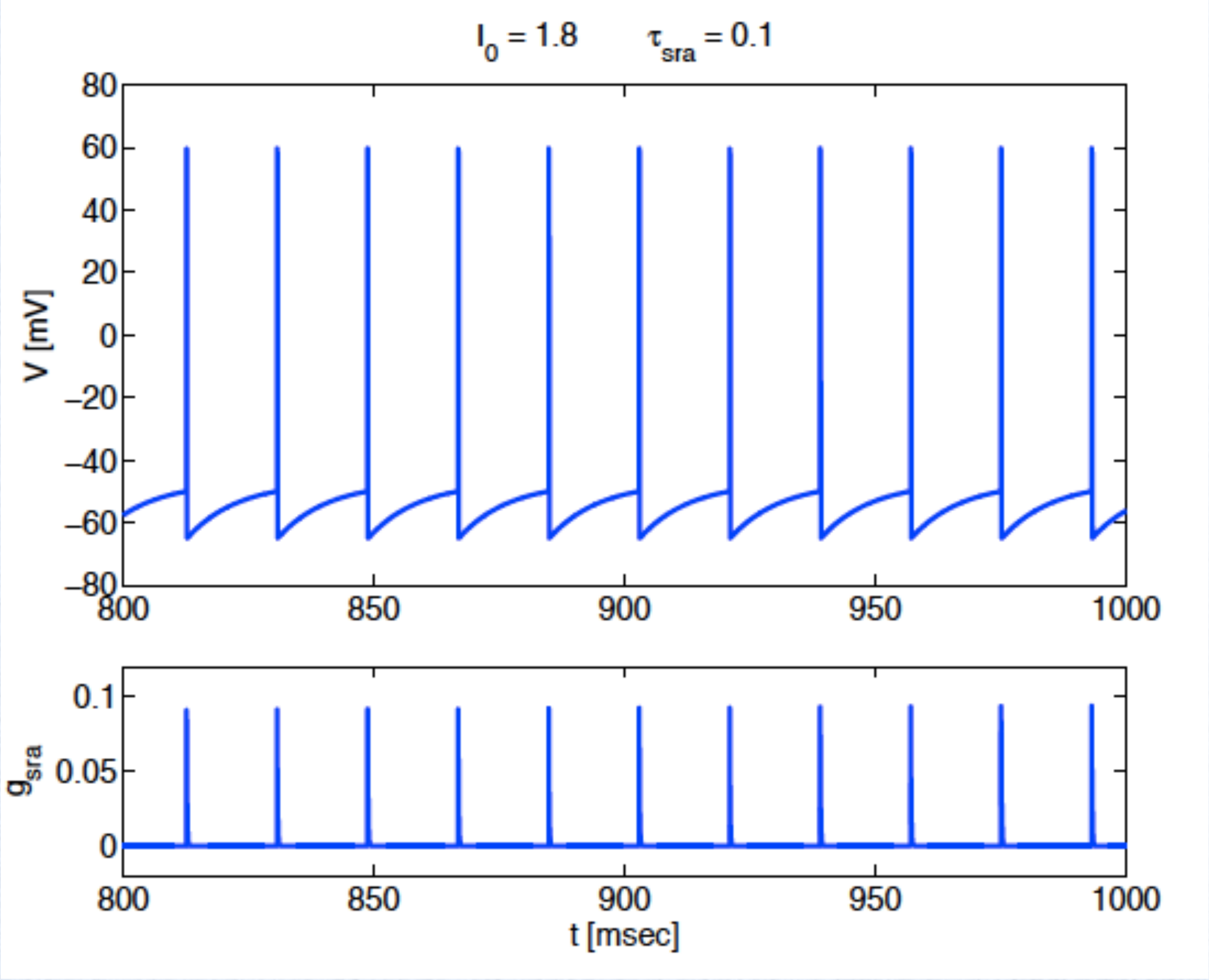

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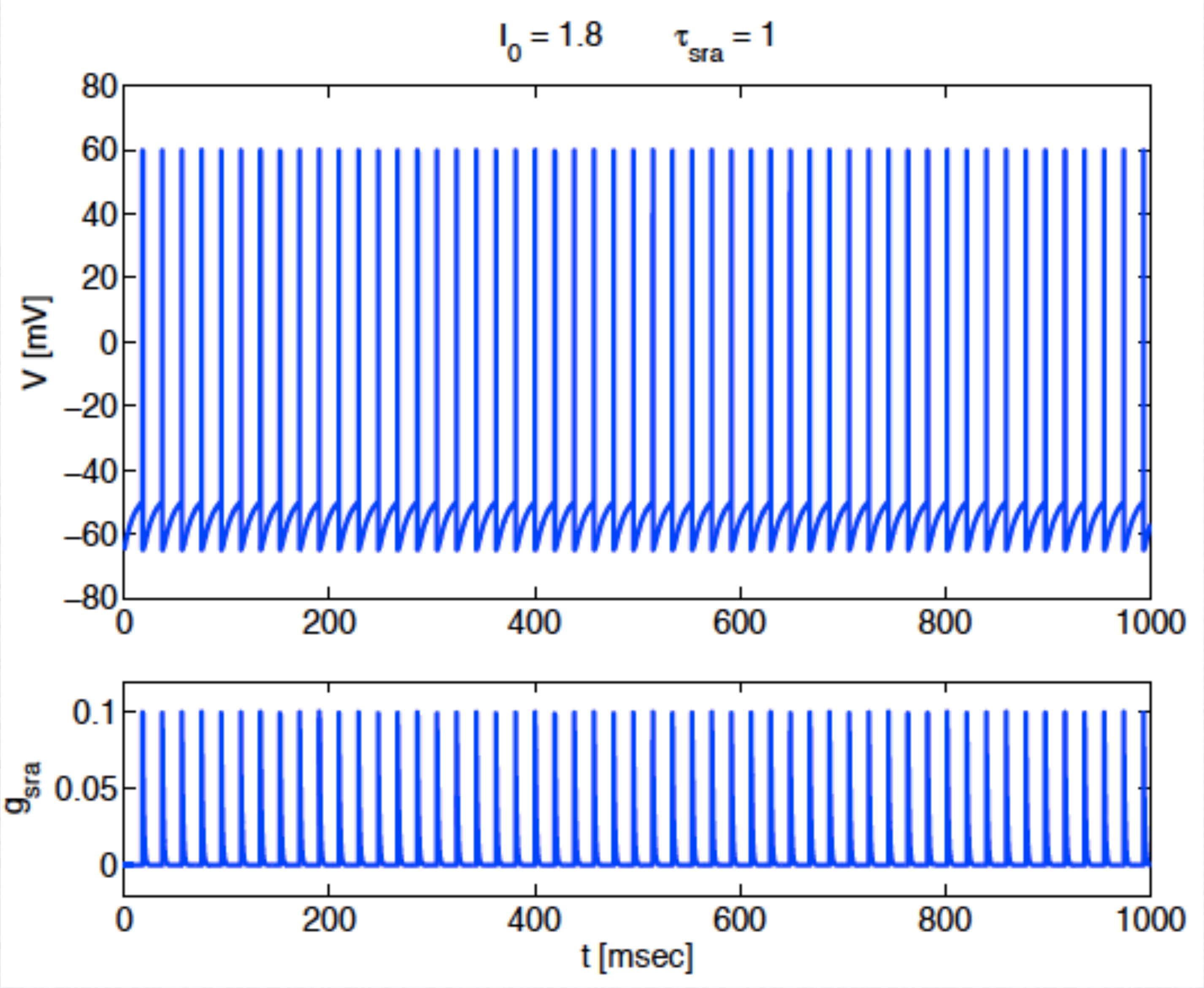

Integrate-and-fire neuron models

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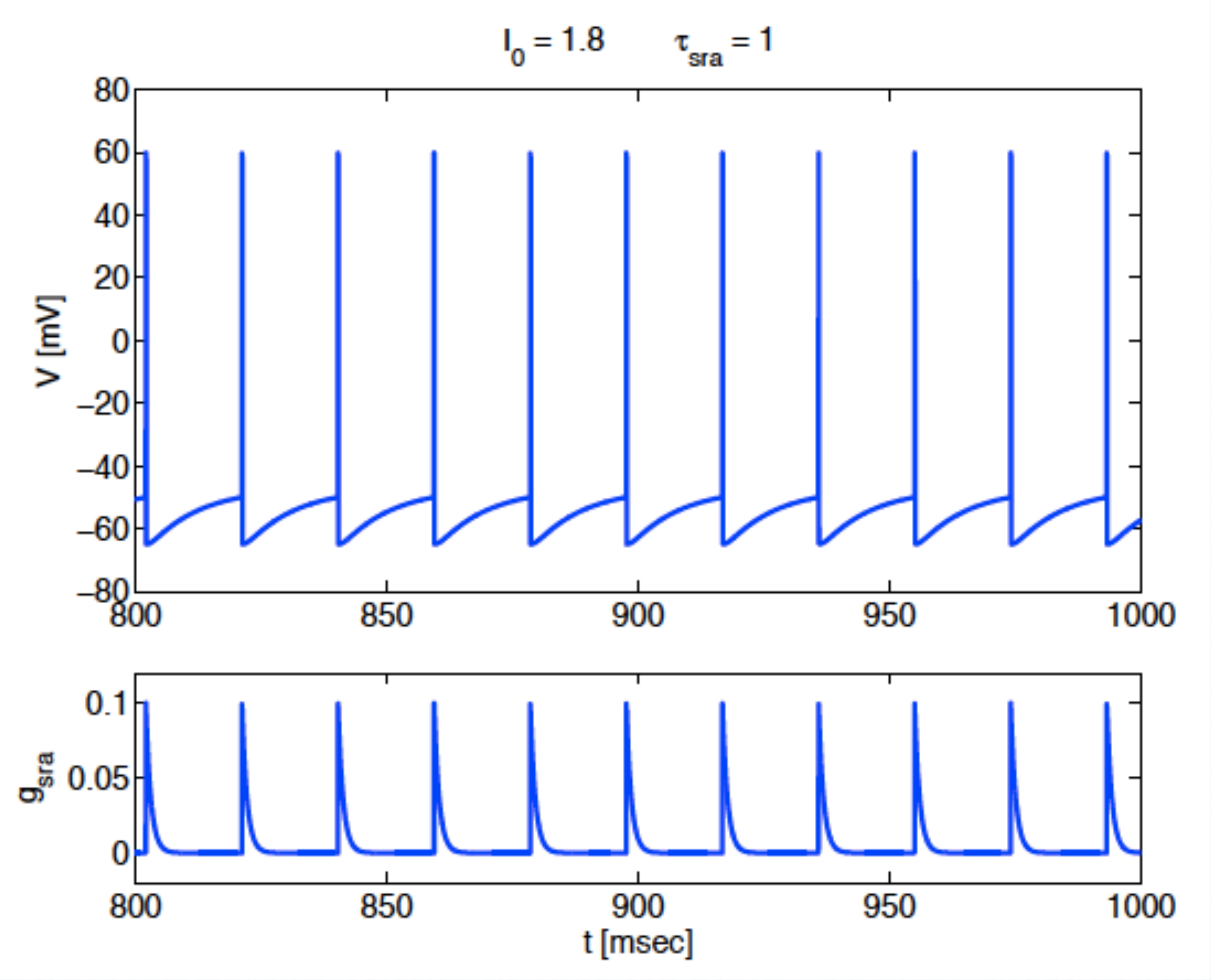
- E_K= -85 $\Delta\ g_{sra}$= 0.1   (Units: mV, nA, msec, MΩ)



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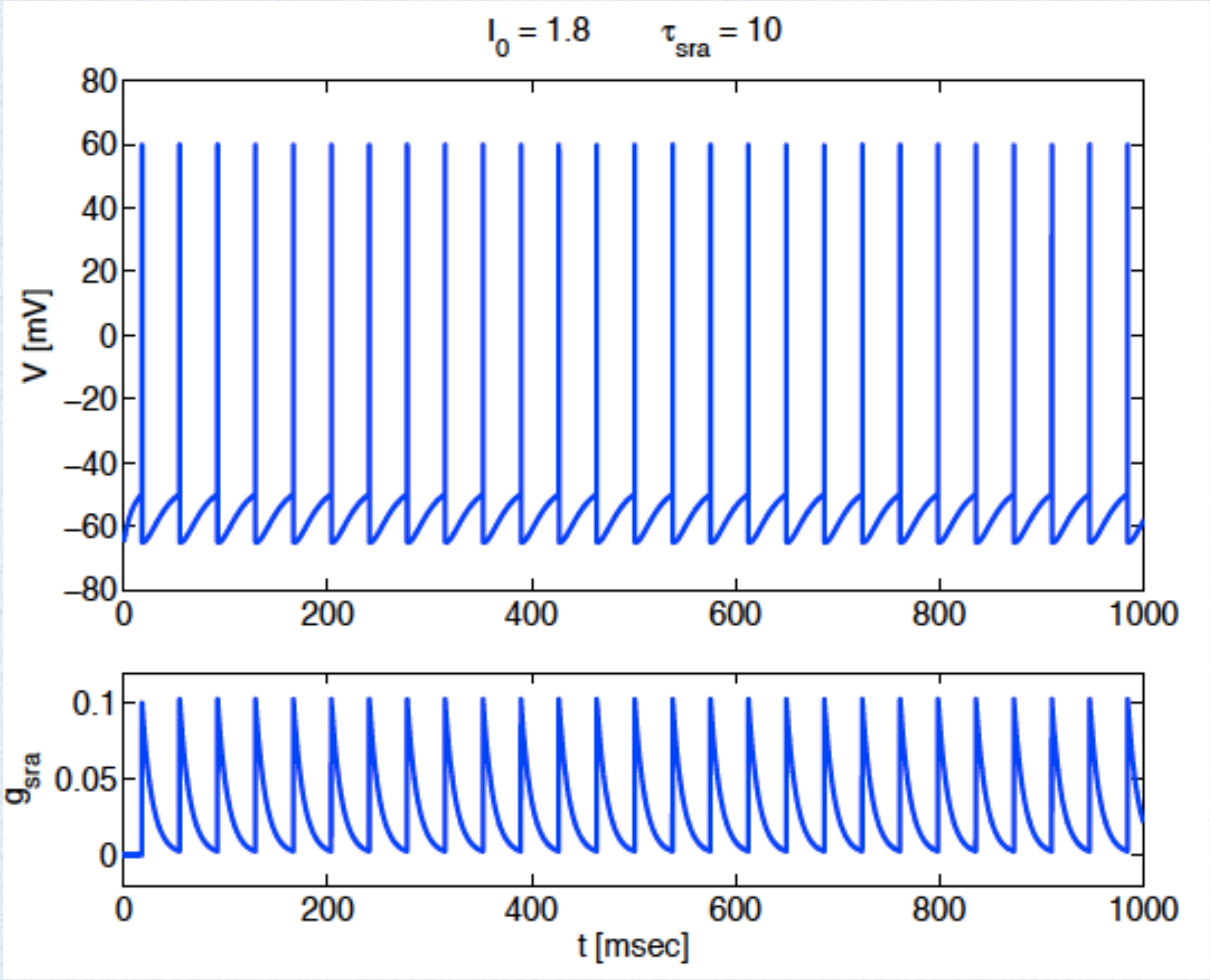

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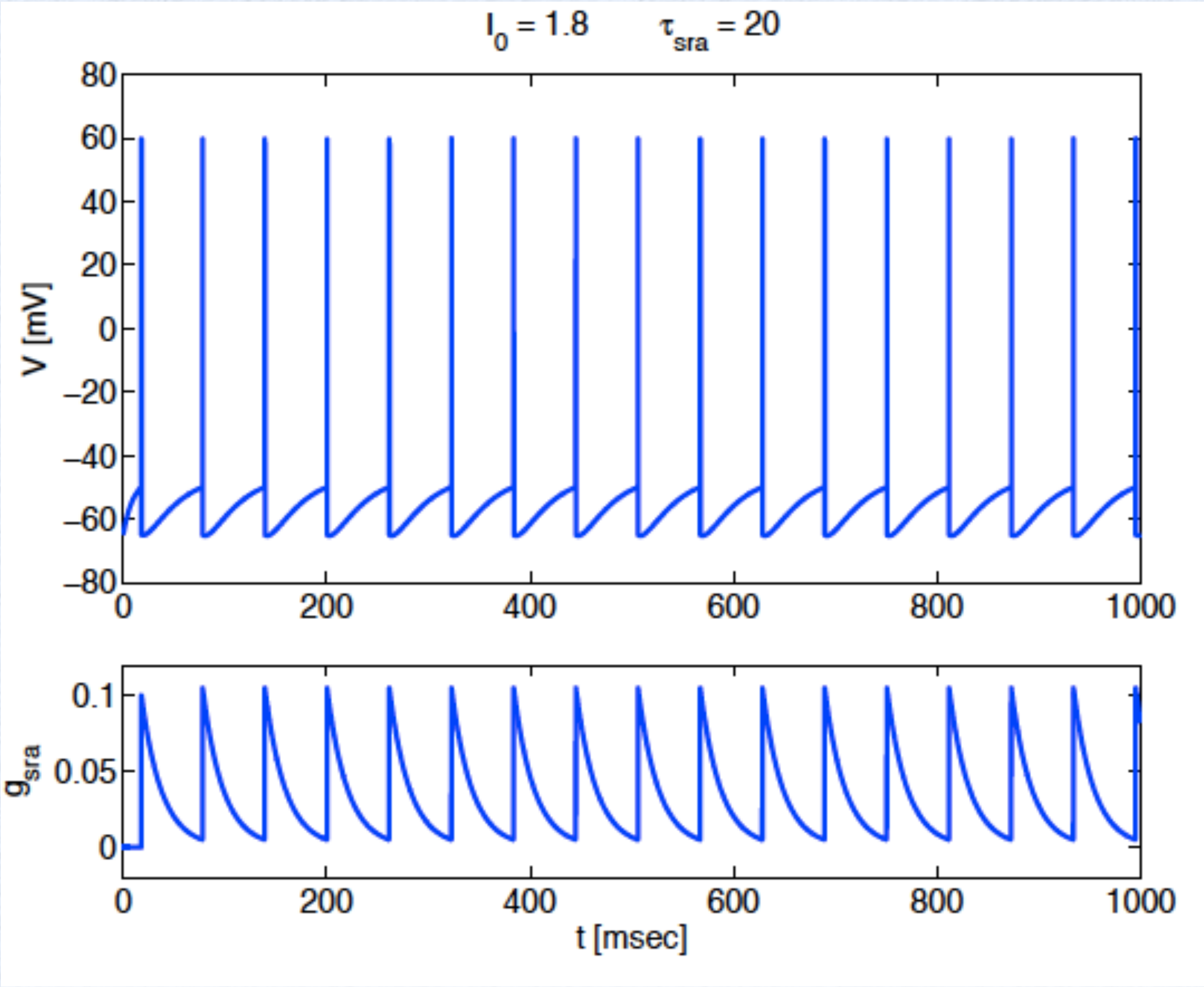

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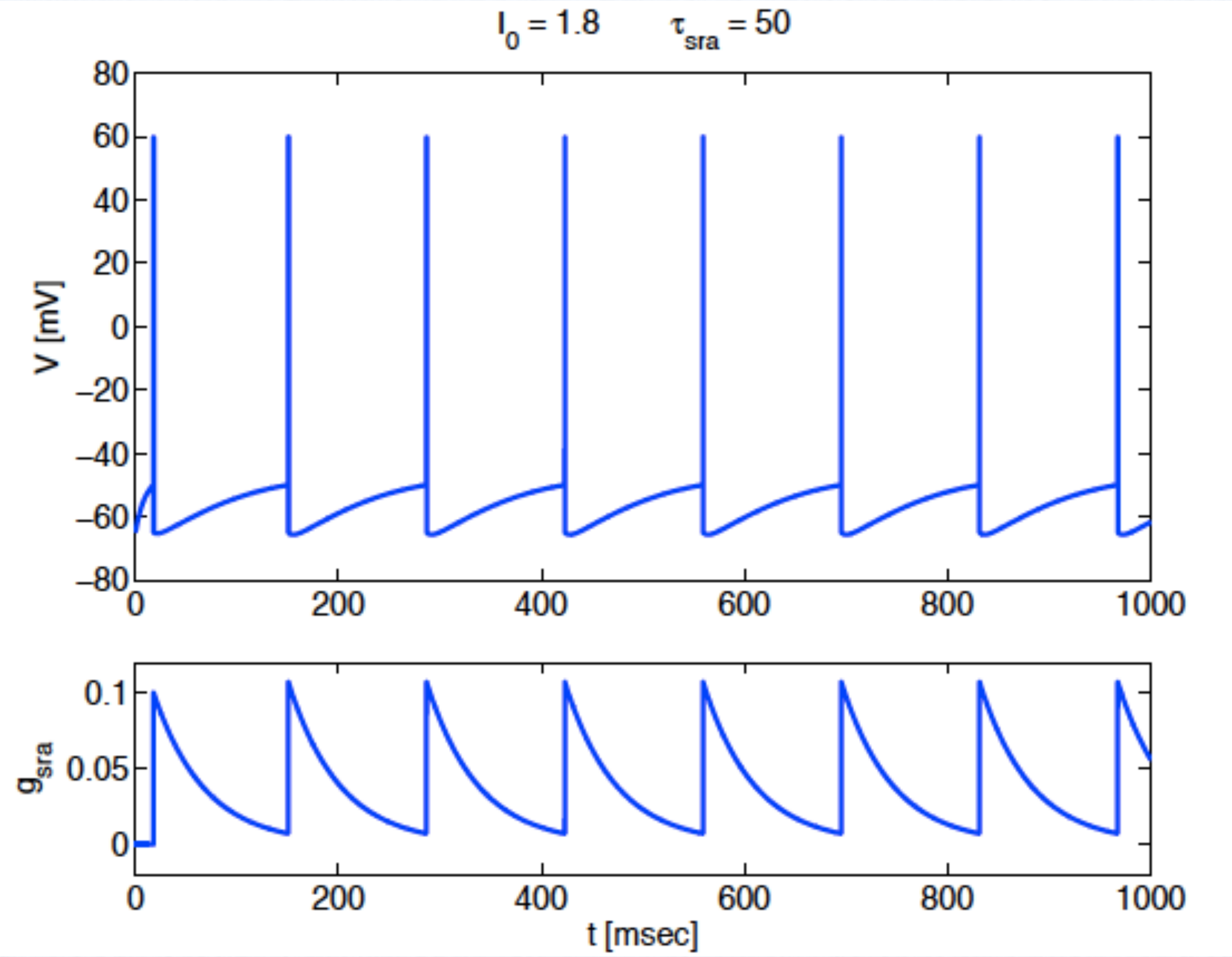

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