Introduction to Computational Neuroscience

Biol 698
Math 635
Biol 498
Math 430

Neuronal Networks

Reference:

- *Mathematical Foundations of Neuroscience*, by G. B. Ermentrout & D. H. Terman - Springer (2010), 1st edition. ISBN 978-0-387-87707-5

Overview

- ☑ Models for neuronal networks
- ☑ Geometric singular perturbation theory
- ☑ Synchronous spiking
- ☑ Out of phase spiking
- ☑ Clustering
- ☑ Waves

Neuronal Network Activity

Spatiotemporal structure of spiking activity

☑ Neurons may fire action potentials in a synchronous or partially synchronous manner

☑ Spiking of different neurons may be uncorrelated

☑ Activity may propagate through the population in a wavelike manner

☑ Activity may remain localized

Neuronal Network Activity

Population rhythms arise through the interactions between

- ☑ Intrinsic properties of cells within the network
- ☑ Synaptic properties of connections between neurons
- ☑ Topology of the network connectivity

Neuronal Network Activity

Traditional view

- ☑ Excitatory synapses promote in phase synchrony
- ☑ Inhibitory synapses promote out-of-phase activity

Examples demonstrate that it is not always the case

- ☑ Additional dependence on the synaptic properties (rise and decay times)
- ☑ Additional dependence on the interaction between intrinsic and synaptic properties

Mathematical Models

- ☑ Individual cells

$$\frac{dv}{dt} = f(v, w),$$
$$\frac{dw}{dt} = \epsilon g(v, w).$$

$\{f = 0\}$ defines a cubic-shaped curve

$\{g = 0\}$ is a monotonically increasing curve.

$f > 0$ ($f < 0$) below(above) the v-nullcline and $g > 0$ ($g < 0$) below (above) the w-nullcline

there is a threshold, V_T, for the synapses so that if v is in the active (silent) phase, it will be larger (smaller) than V_T

Mathematical Models

- ☑ Individual cells

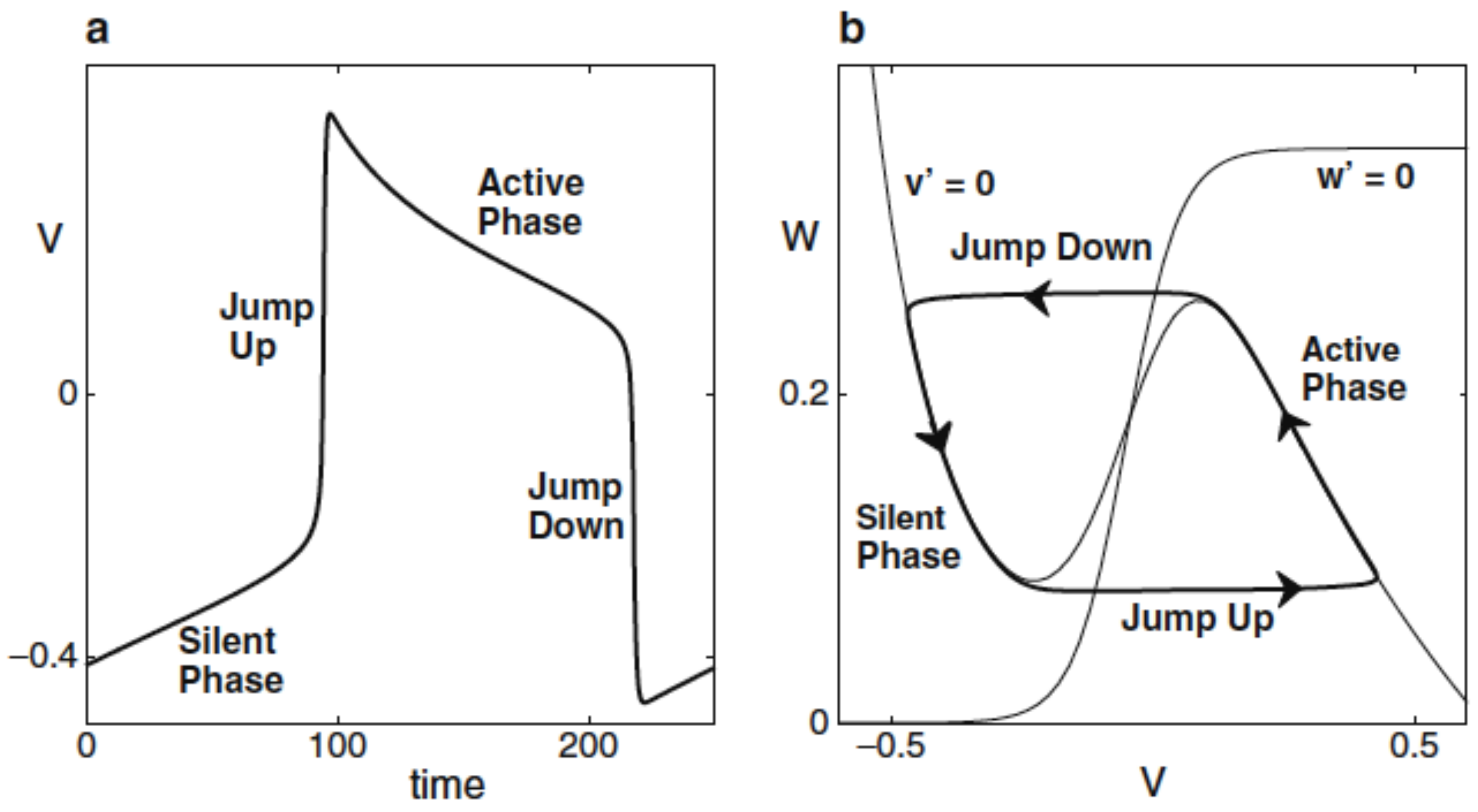


Fig. 9.4 (a) Periodic solution of the Morris–Lecar equations corresponding to an action potential. The projection of this solution onto the (v, w)-phase plane is shown in (**b**)

Mathematical Models

- Synaptic connections

$$I_{\text{syn}} = g_{\text{syn}} s (V_{\text{post}} - v_{\text{syn}})$$

$$\frac{\mathrm{d}s}{\mathrm{d}t} = \alpha(1 - s) H_{\infty}(V_{\text{pre}} - V_{\text{T}}) - \beta s$$

Here, α and β represent the rates at which the synapse turns on and turns off, respectively. Recall that different types of synapses may turn on or turn off at very different rates. For example, GABA_B synapses are slow to activate and slow to turn off, compared with GABA_A and AMPA synapses. We assume H_{∞} is a smooth approximation of the Heaviside step function (or actually is the Heaviside step function) and V_{T} is some threshold.

Mathematical Models

- Pair of mutually coupled neurons

$$\frac{\mathrm{d}v_i}{\mathrm{d}t} = f(v_i, w_i) - g_{\text{syn}} s_j (v_i - v_{\text{syn}}),$$

$$\frac{\mathrm{d}w_i}{\mathrm{d}t} = \epsilon g(v_i, w_i),$$

$$\frac{\mathrm{d}s_i}{\mathrm{d}t} = \alpha(1 - s_i) H_\infty(v_i - V_{\mathrm{T}}) - \beta s_i.$$

i and j are 1 or 2 with $i \neq j$

AMPA

$\mathrm{GABA_A}$

$\mathrm{GABA_B}$

Mathematical Models

- Direct synapses

$$\frac{ds_i}{dt} = \alpha(1 - s_i)H_\infty(v_i - V_T) - \beta s_i$$

- Indirect synapses (secondary processes)

$$\frac{dx_i}{dt} = \epsilon\alpha_x(1 - x_i)H_\infty(v_i - V_T) - \epsilon\beta_x x_i\,,$$
$$\frac{ds_i}{dt} = \alpha(1 - s_i)H(x_i - \theta_x) - \beta s_i\,.$$

α, β, α_x, and β_x are assumed to be independent of ϵ

$GABA_B$ NMDA

Mathematical Models

- Network architecture

$$\frac{dv_i}{dt} = f_i(v_i, w_i) - g^i_{syn}\left(\sum_j W_{ij}s_j\right)\left(v_i - v^i_{syn}\right),$$

$$\frac{dw_i}{dt} = \epsilon g_i(v_i, w_i),$$

$$\frac{ds_i}{dt} = \alpha_i(1 - s_i)H_\infty(v_i - V_T) - \beta_i s_i.$$

W_{ij} represent synaptic weights

probability that there is a connection from cell j to cell i

Mathematical Models

- Examples of firing patterns: Morris-Lecar equations

$$\frac{dv_i}{dt} = I - I_{\text{ion}}(v_i, w_i) - g_{\text{syn}} s_j (v_i - v_{\text{syn}}),$$

$$\frac{dw_i}{dt} = (w_\infty(v_i) - w_i)/\tau_w(v_i),$$

$$\frac{ds_i}{dt} = \alpha(1 - s_i) H_\infty(v_i - V_T) - \beta s_i,$$

i and j are 1 or 2 and $i \neq j$

Mathematical Models

- Examples of firing patterns: Morris-Lecar equations (excitation)

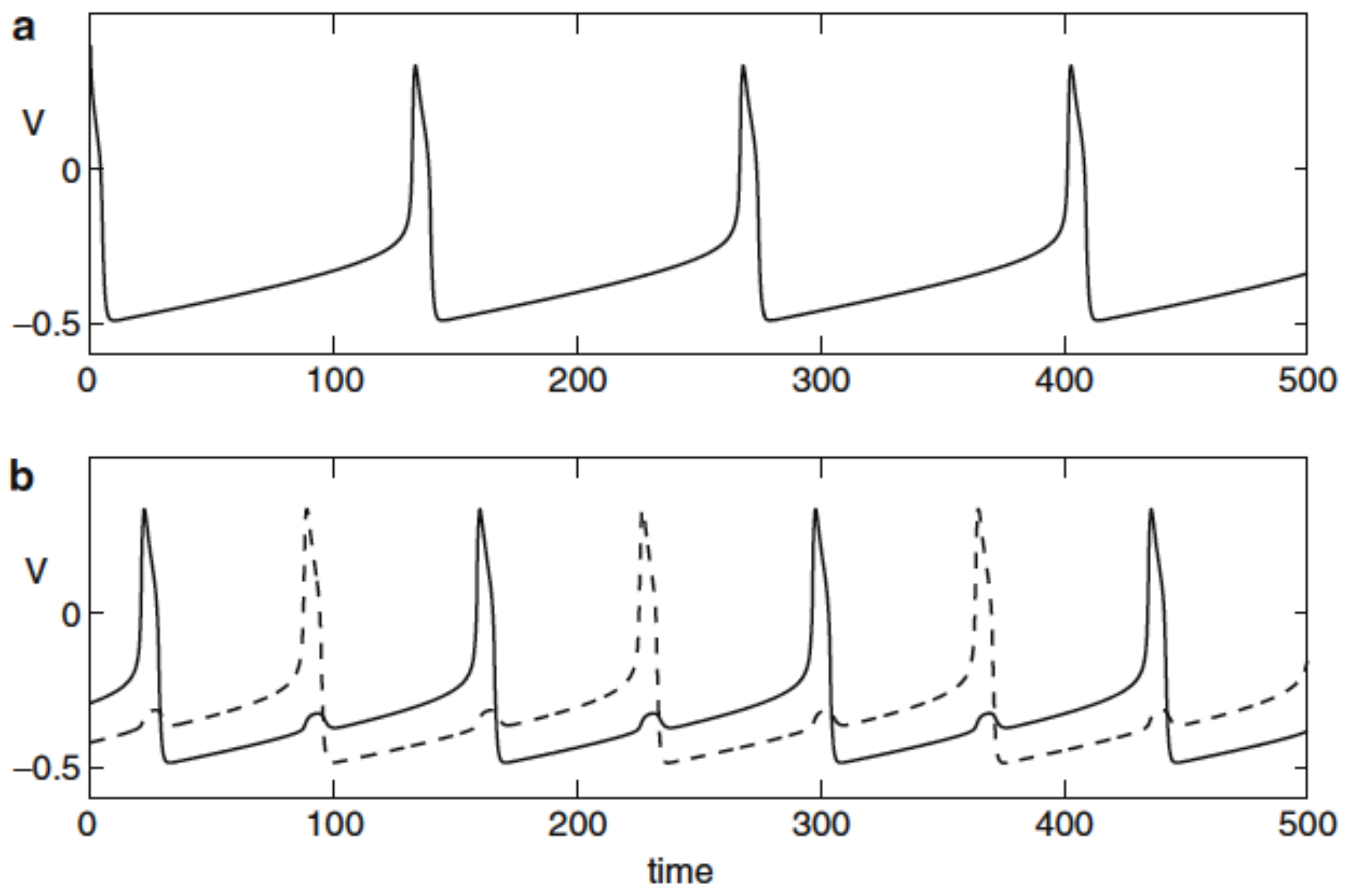


Fig. 9.1 Solutions of a network of two mutually coupled Morris–Lecar neurons with excitatory coupling. (**a**) Synchronous solution. The membrane potentials are equal so only one is shown. (**b**) Antiphase behavior. The solutions shown in (**a**) and (**b**) are for the same parameter values but different initial conditions. Hence, the system is bistable

Mathematical Models

- Examples of firing patterns: Morris-Lecar equations (inhibition)

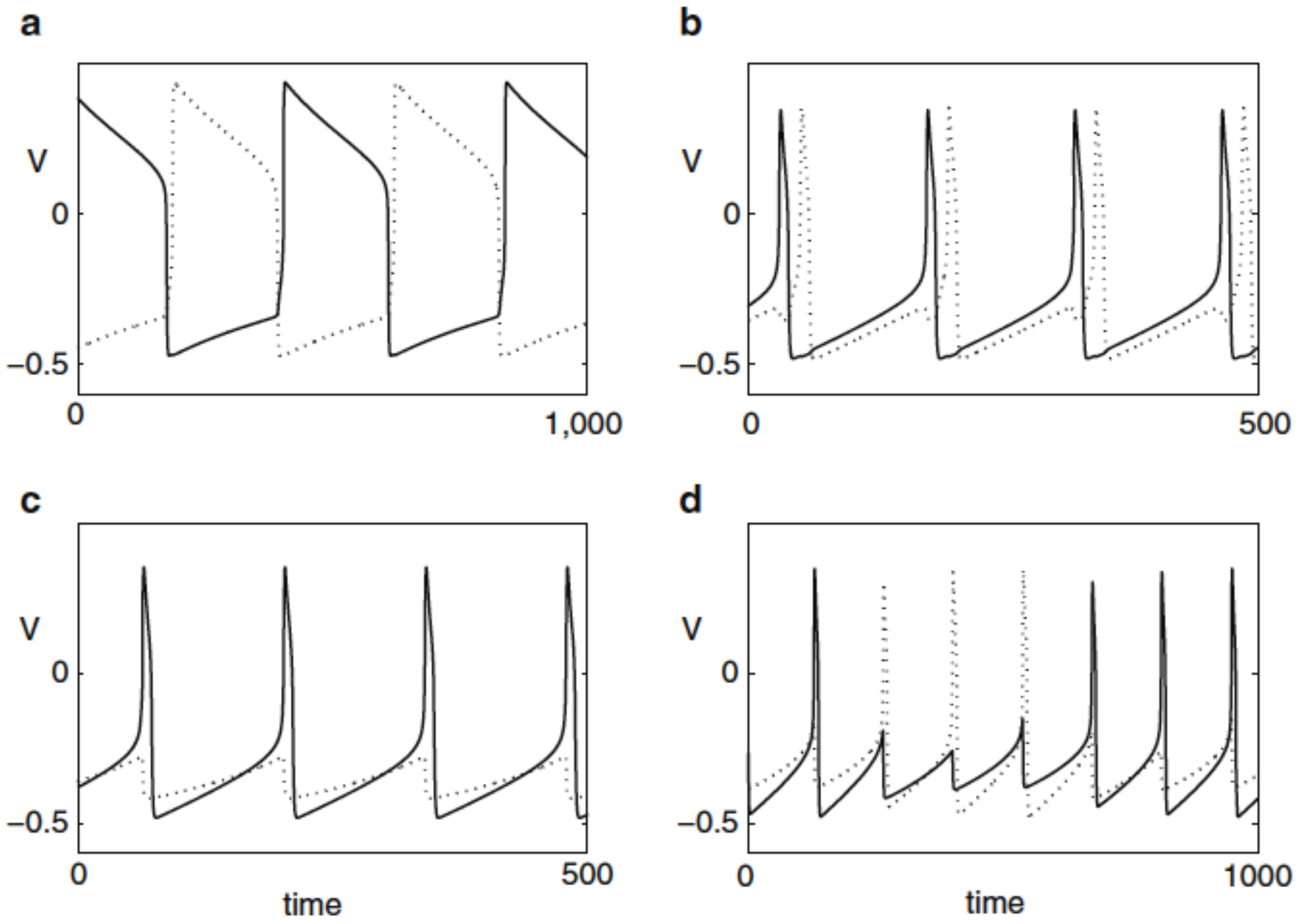


Fig. 9.2 Solutions of a network of two mutually coupled Morris–Lecar neurons with inhibitory coupling. (**a**) Each cell fires owing to postinhibitory rebound. (**b**) An almost-synchronous solution. (**c**) A suppressed solution. (**d**) The cells take turns firing three spikes while the other cell is silent

Mathematical Models

- Examples of firing patterns: clustering (all to all coupling)

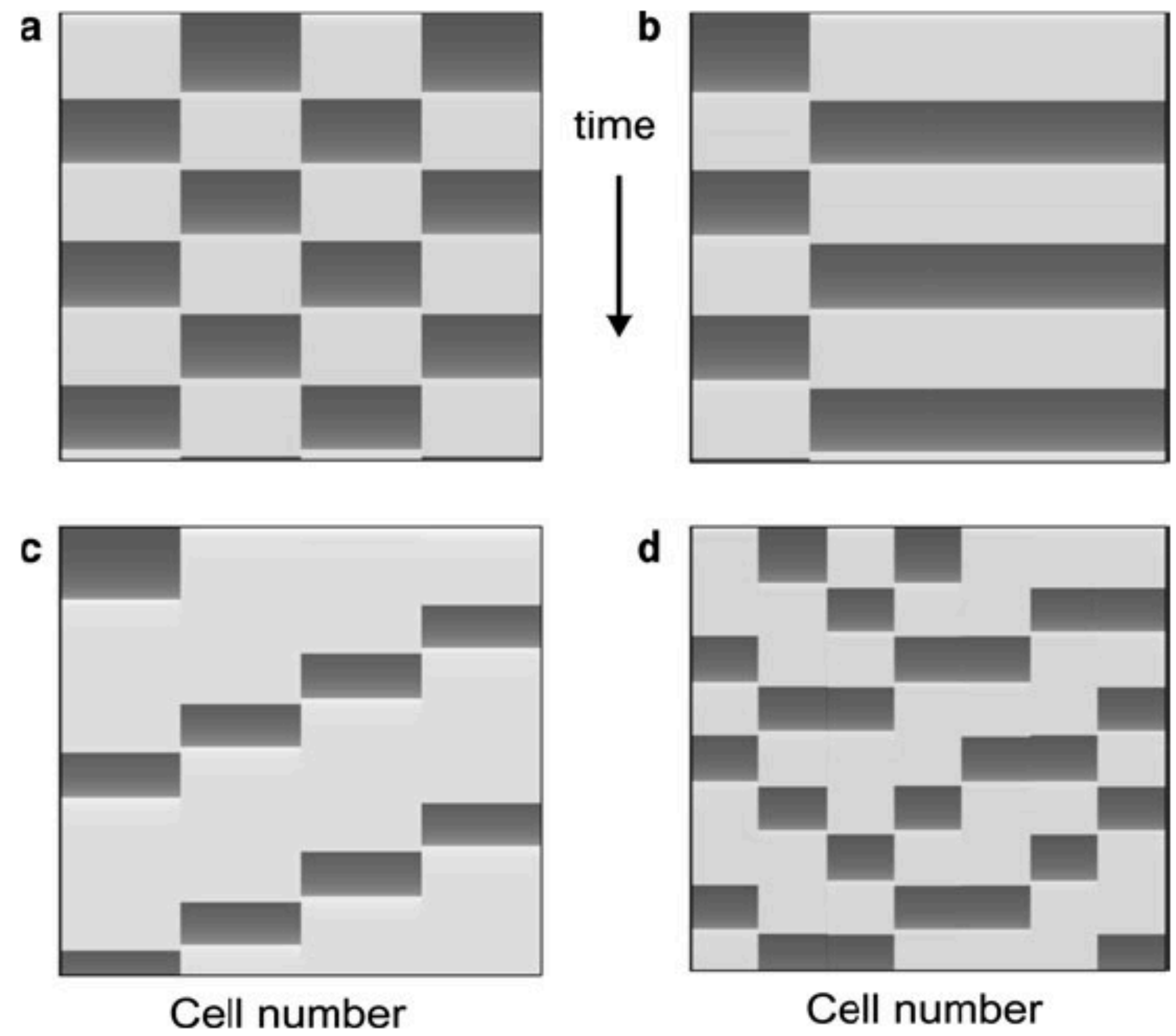


Fig. 9.3 Firing patterns in inhibitory networks. (**a**) and (**b**) show examples of clustering. Wavelike behavior is shown in (**c**) and dynamic clustering in (**d**). The *columns* represent the time evolution of a single cell; a *dark rectangle* corresponds to when the cell is active

Mathematical Models

- Singular construction of the action potential

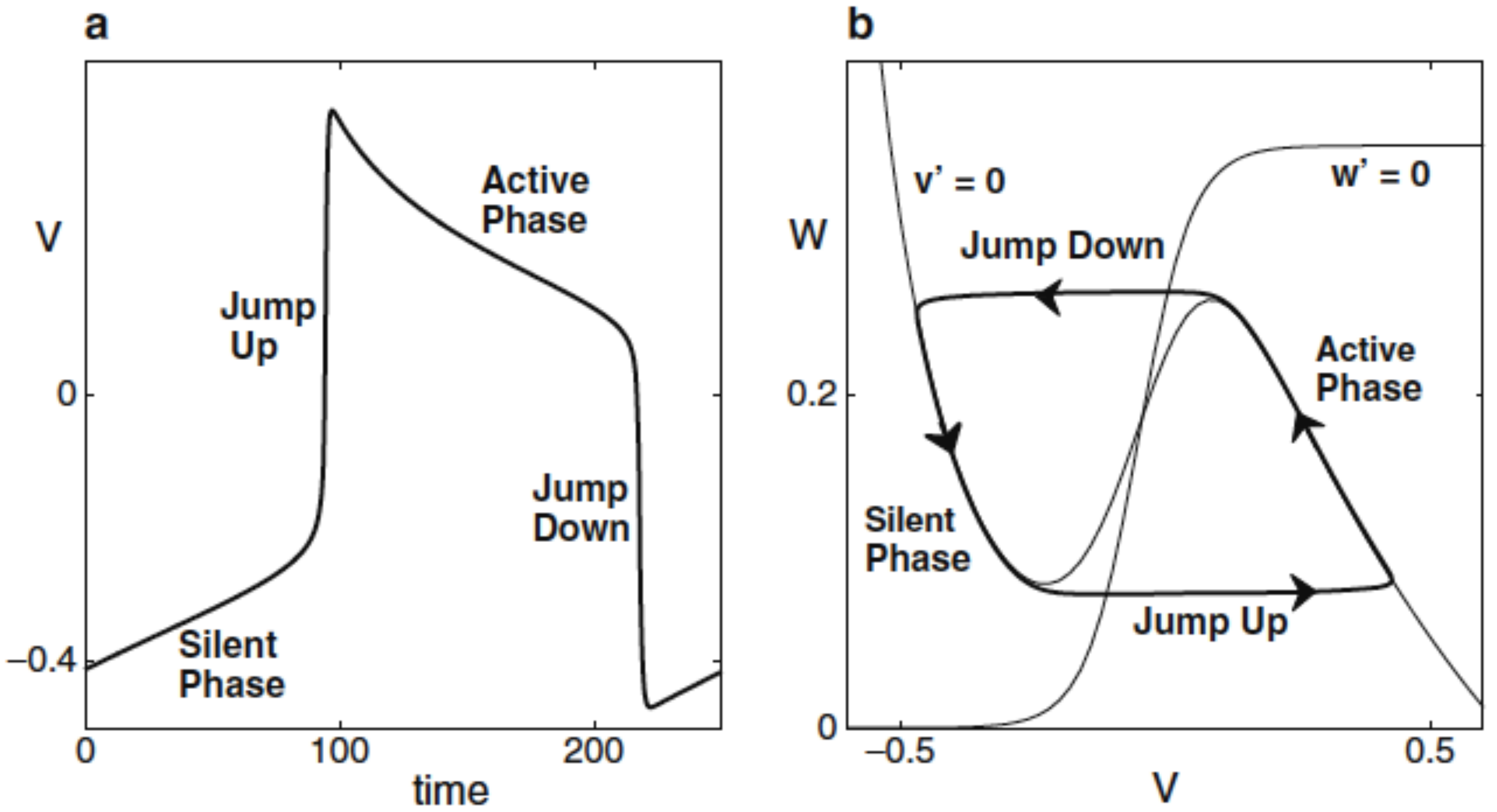


Fig. 9.4 (**a**) Periodic solution of the Morris–Lecar equations corresponding to an action potential. The projection of this solution onto the (v, w)-phase plane is shown in (**b**)

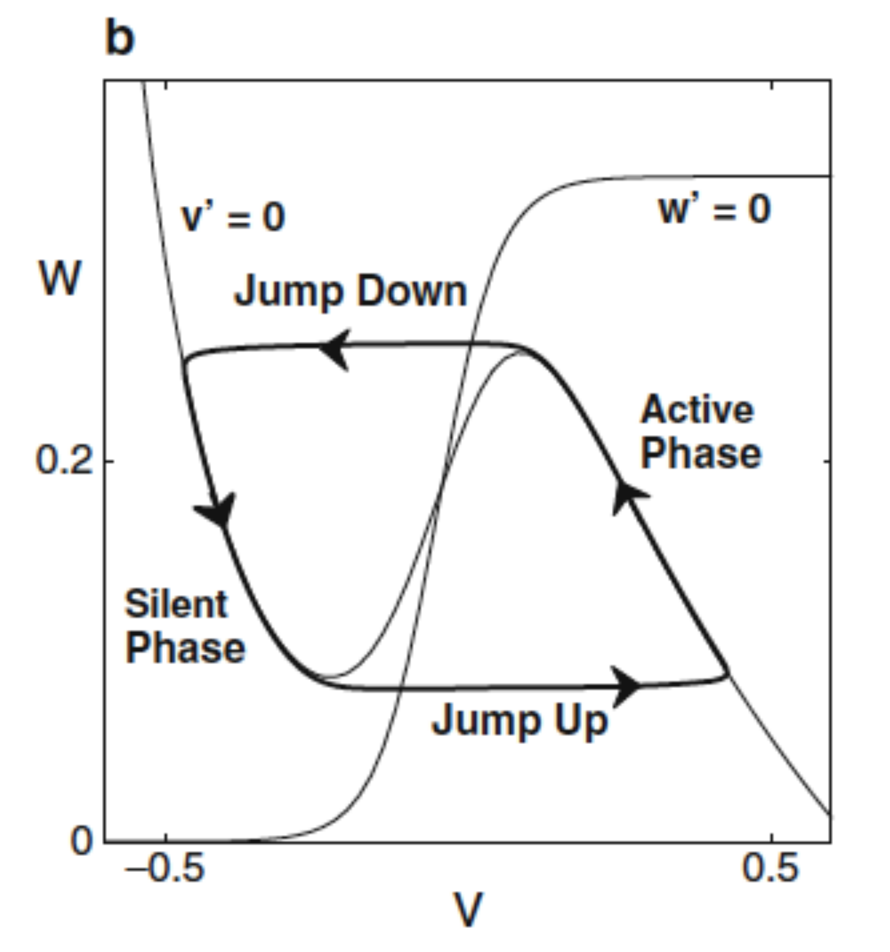
MATHEMATICAL MODELS

☑ Singular construction of the action potential

Geometric singular perturbation theory

$$\frac{dv}{dt} = f(v, w),$$
$$\frac{dw}{dt} = \epsilon g(v, w).$$

t	fast timescale
$\tau = \epsilon t$	slow timescale



Mathematical Models

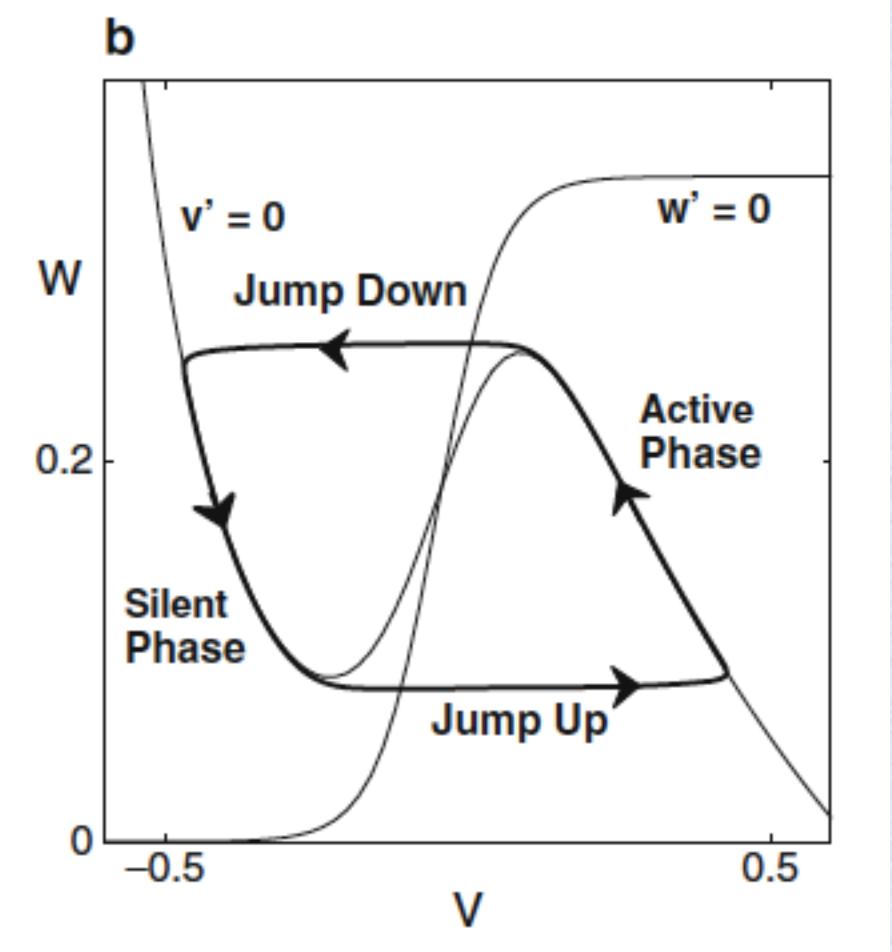
- Singular construction of the action potential

Geometric singular perturbation theory

$\epsilon = 0$

$$\frac{dv}{dt} = f(v, w),$$
$$\frac{dw}{dt} = 0.$$

$$\frac{dv}{dt} = f(v, w),$$
$$\frac{dw}{dt} = 0.$$



Mathematical Models

☑ Singular construction of the action potential

Example: square pulse of current

$$\frac{dv}{dt} = f(v, w) + I(t),$$
$$\frac{dw}{dt} = \epsilon g(v, w).$$

We assume

the system is excitable when $I(t) = 0$;

there exist I_0 and $T_{on} < T_{off}$ such that

$$I(t) = \begin{cases} I_0 & \text{if } T_{on} < t < T_{off} \\ 0 & \text{otherwise.} \end{cases}$$

Mathematical Models

- ☑ Singular construction of the action potential

Example: square pulse o current

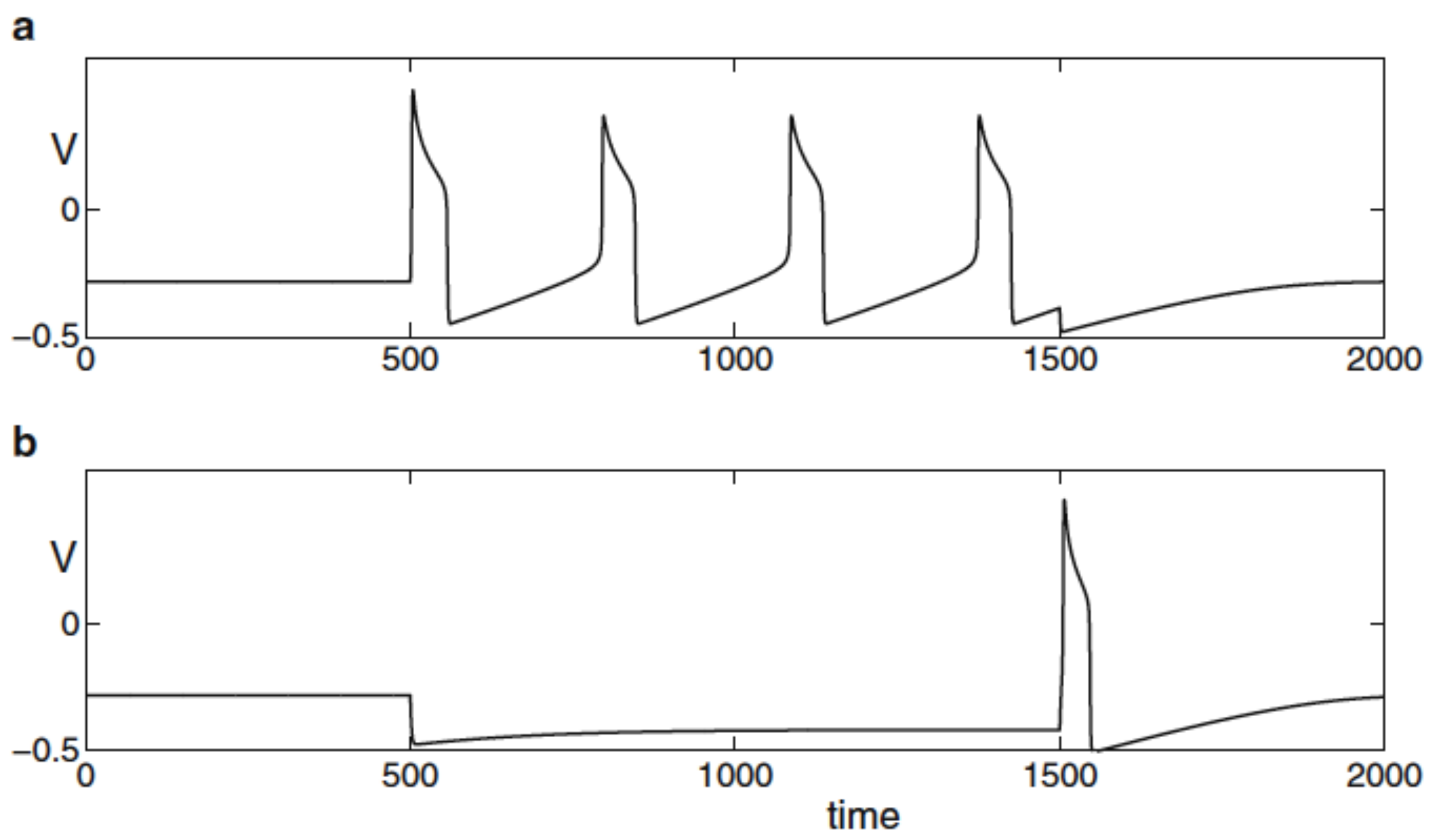


Fig. 9.5 Response of a model neuron to applied current. Current is applied at $t = 500$ and turned off at $t = 1{,}500$. **(a)** The current is depolarizing and the neuron fires a series of action potentials. **(b)** The current is hyperpolarizing and the neuron exhibits postinhibitory rebound

Mathematical Models

- ☑ Singular construction of the action potential

Example: square pulse o current

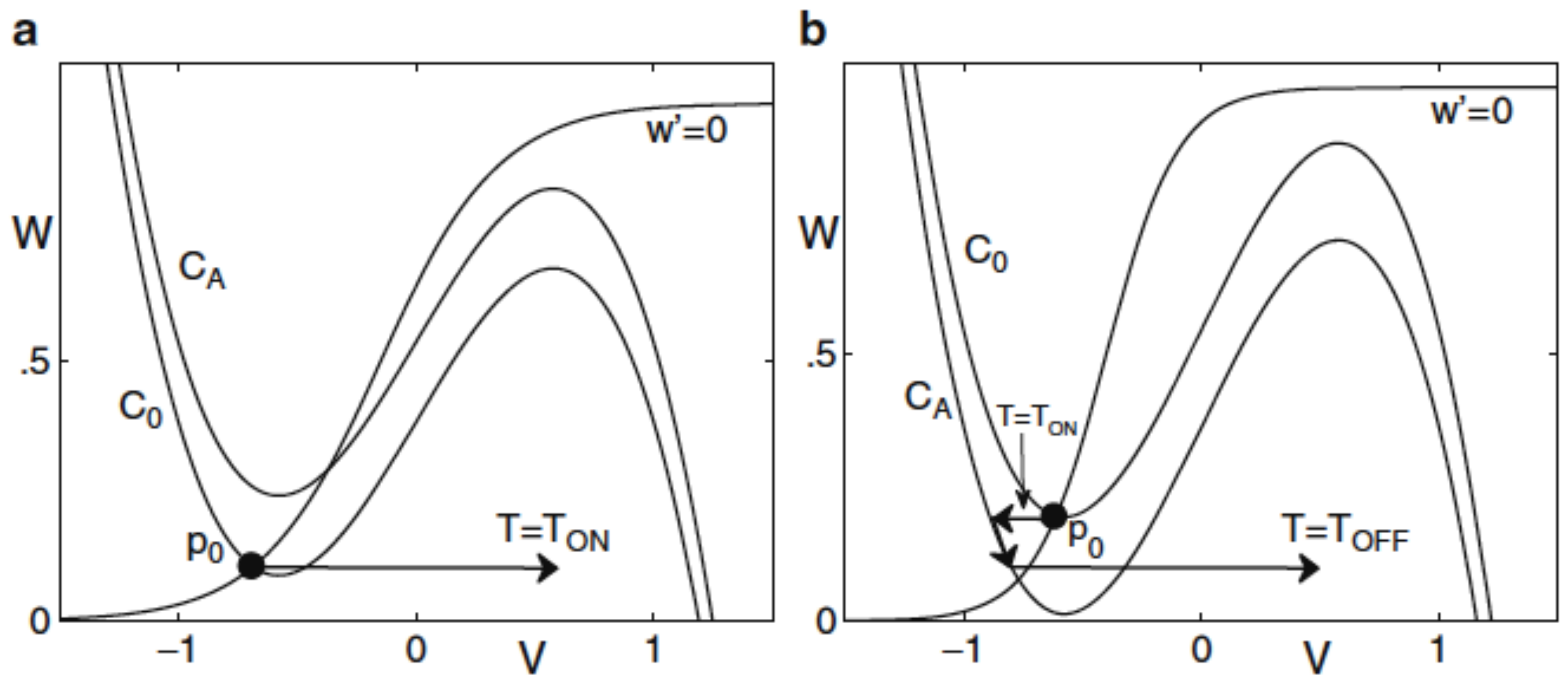


Fig. 9.6 Phase-space representation of the response of a model neuron to applied current. Current is applied at time $t = T_{on}$ and turned off at $t = T_{off}$. **(a)** Depolarizing current. The cell jumps up as soon as the current is turned on. **(b)** Hyperplorizing current. The cell jumps to the left branch of C_A when the current is turned on and jumps up to the active phase owing to postinhibitory rebound when the current is turned off

Mathematical Models

- Synchrony with excitatory synapses

$$(v_1, w_1, s_1) = (v_2, w_2, s_2) \equiv (v, w, s)$$

$$\frac{dv}{dt} = f(v,w) - g_{syn}s(\ldots$$
$$\frac{dw}{dt} = \epsilon g(v,w),$$
$$\frac{ds}{dt} = \alpha(1-s)H_\infty(v - \ldots$$

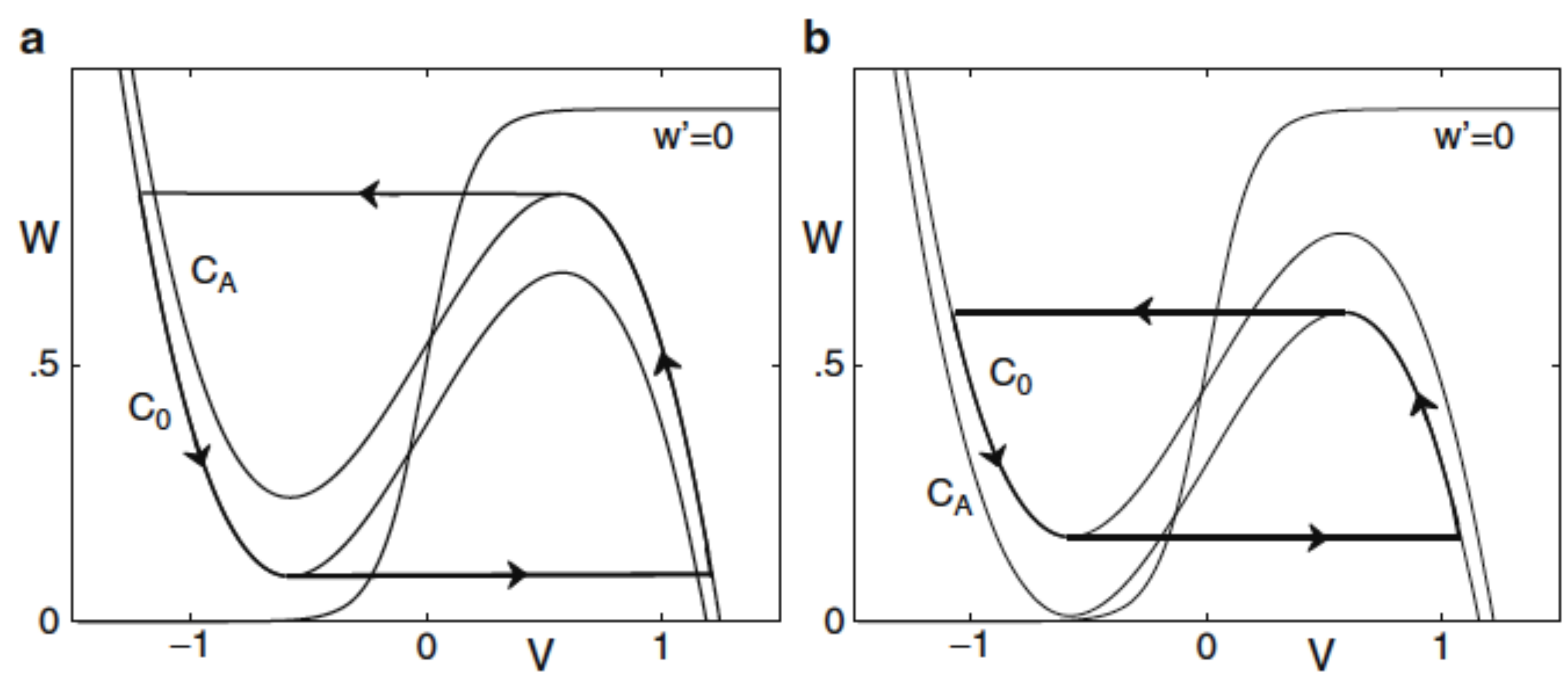


Fig. 9.7 Synchronous singular trajectories corresponding to (a) excitatory synapses and (b) inhibitory synapses

Mathematical Models

- Synchrony with excitatory synapses

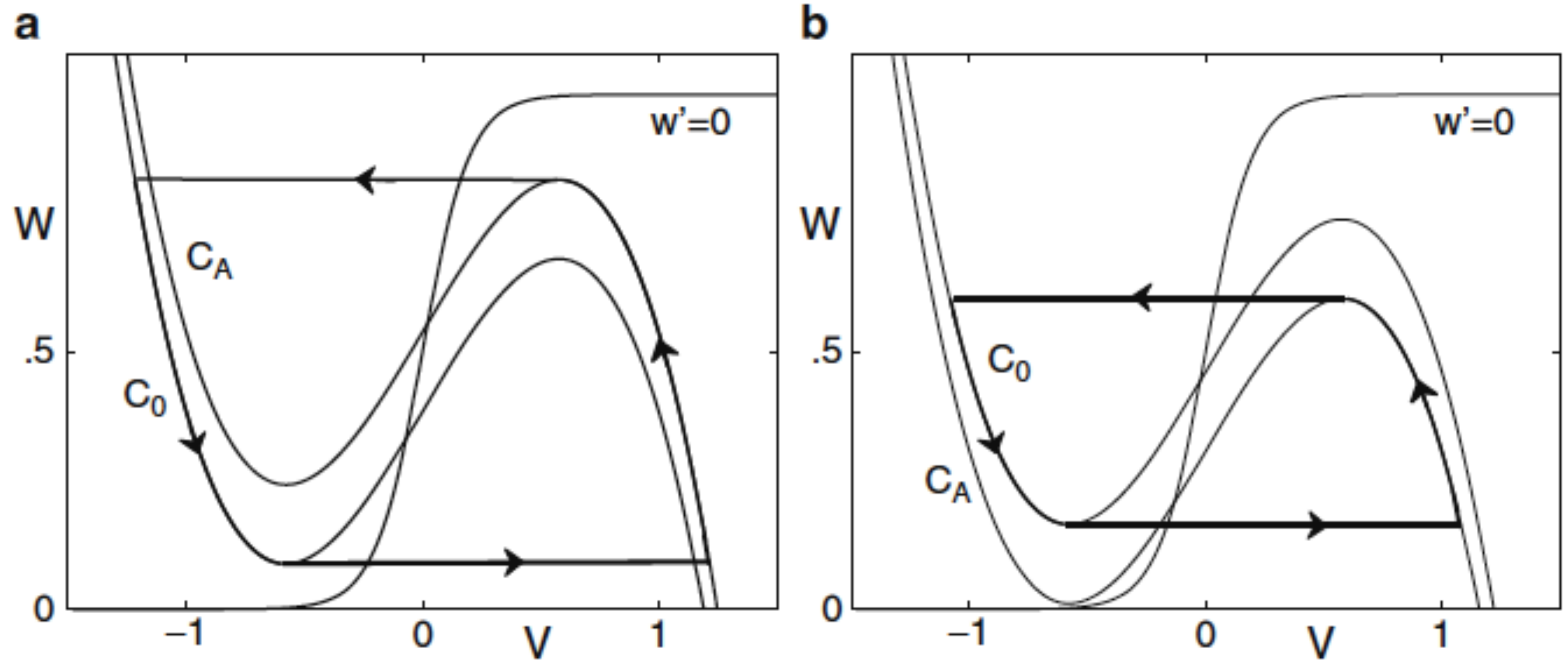
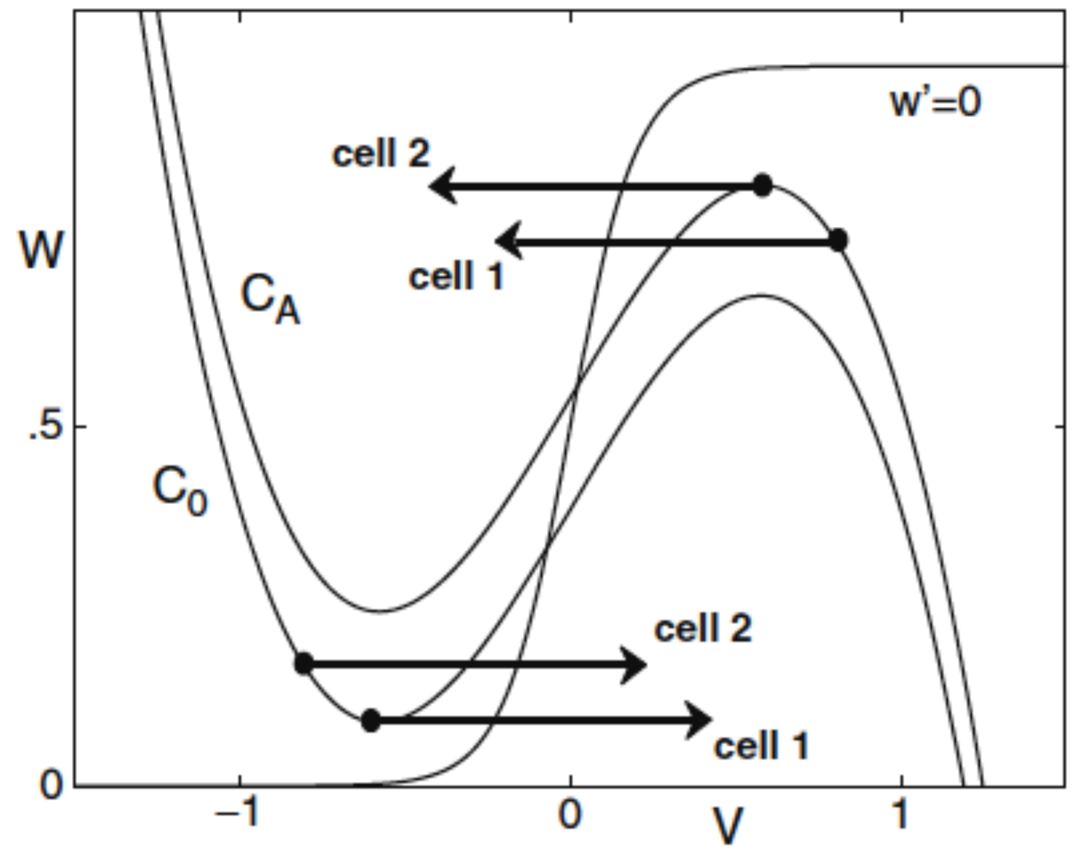


Fig. 9.7 Synchronous singular trajectories corresponding to (**a**) excitatory synapses and (**b**) inhibitory synapses

Mathematical Models

- ☑ Synchrony with excitatory synapses

Fig. 9.8 Fast threshold modulation



Mathematical Models

- Post-inhibitory rebound: two mutually coupled cells

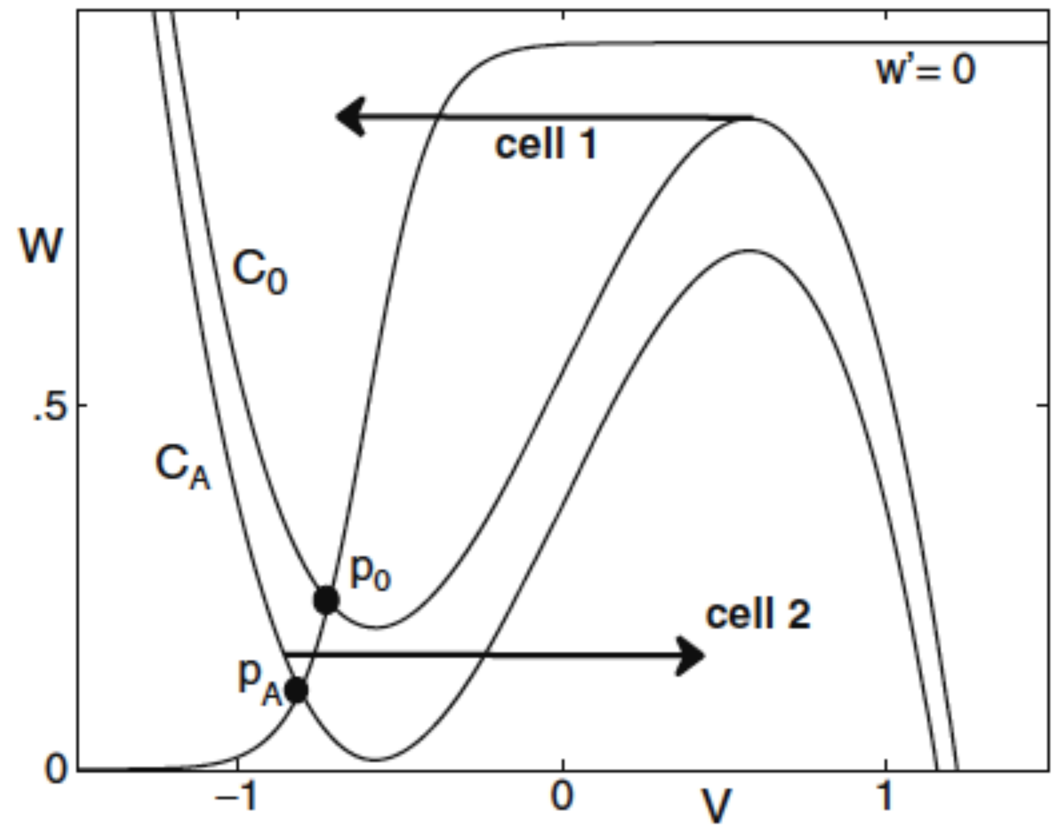


Fig. 9.9 Postinhibitory rebound

Mathematical Models

- Post-inhibitory rebound: two mutually coupled cells

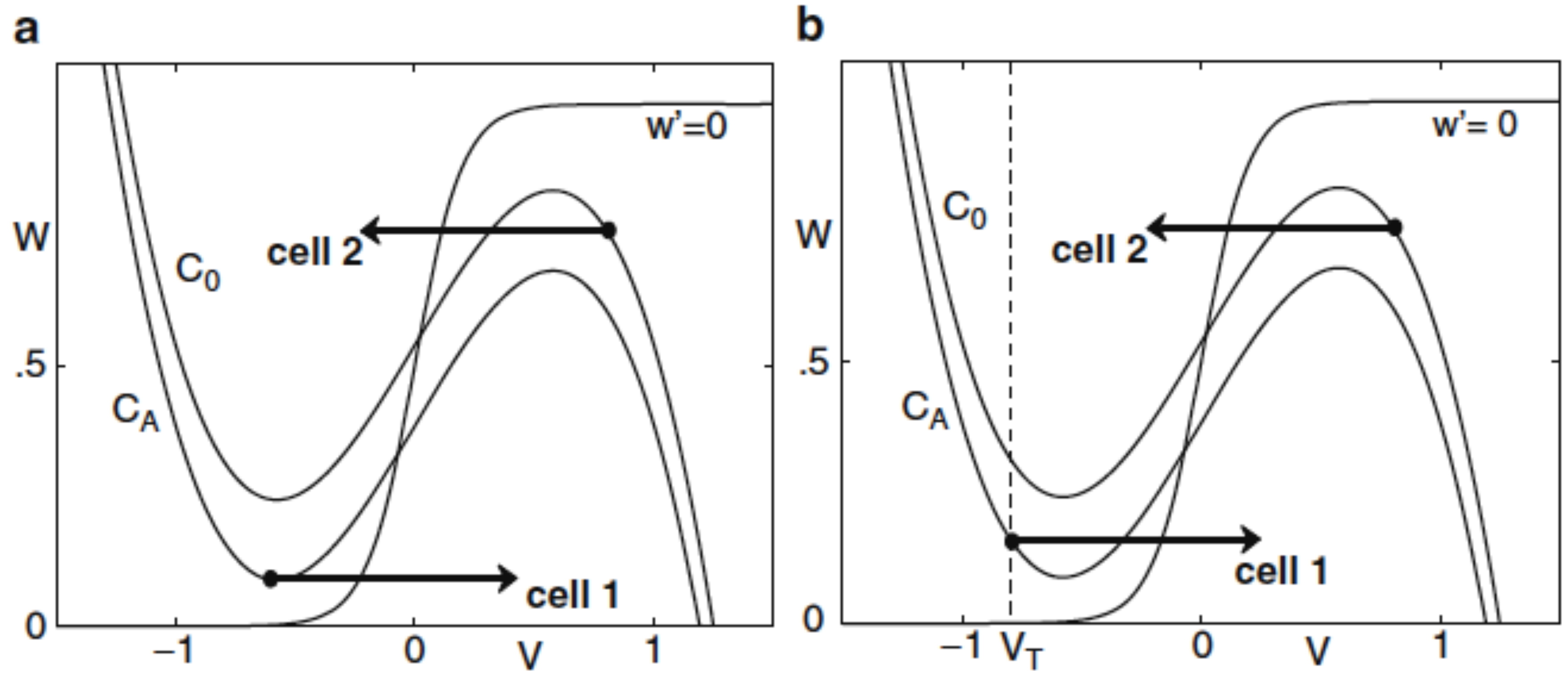


Fig. 9.10 (**a**) Cellular and (**b**) synaptic escape mechanisms

Mathematical Models

- Antiphase oscillations with excitatory coupling

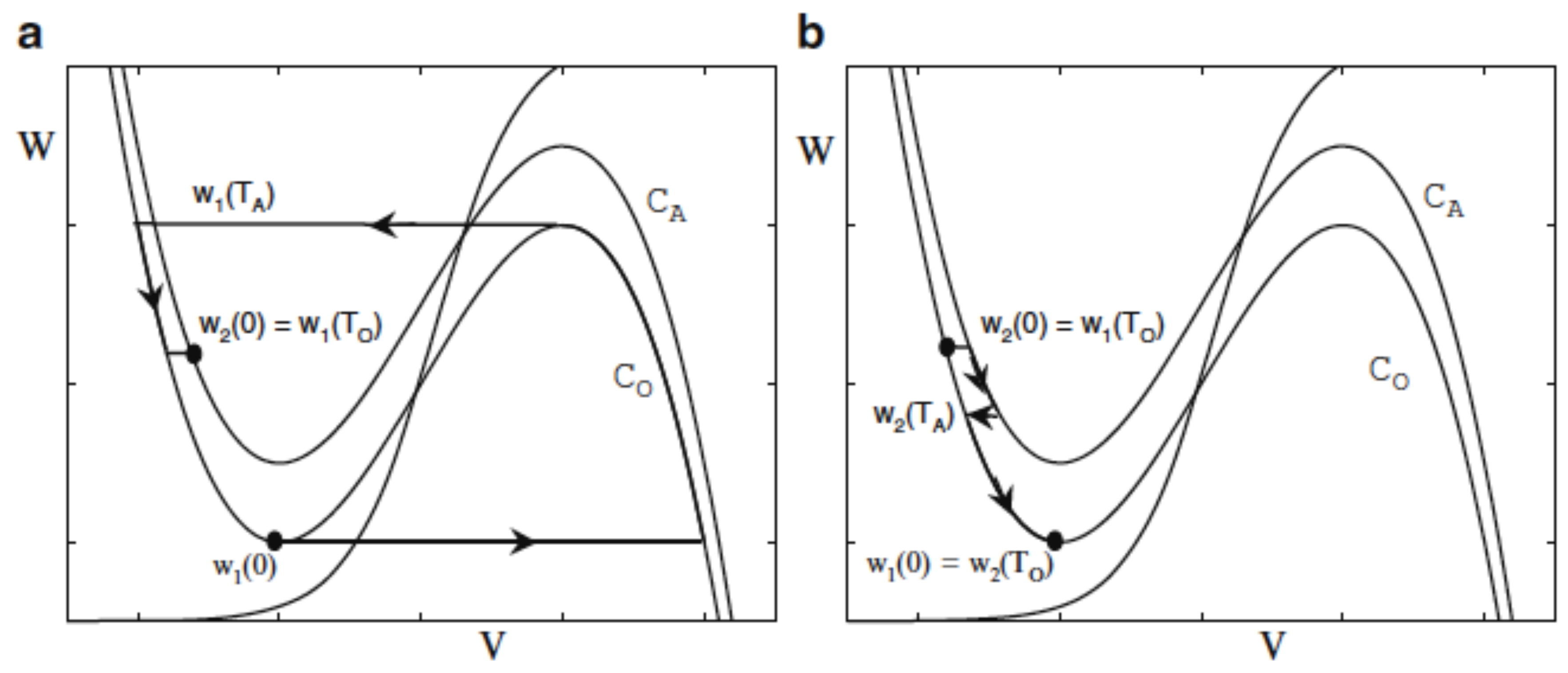


Fig. 9.13 Singular construction of antiphase solution. (**a**) Cell 1 and (**b**) cell 2

Mathematical Models

- ☑ Propagating waves:

thalamic network model

$$\frac{dv}{dt} = f(v,h),$$
$$\frac{dh}{dt} = \epsilon g(v,h)/\tau(v).$$

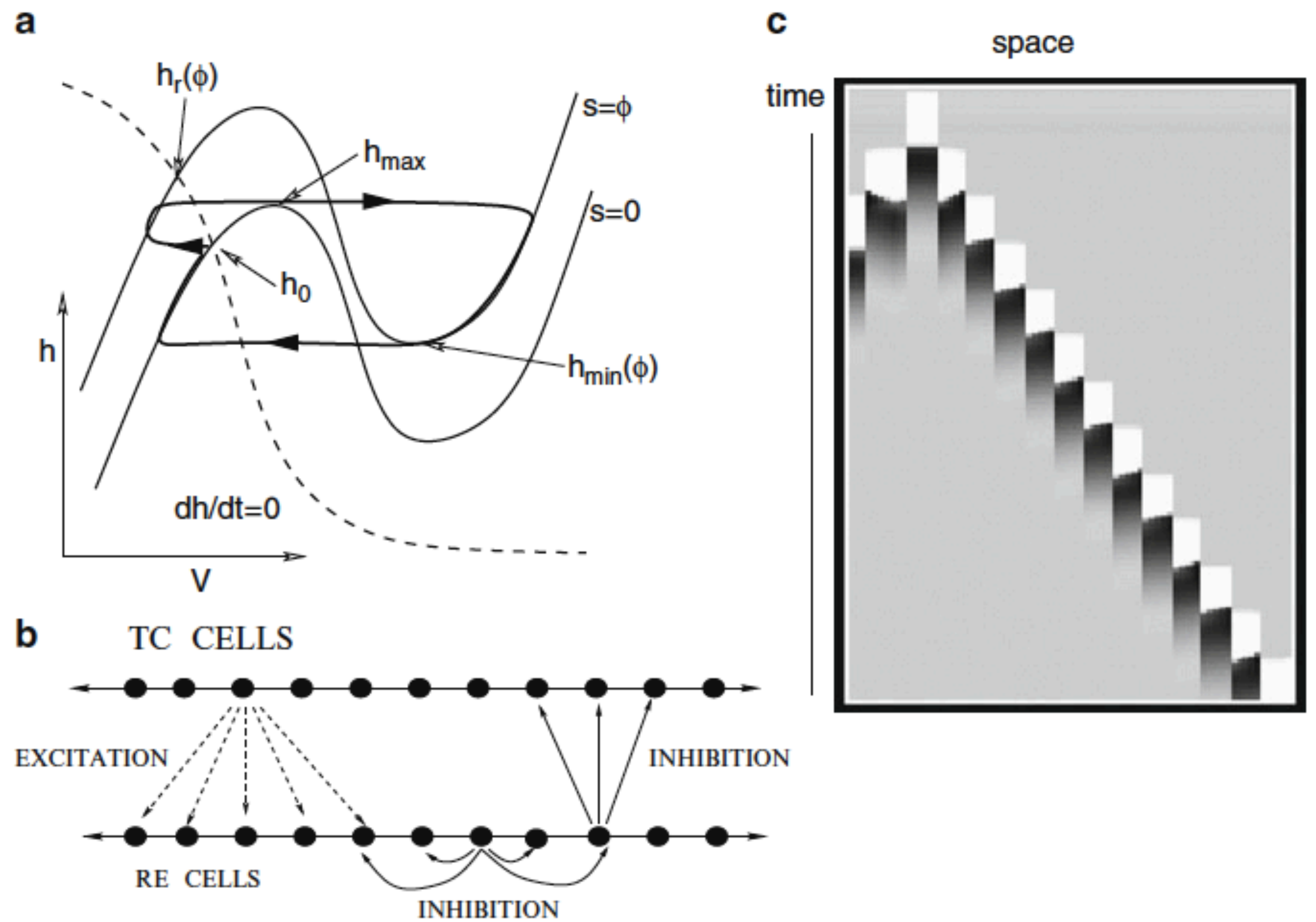


Fig. 9.18 Thalamic network model. **(a)** Phase plane showing the h-nullcline (*dashed line*) and v-nullcline at rest ($s = 0$). Several important values of h are shown. The approximate singular trajectory of a lurching wave is drawn in *thick lines*. **(b)** The architecture of the full model. **(c)** A simulation of a lurching wave. The *gray scale* depicts voltage; *white* corresponds to −40 mV and *black* corresponds to −90 mV. *TC* thalamocortical, *RE* reticular nucleus