Methods of Applied Mathematics II (Math451H): Neuronal Dynamics

Sun, Nov 23, 2014

Integrate-and-fire neuron models

- A neuron will typically fire and action potential (or spike) when its membrane potential reaches a threshold value of about -55 to -50 mV

- Action potentials:

  - V increases rapidly and then decreases to a value below threshold

  - Stereotypical trajectory

  - The mechanism involves the activation of Na^+ and K^+ conductances

Integrate-and-fire neuron models

- Passive membrane equation

$$\tau \frac{dV}{dt} = - V + E_L + R\, I_{app}(t) \qquad V(0) = V_0$$

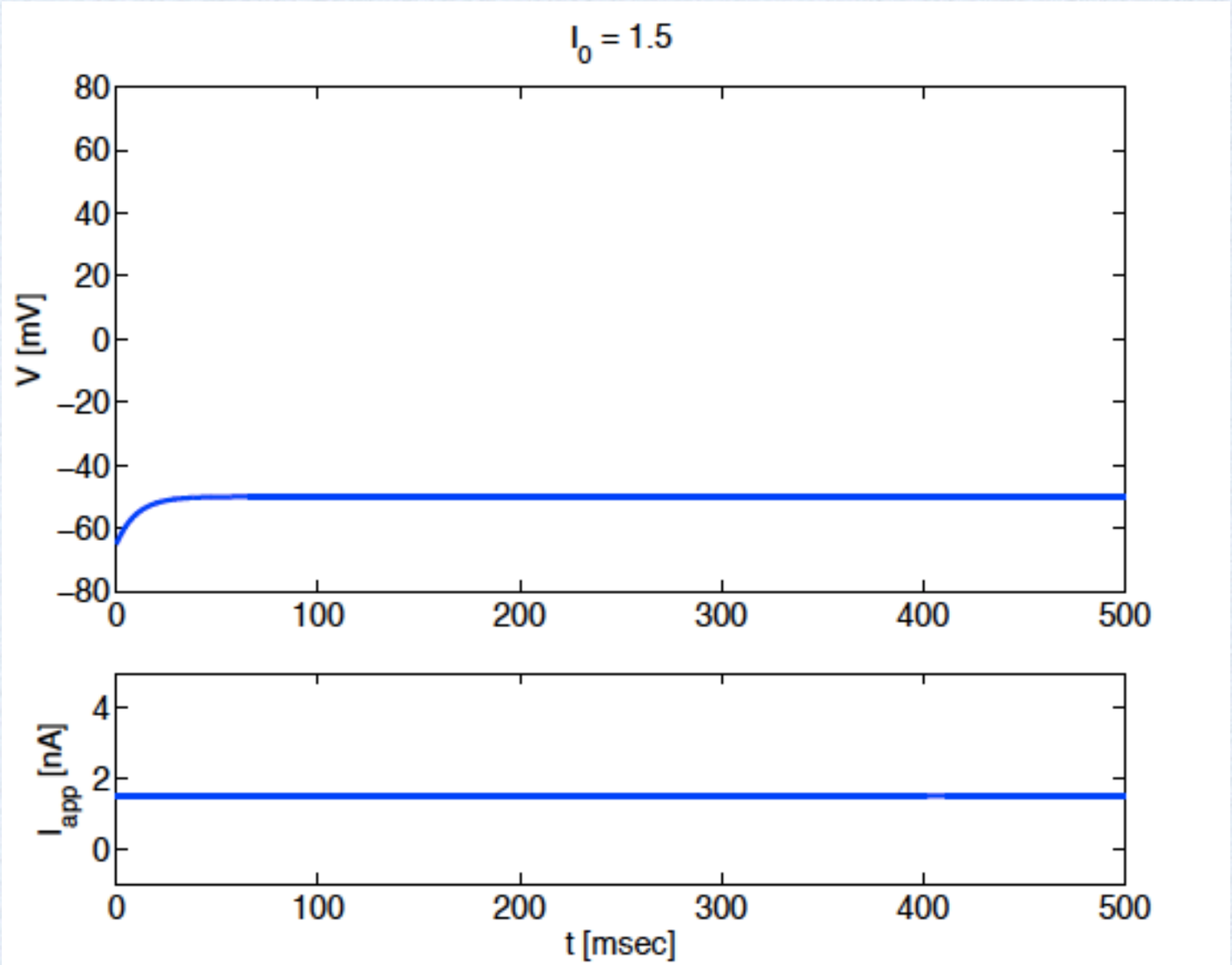
- An action potential occurs whenever $V = V_{th}$ (voltage threshold for spike generation)

- After the action potential, V is reset to a value $V = V_{rst}$ (voltage reset value)

Integrate-and-fire neuron models

- E_L= -65 V_{th}= -50 V_{rst} = -65 τ = 10 R = 10

(Units: mV, nA, msec, MΩ)



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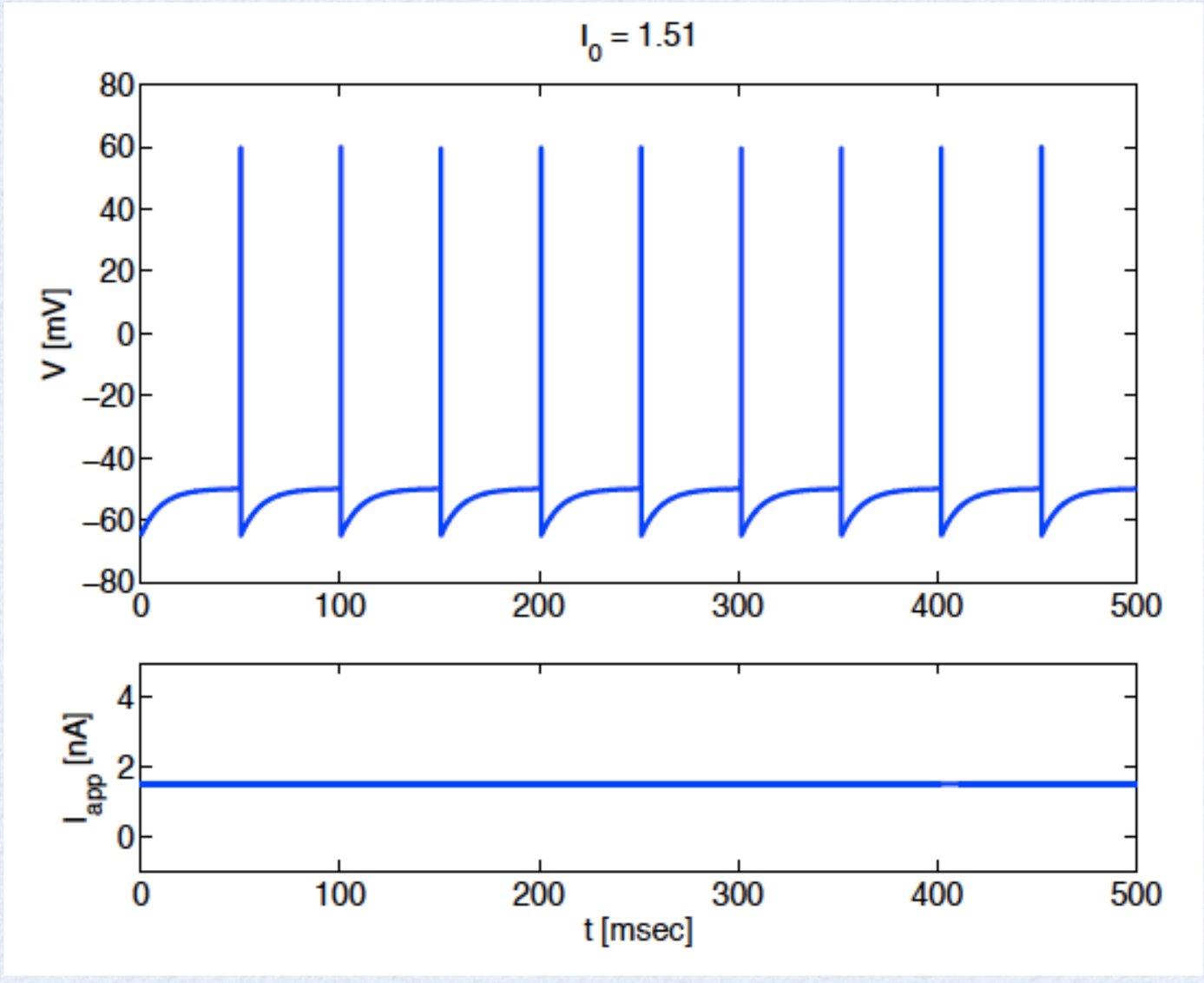

Integrate-and-fire neuron models

- Calculation of the firing frequency (r_{isi}) and inter-spike interval (ISI)

$$V(t) = \boxed{V_\infty + E_L} + (V_0 - E_L - V_\infty)\, e^{-t/\tau}$$

$$t_{isi} = \tau \ln \frac{V_{rst} - V_\infty - E_L}{V_{th} - V_\infty - E_L}$$

$$r_{isi} = \frac{1}{t_{isi}} = \frac{1}{\tau} \left[\ln \frac{V_{rst} - V_\infty - E_L}{V_{th} - V_\infty - E_L} \right]^{-1}$$

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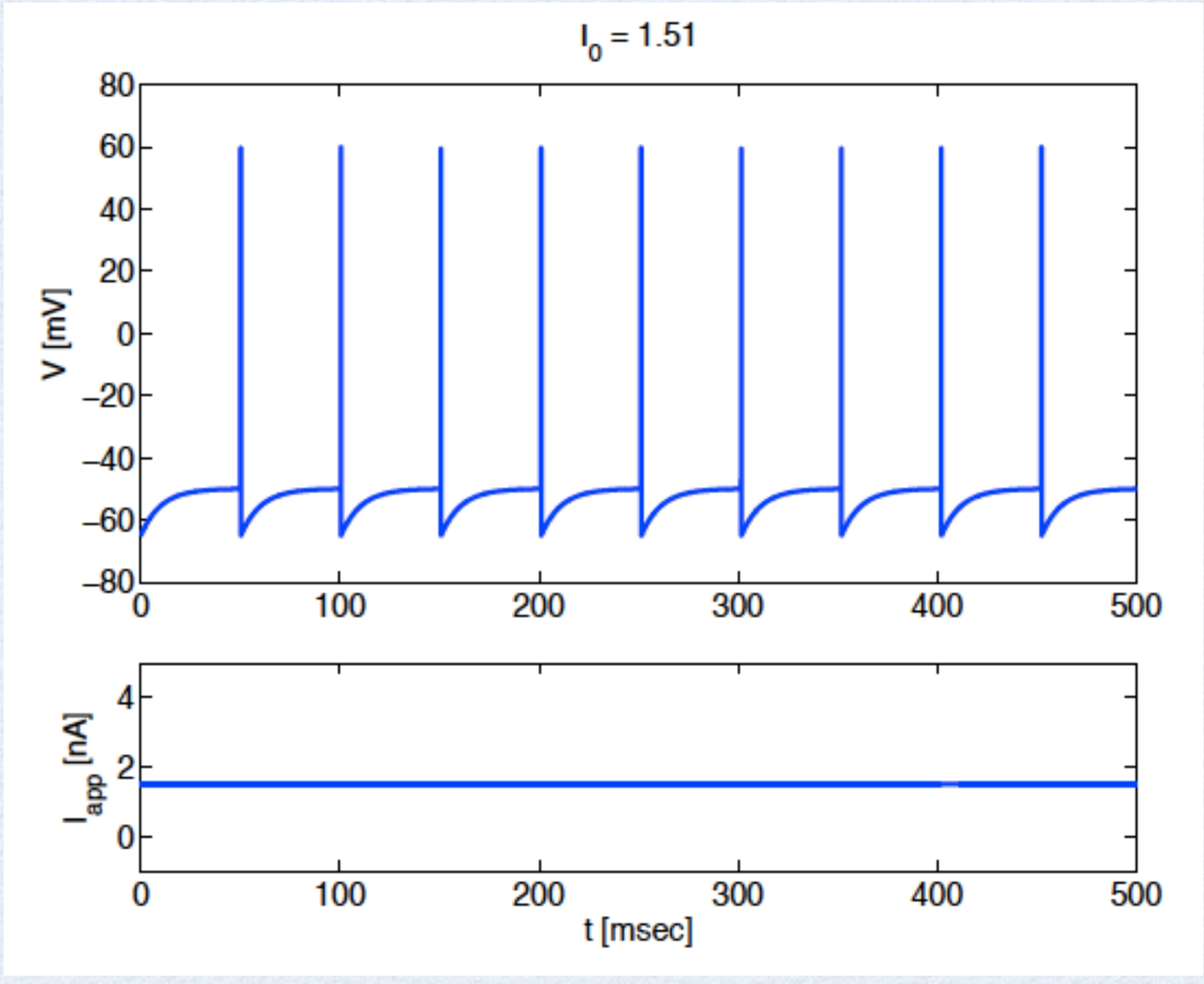
t_{isi} ~ 50 msec

r_{isi} ~ 20 spk /sec



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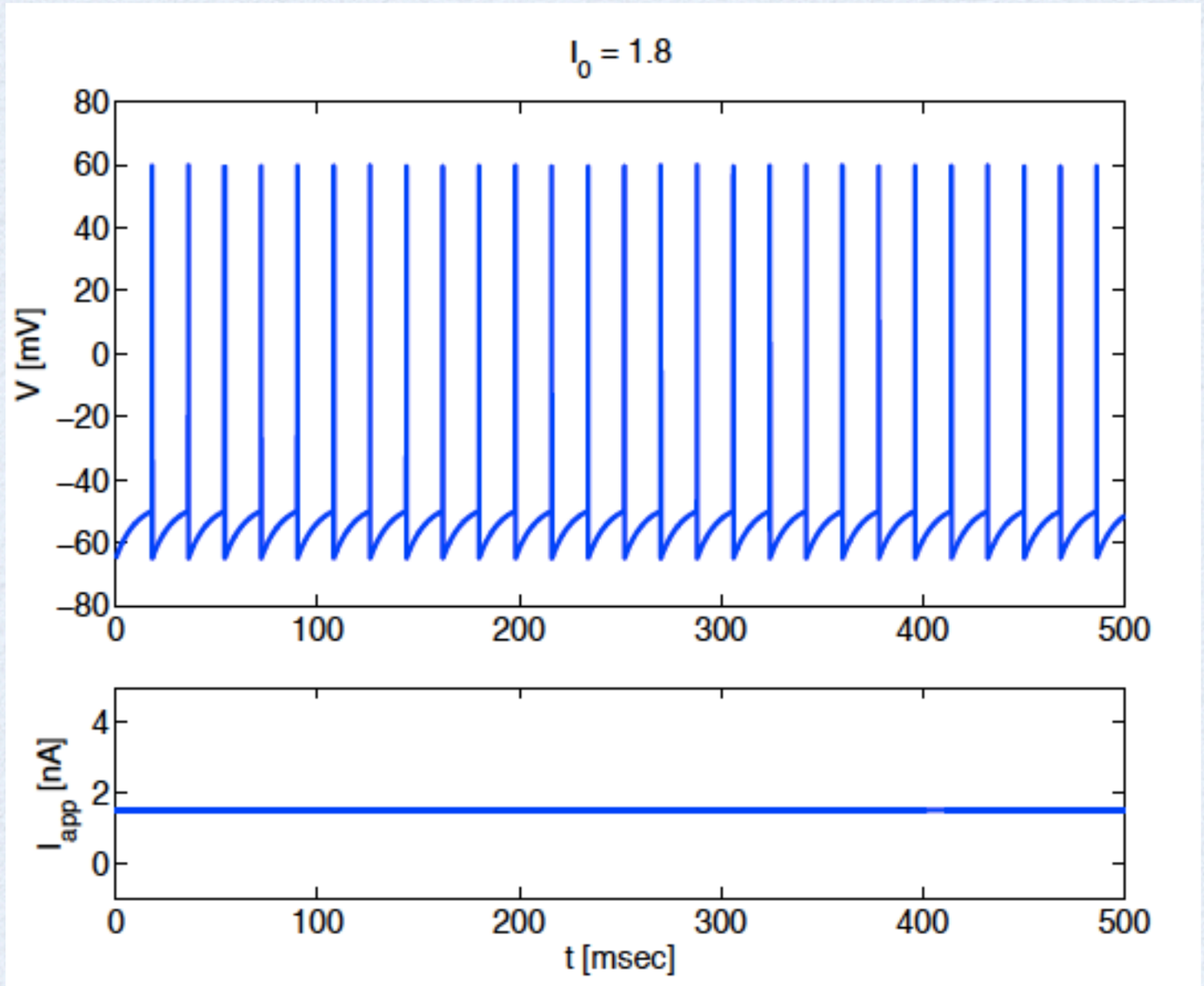

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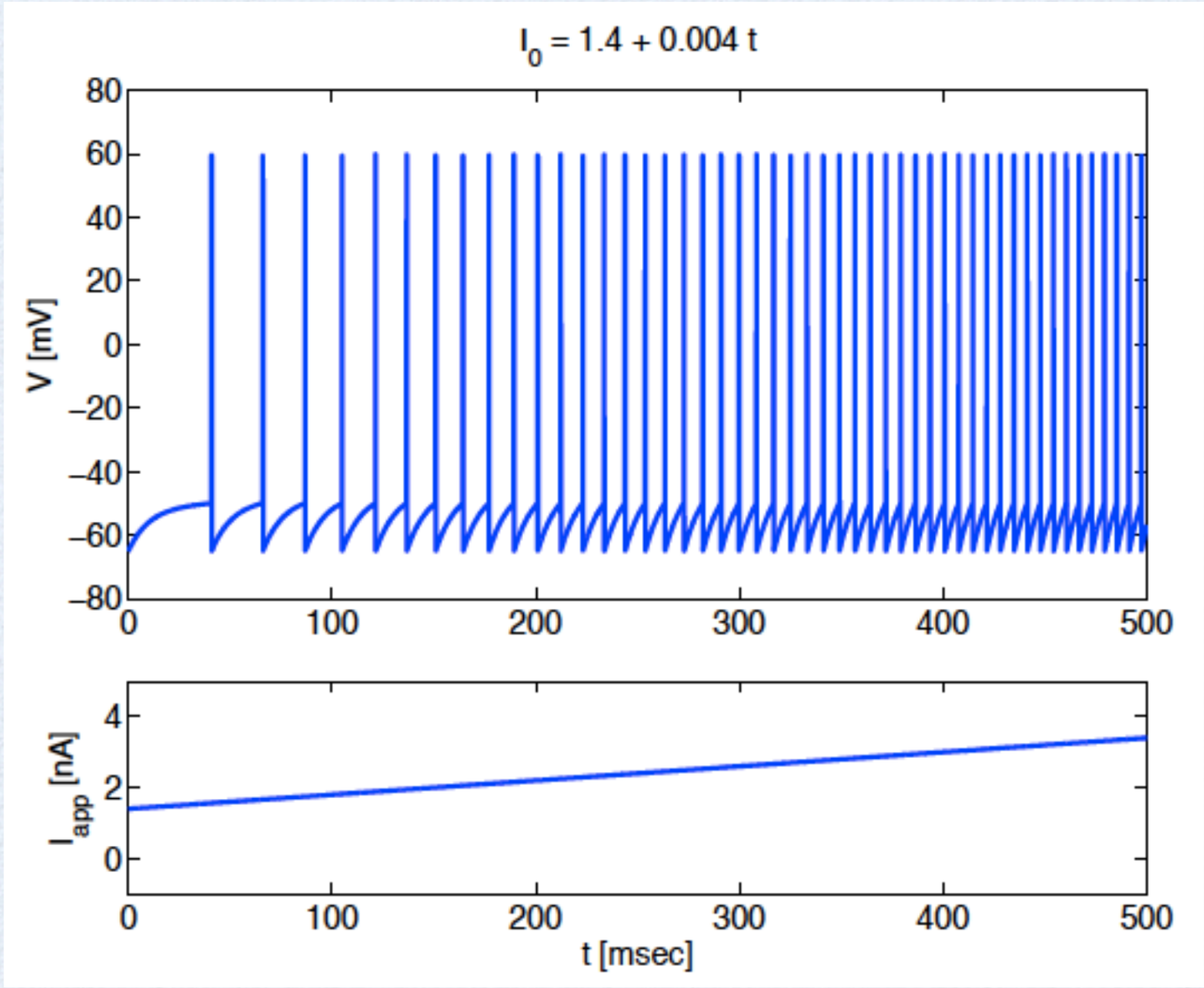

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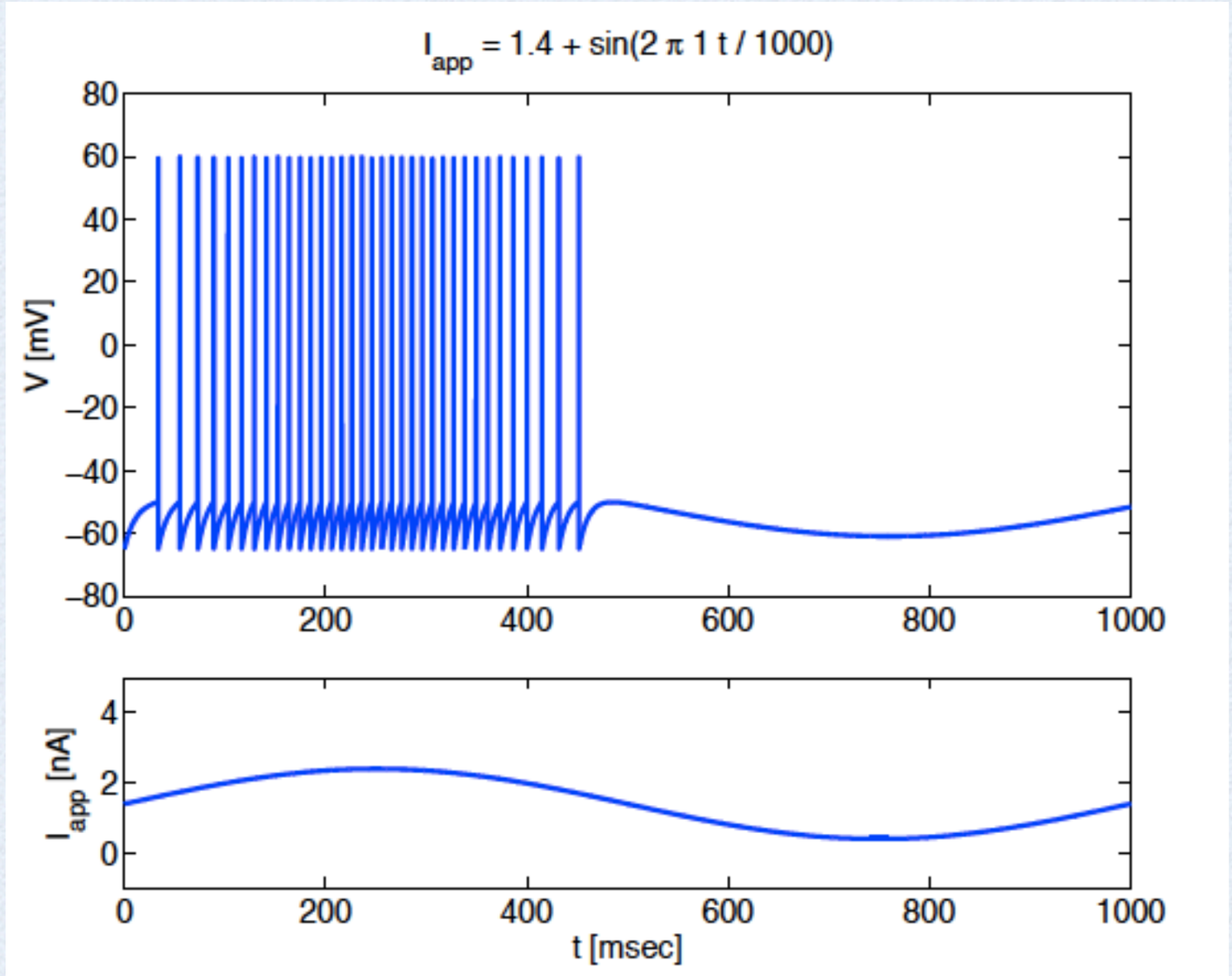

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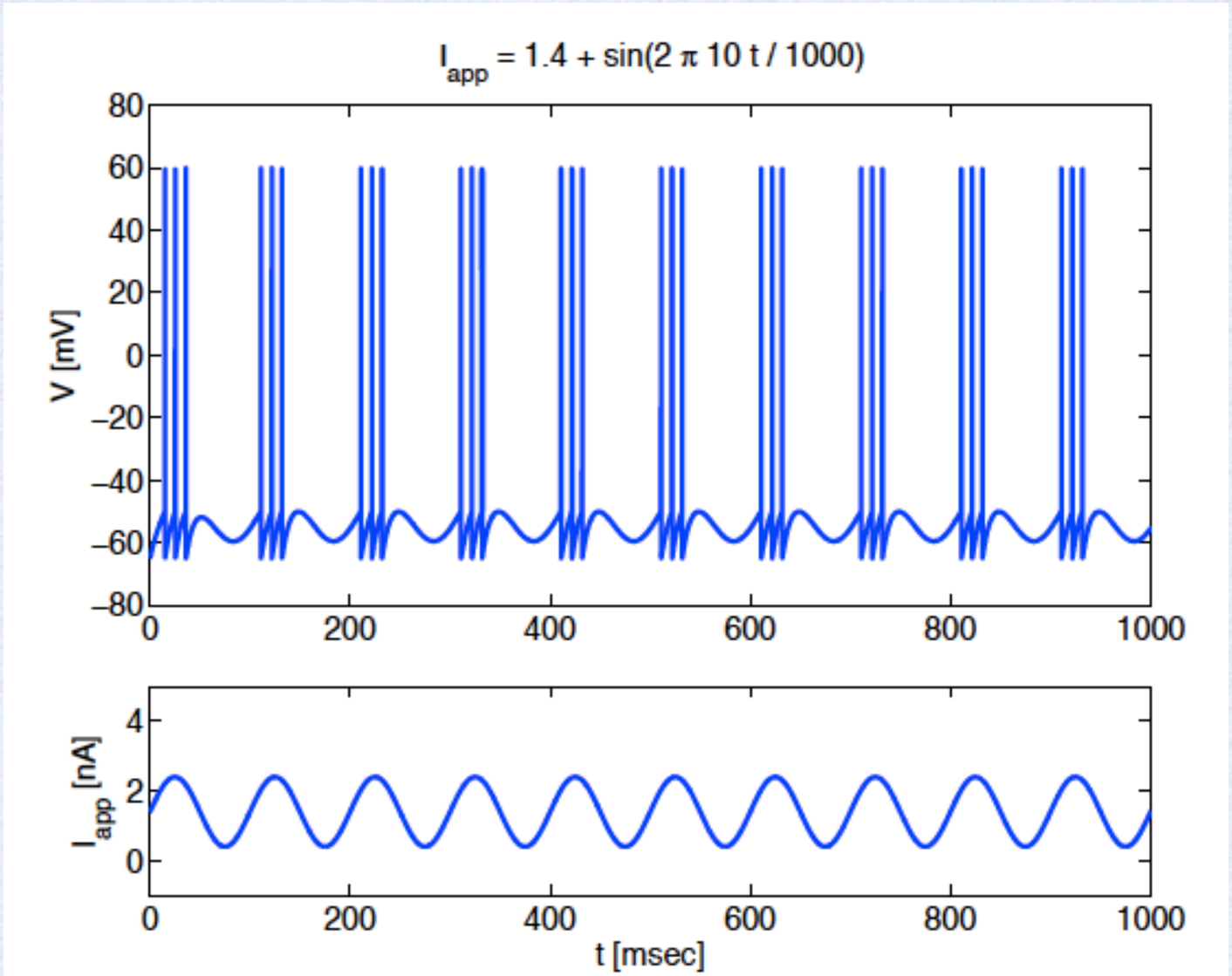

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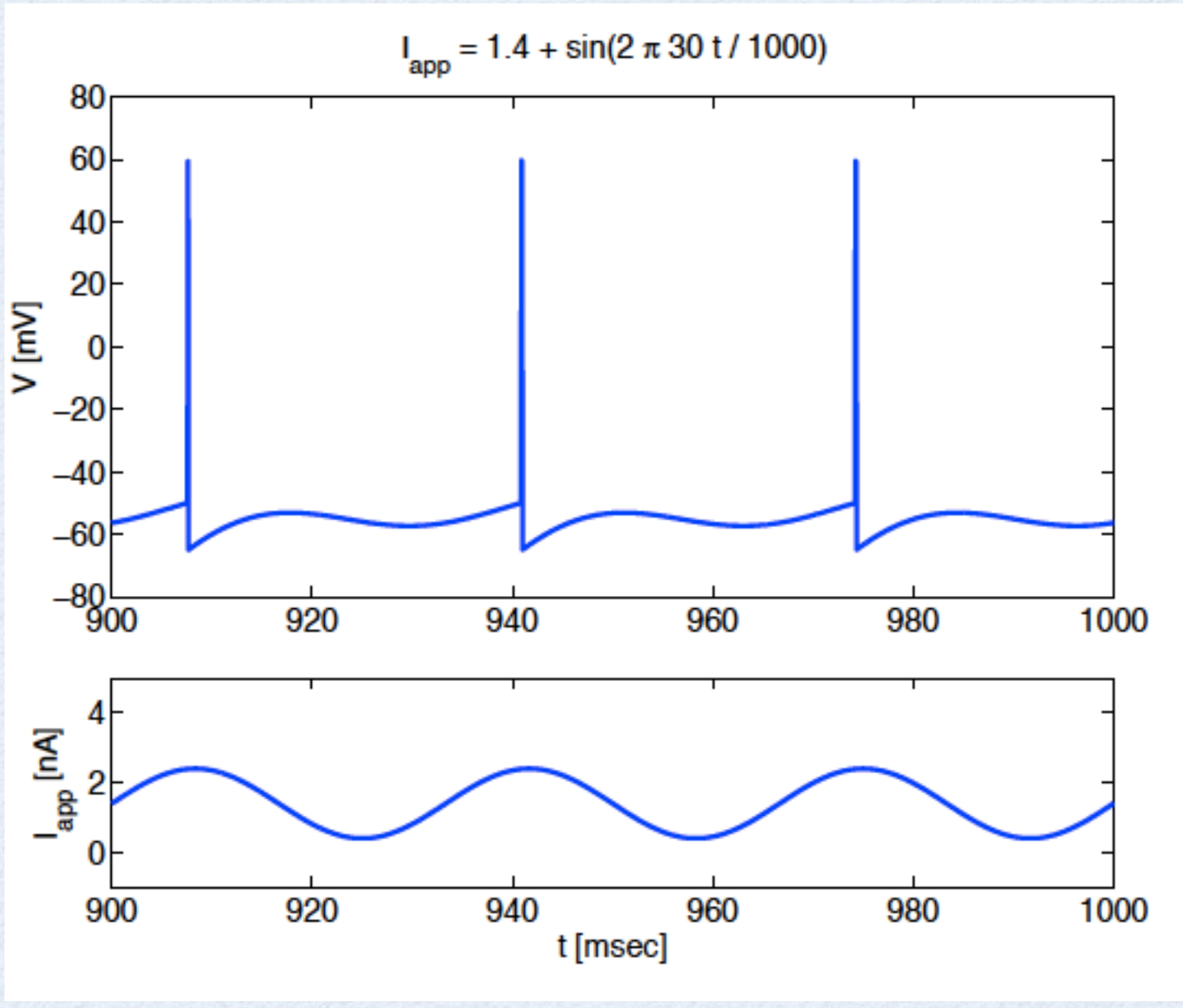

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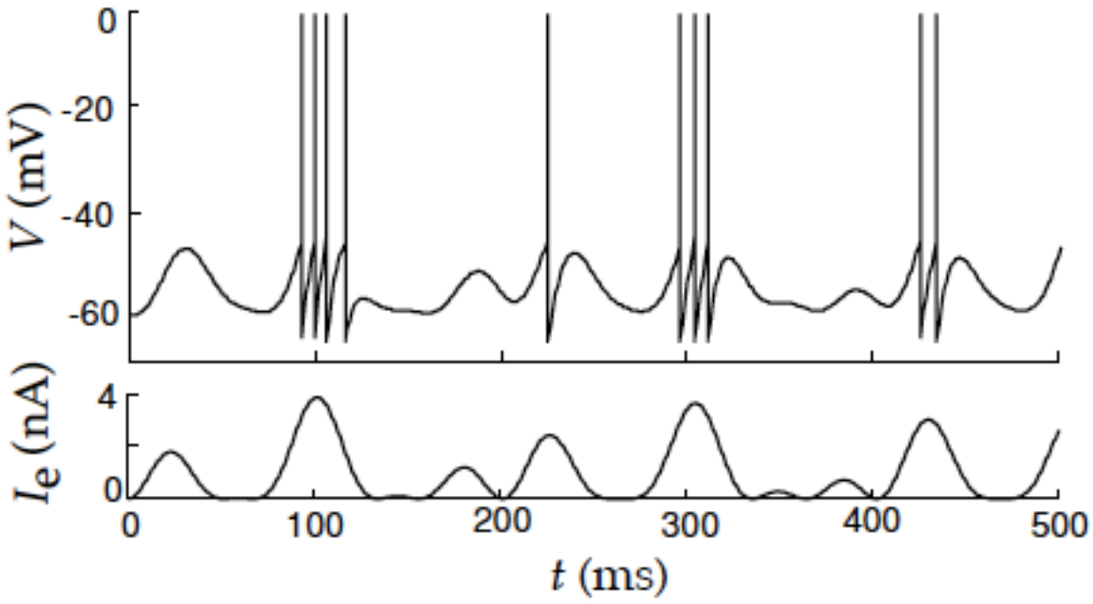

Figure 5.5 A passive integrate-and-fire model driven by a time-varying electrode current. The upper trace is the membrane potential, and the bottom trace the driving current. The action potentials in this figure are simply pasted onto the membrane potential trajectory whenever it reaches the threshold value. The parameters of the model are $E_L = V_{reset} = -65$ mV, $V_{th} = -50$ mV, $\tau_m = 10$ ms, and $R_m = 10$ MΩ.

Dayan & Abbott

Integrate-and-fire neuron models

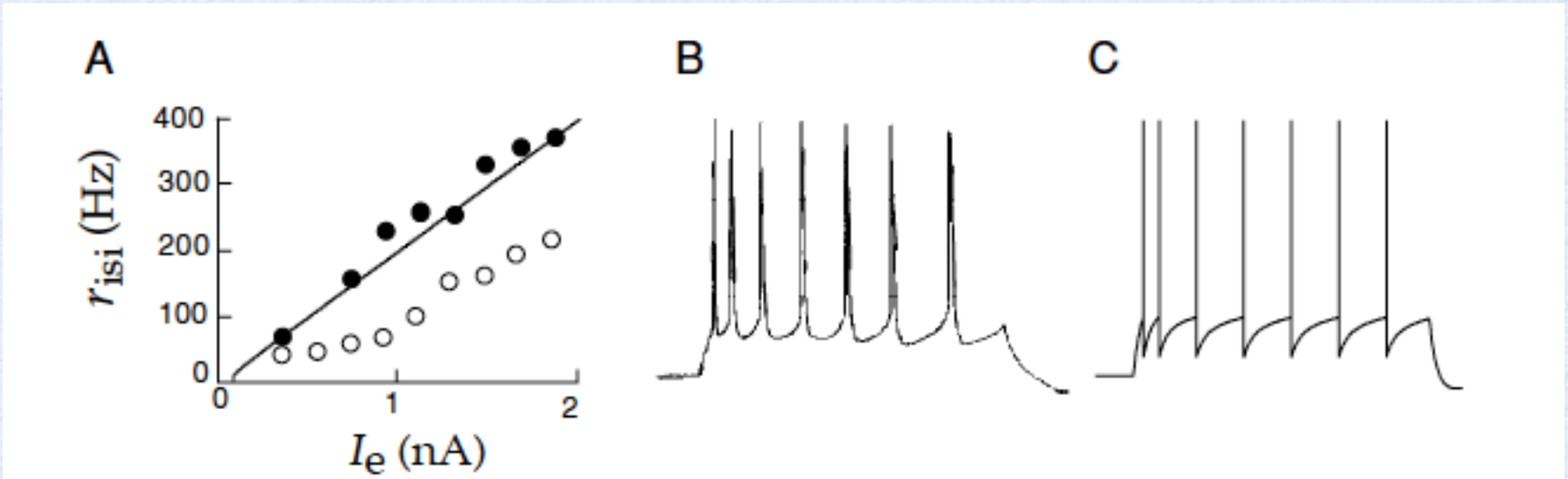

Figure 5.6 (A) Comparison of interspike-interval firing rates as a function of injected current for an integrate-and-fire model and a cortical neuron measure in vivo. The line gives r_{isi} for a model neuron with $\tau_m = 30$ ms, $E_L = V_{reset} = -65$ mV, $V_{th} = -50$ mV, and $R_m = 90$ MΩ. The data points are from a pyramidal cell in the primary visual cortex of a cat. The filled circles show the inverse of the interspike interval for the first two spikes fired, and the open circles show the steady-state interspike-interval firing rate after spike-rate adaptation. (B) A recording of the firing of a cortical neuron under constant current injection, showing spike-rate adaptation. (C) Membrane voltage trajectory and spikes for an integrate-and-fire model with an added current, with $r_m \Delta g_{sra} = 0.06$, $\tau_{sra} = 100$ ms, and $E_K = -70$ mV (see equations 5.13 and 5.14). (Data in A from Ahmed et al., 1998; B from McCormick, 1990.)

Dayan & Abbott

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- E_K= -85 Δ g_{sra}= 0.1

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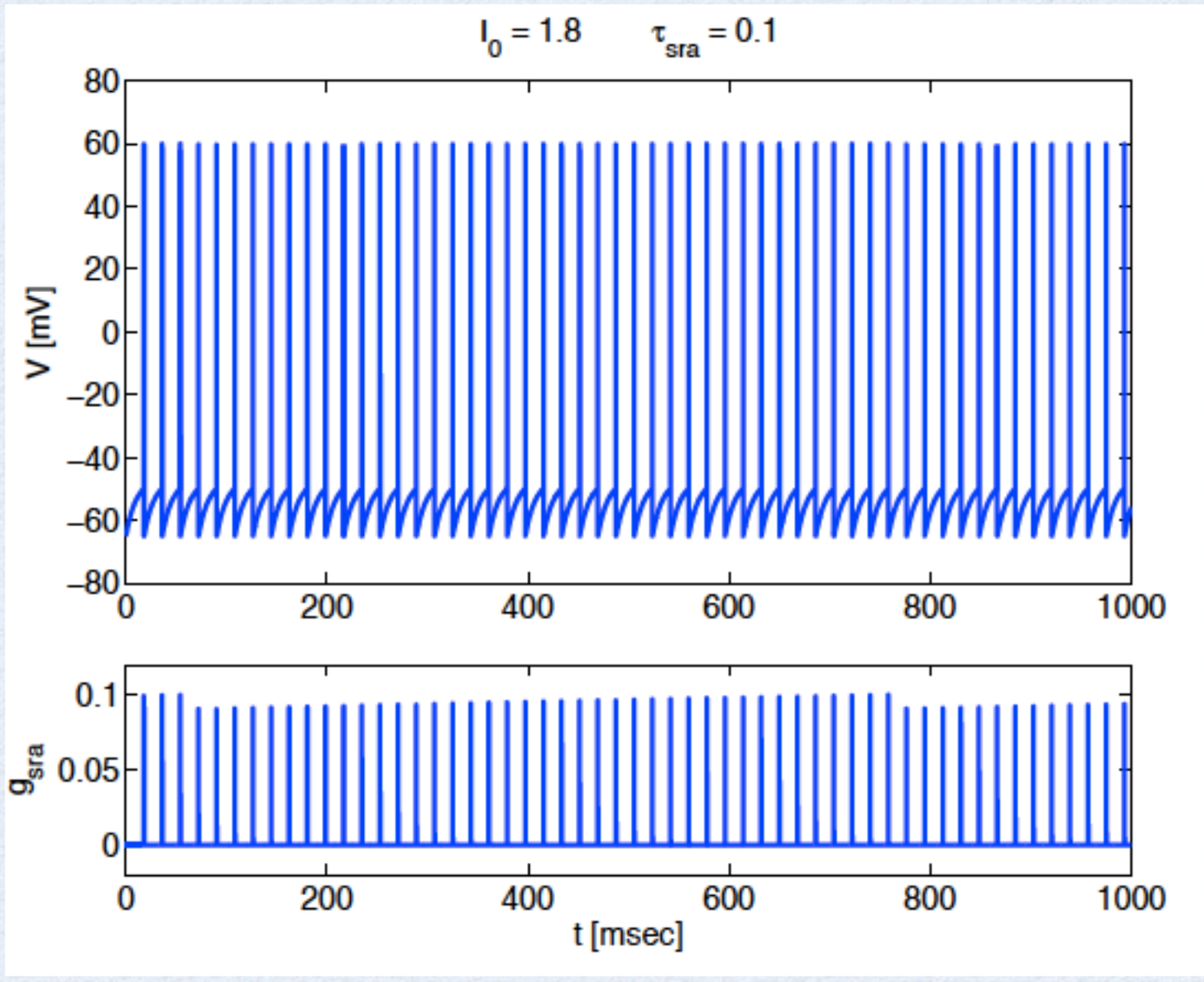

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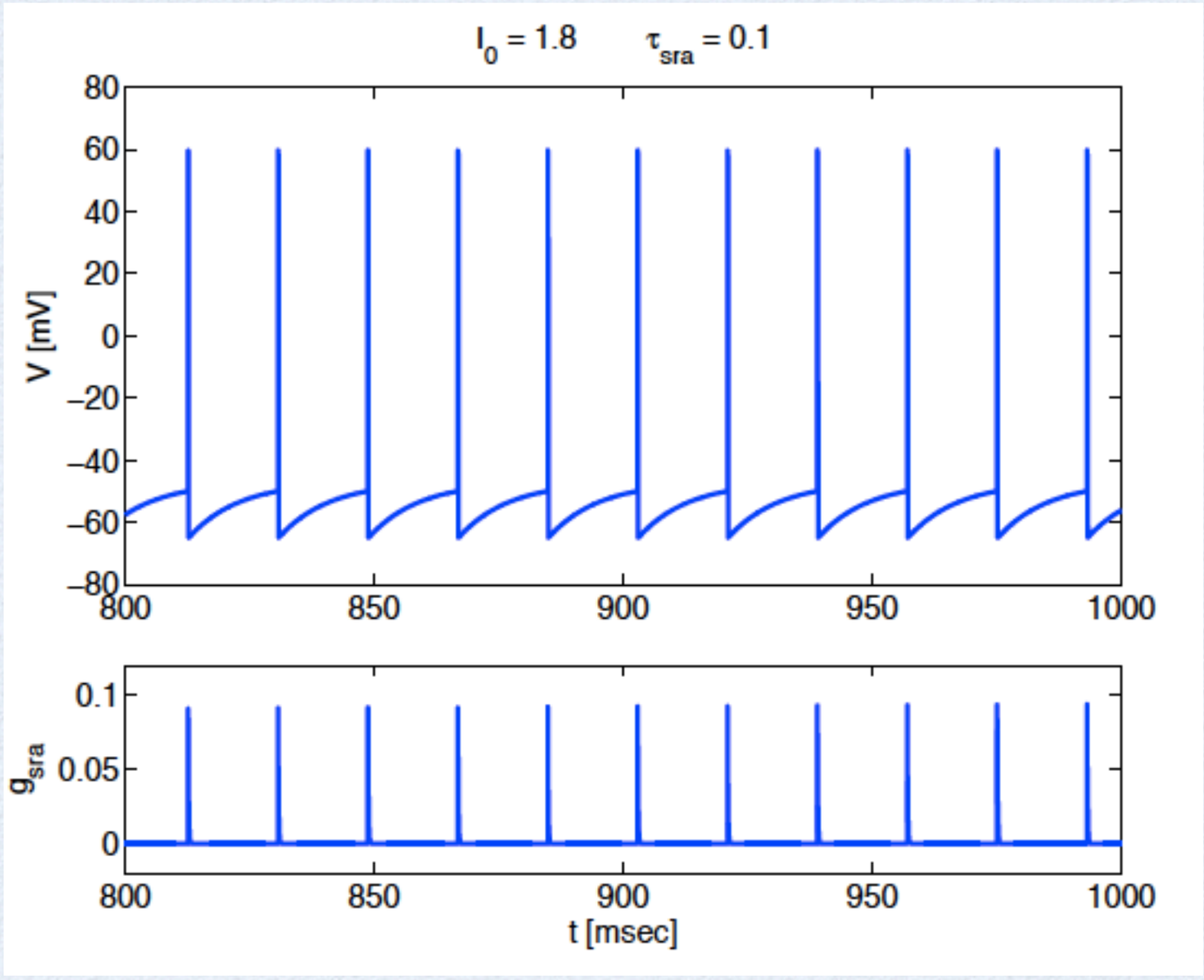

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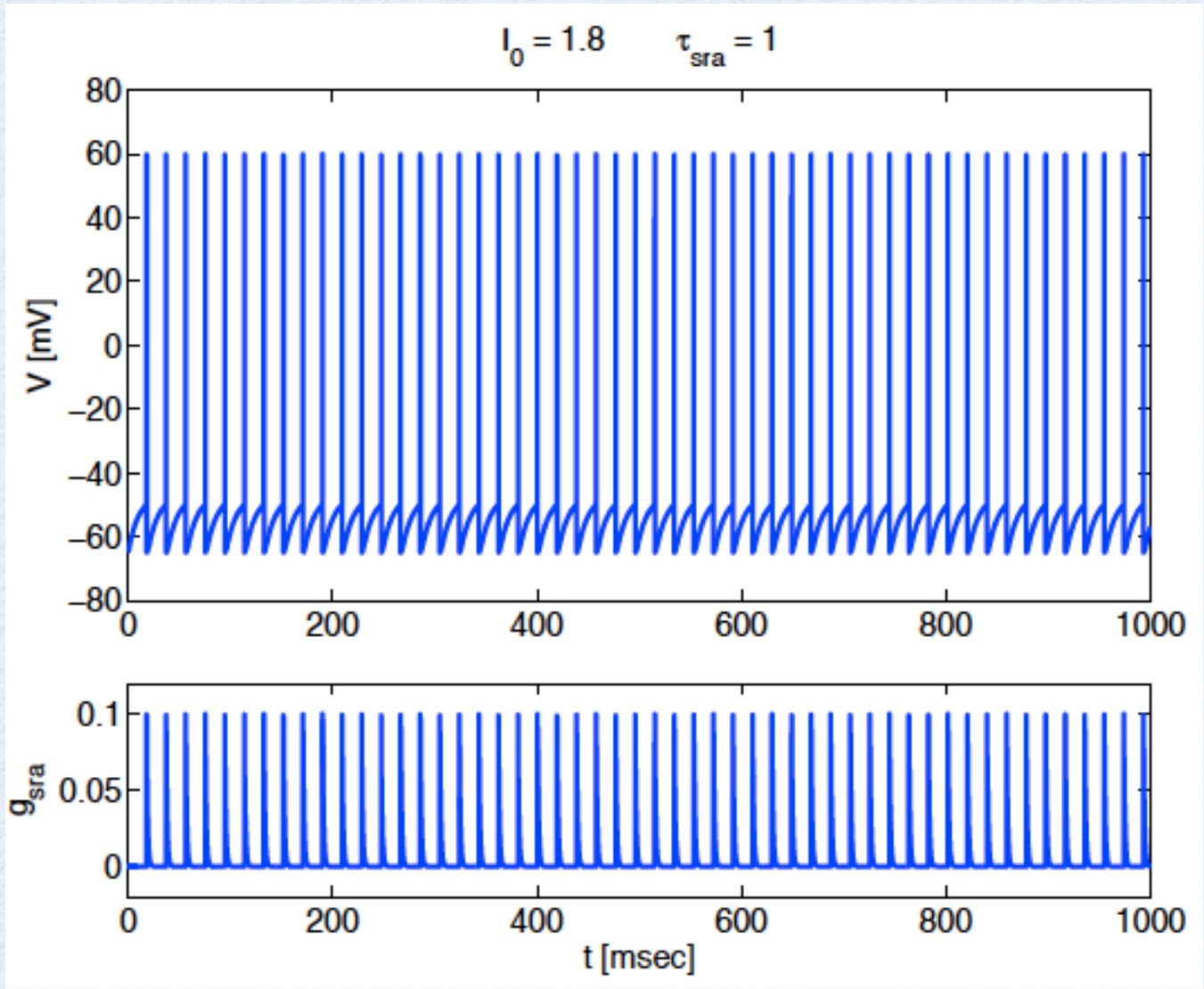

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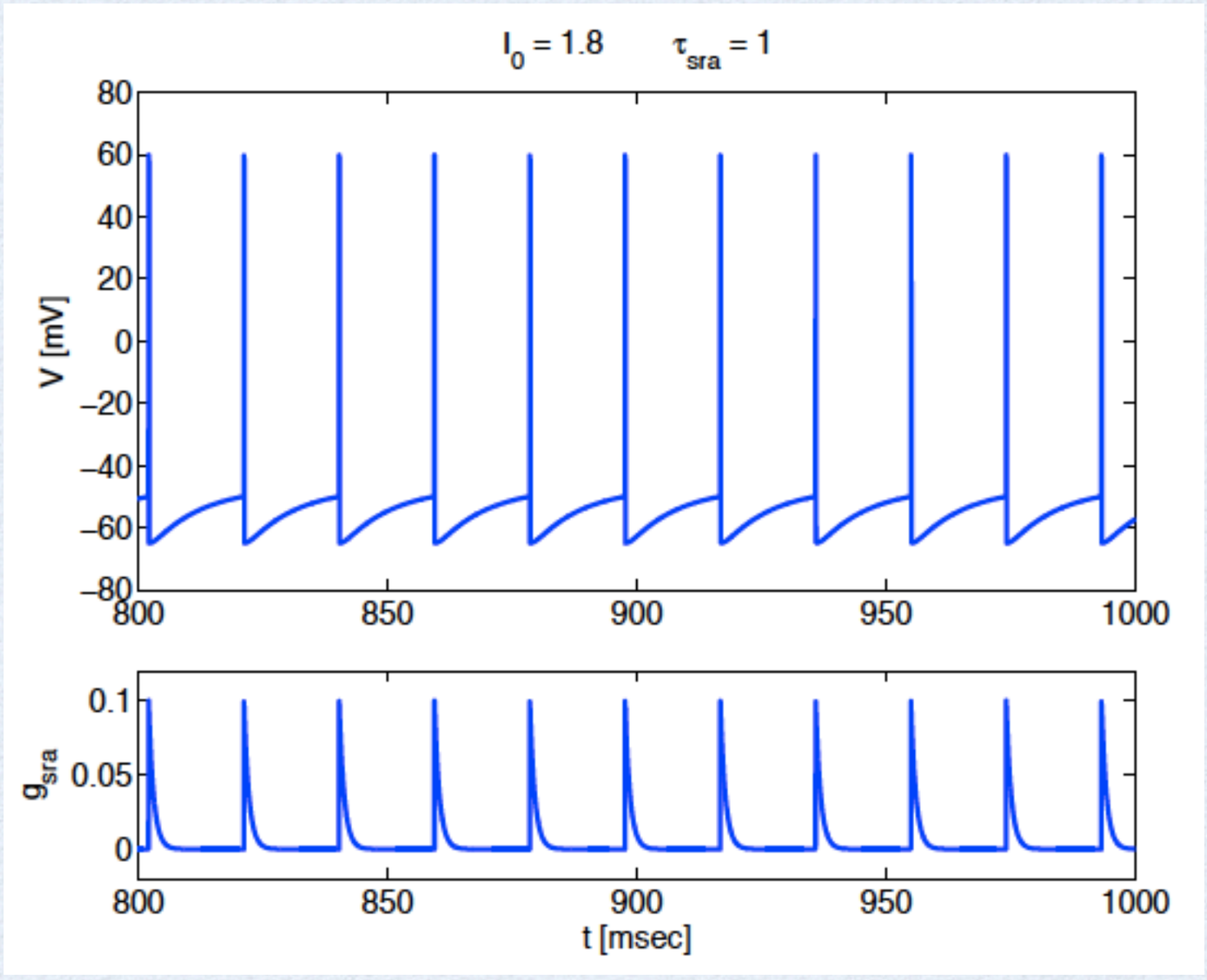

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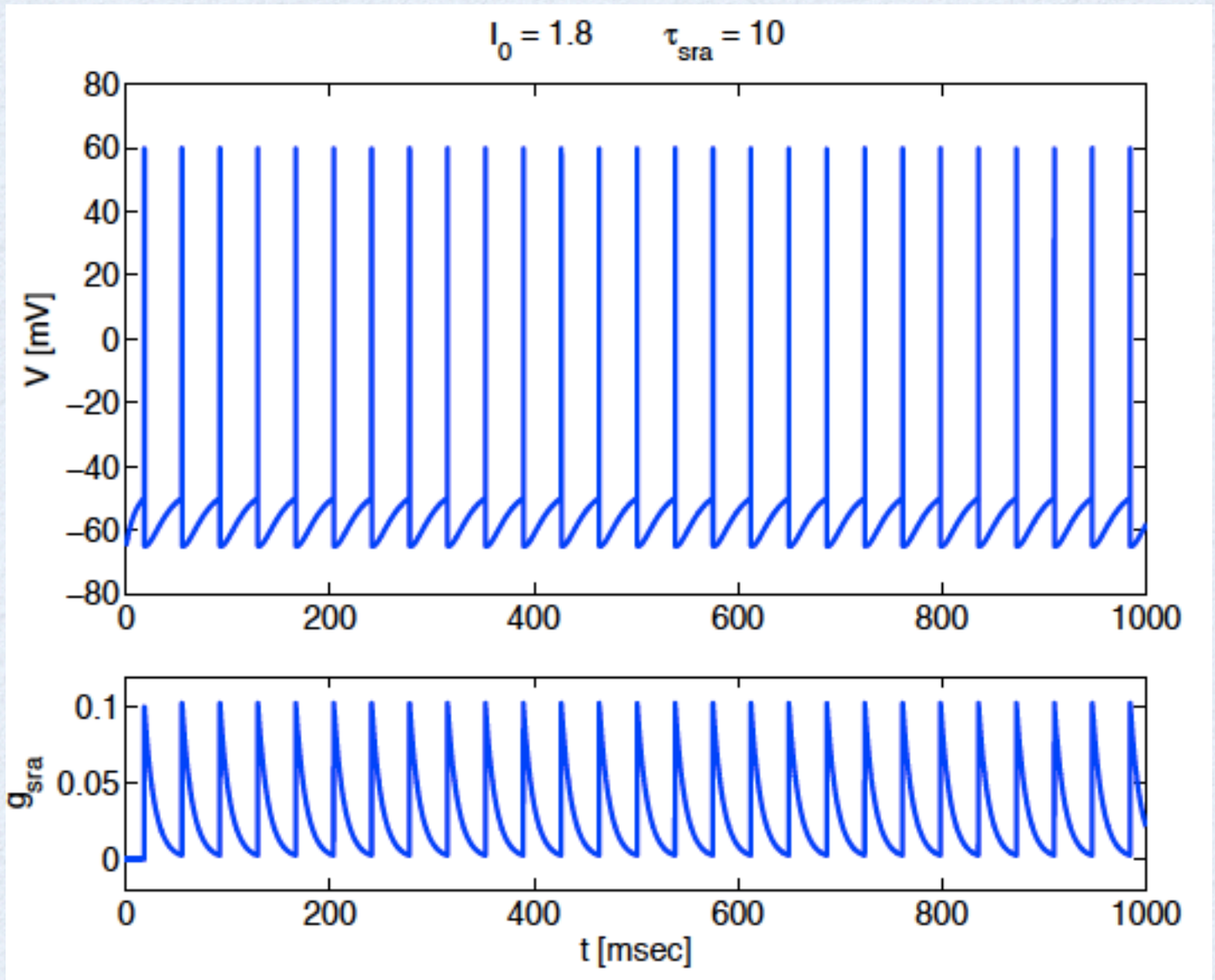

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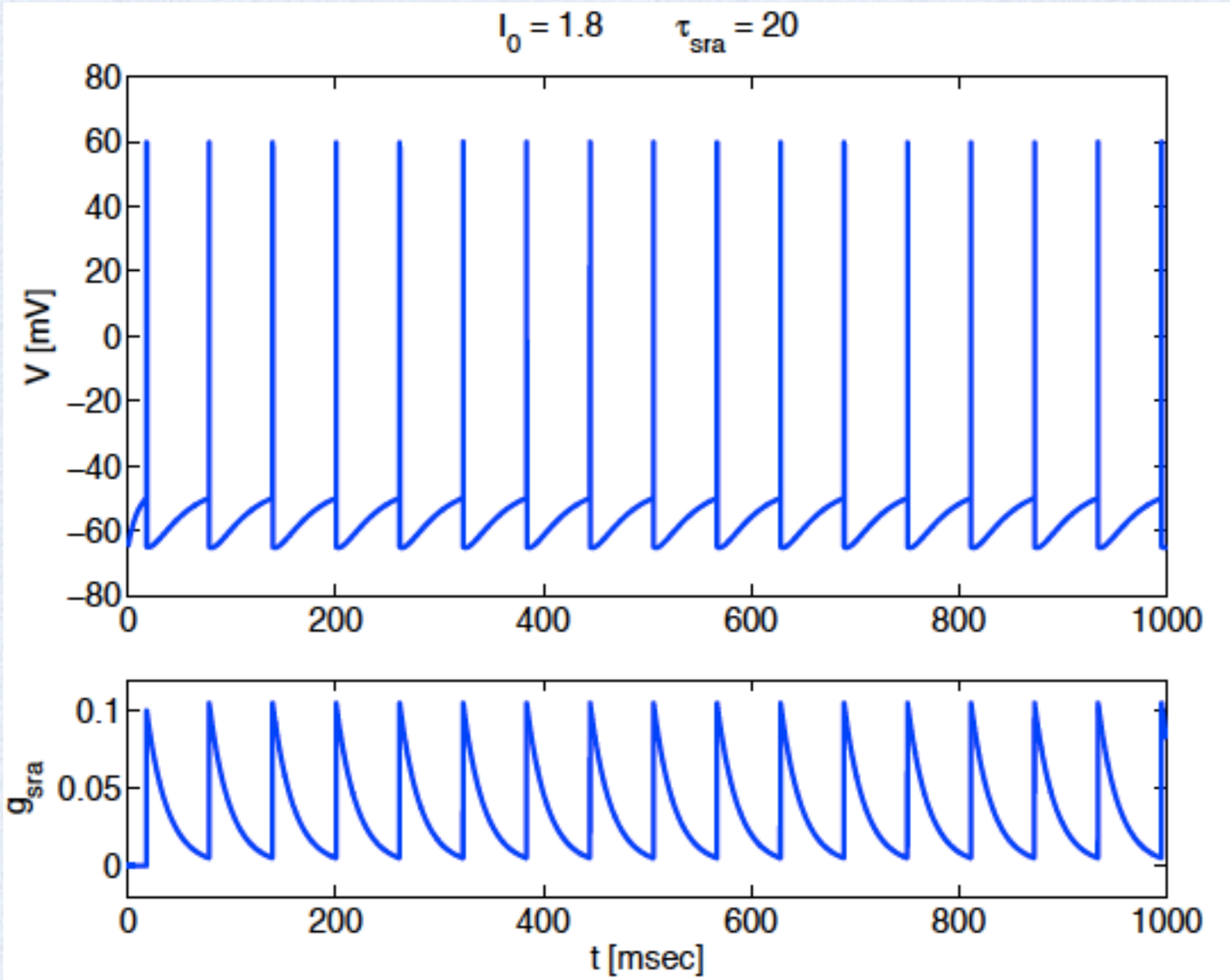

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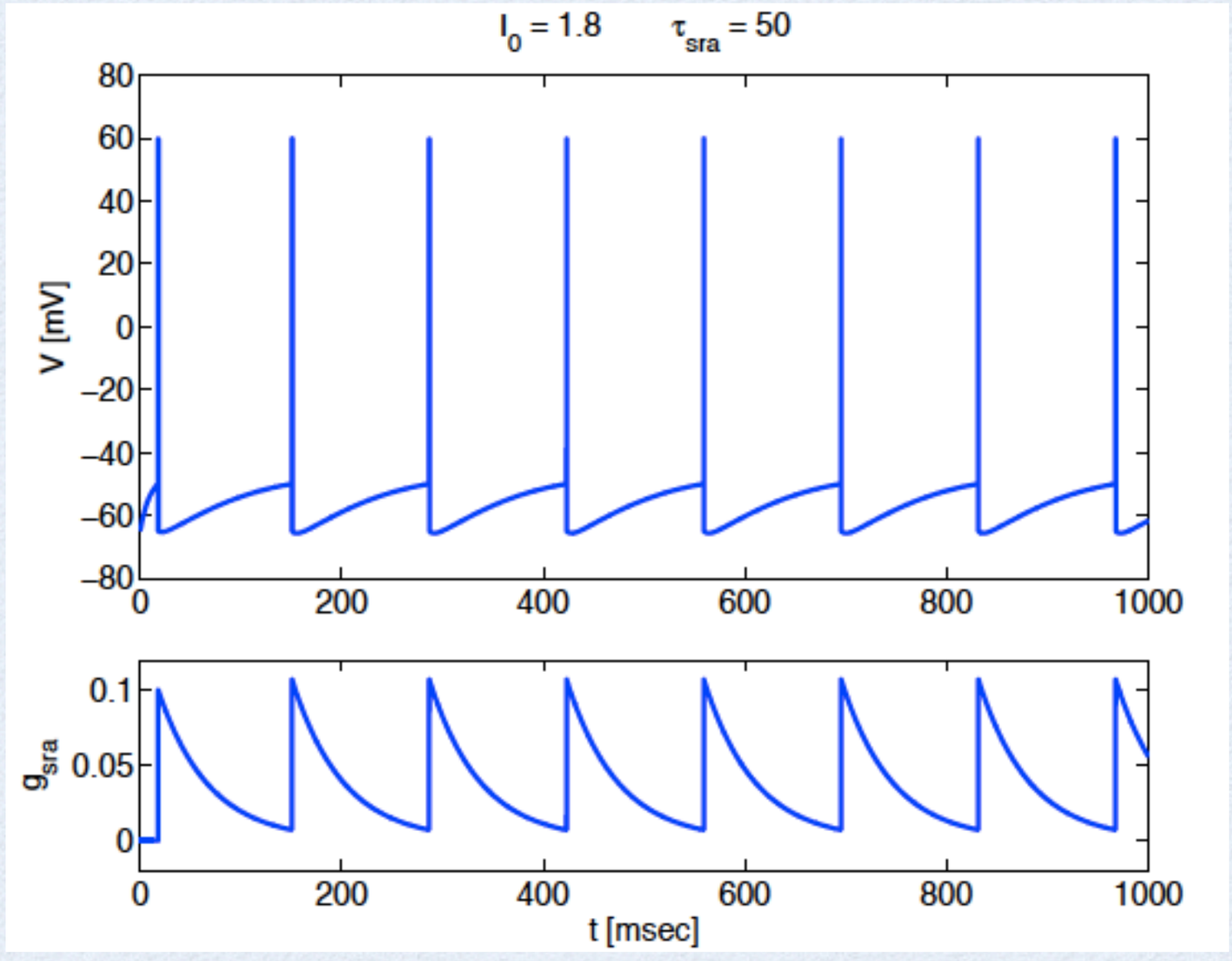

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