Methods of Applied Mathematics II (Math 451H)
Spring 2014

Modeling Assignment I

Consider the following passive membrane equation

$$C\,\frac{dV}{dt} = -G_L\,(V - E_L) + I_{app} + I_{in}(t) \tag{1}$$

where V is the membrane potential (mV), t is time measured in msec , C is the membrane capacitance (μF/cm^2), I_{app} is the applied bias (DC) current (μA/cm^2), $I_{in}(t)$ is a time-dependent input current (μA/cm^2),

Write a code to solve eq. (1) using the modified Euler method (Runge-Kutta, order 2) with time step $\Delta t = 0.1$.

Consider the following parameter values: $E_L = -65$, $G_L = 0.1$, $C = 1$

1. Plot the numerical solution to eq. (1) for $I_{in}(t) = 0$ and
   (a) $I_{app} = 0.5$.
   (b) $I_{app} = 1$.
2. Plot the numerical solution to eq. (1) for $I_{app} = 1$ and

   $$I_{in}(t) = I_{app}\,Heav(t - t_i) * Heav(t_f - t),$$

   where $Heav(t)$ is the Heaviside function, with
   (a) $t_i = 100$ and $t_f = 200$.
   (b) $t_i = 100$ and $t_f = 400$.
3. Plot the numerical solution to eq. (1) for $I_{app} = 1$ and

   $$I_{in}(t) = I_{app}\ \sin(2\,\pi\,\omega\,t/1000),$$

   with
   (a) $\omega = 1$
   (b) $\omega = 5$

(c) $\omega = 10$

(d) $\omega = 20$

(e) $\omega = 100$

4. (a) Plot a graph relating the output frequency (y-axis) vs. the input frequency (x-axis) for $\omega \in [0, 100]$.

   (b) Plot a graph relating the amplitude of the output oscillations (y-axis) vs. the input frequency (x-axis) or $\omega \in [0, 100]$.

5. Calculate the time constant $\tau = C \,/\, G_L$. What information does the time constant provide about the solution?

6. Design an numerical experiment that allows you to infer the values of C, G_L and E_L from a blind code that generates data using these parameters. (You can do simulations, but you do not have access to the parameters.) Use $I_{in}(t) = 0$ and change I_{app} according to the experimental needs.