Methods of Applied Mathematics II (Math 451H)
Spring 2014

Modeling Assignment III

Build an isopotential model of Hodgkin-Huxley type for the generation of action potentials.

The current balance equation is given by

$$C\,\frac{dV}{dt} = I_{app} - G_L\,(V - E_L) - G_{Na}\,m^3\,h\,(V - E_{Na}) - G_K\,n^4\,(V - E_k). \tag{1}$$

Use the following parameter values: $C = 1\,\mu F/cm^2$, $E_L = -52\,mV$, $E_{Na} = 55\,mV$, $E_K = -75\,mV$, $G_L = 0.3$, $G_{na} = 120\,mS/cm^2$ and $G_K = 36\,mS/cm^2$, $\phi = 1$.

The gating variables x ($x = m, h, n$) obey differential equations of the form

$$\frac{dx}{dt} = \phi\,\frac{x_\infty(V) - x}{\tau_x(V)}. \tag{2}$$

The voltage-dependent activation/inactivation curves $x_\infty(V)$ are given by

$$m_\infty(V) = \frac{1}{1 + e^{-(V+40)/9}},$$

$$h_\infty(V) = \frac{1}{1 + e^{(V+62)/10}},$$

and

$$n_\infty(V) = \frac{1}{1 + e^{-(V+53)/16}}.$$

The voltage-dependent time constants $\tau_x(V)$ are given by

$$\tau_m(V) = 0.3,$$

$$\tau_h(V) = 1 + \frac{11}{1 + e^{(V+62)/10}},$$

and

$$\tau_n(V) = 1 + \frac{6}{1 + e^{(V+53)/16}}.$$

1. Plot

   (a) the voltage-dependent activation/inactivation curves (superimposed).

   (b) the voltage-dependent time constants (superimposed).

2. Apply a square pulse of tonic (DC) current (applied current) of 1000 msec duration and amplitude I_{app}. You may want to use $V(0) = -80$, $m(0) = 0$, $h(0) = 0$ and $n(0) = 1$ and wait enough time (conservative value of ~ 1000 msec) until the voltage relaxes to equilibrium before increasing I_{app}.

   (a) Quantify the effect of I_{app}. Plot voltage traces (graphs of V vs. t) for various values of I_{app} in the range $[0, 4]$. What is the threshold value of I_{app} for which the model produces spikes?

   (b) Quantify the dependence of the spiking frequency (inverse of the interspike-interval) as a function of I_{app}. Plot the corresponding graph.

   (c) Quantify the dependence of the spiking frequency as a function of the parameter ϕ for $I_{app} = 4$. Plot the corresponding graph.

3. Use to same protocol as before to quantify the effect of changes in the following parameters

   (a) G_{Na} for $I_{app} = 4$

   (b) G_K for $I_{app} = 4$.

   (c) G_L for $I_{app} = 4$.