Math 473/573
Fall 2016
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Homework 1

1. Consider the following so-called logistic equation

$$\frac{dP}{dt} = r\,P\left(1 - \frac{P}{K}\right), \tag{1}$$

describing the change of size (growth, decrease) of a given population P over time (t), with $r > 0$ and $K > 0$ constants and $P(0) = P_0 > 0$.

(a) What are the units of the parameters r and K?

(b) Show that r provides an appropriate time scale (i.e., r can be used to rescale time).

(c) Are there additional time scales?

(d) Show that either K or P_0 provide appropriate population scales (i.e., both can be used to rescale P).

(e) Are there additional population scales?

(f) If your NJIT ID is even, consider K as a population scale, rescale the system (both t and P), solve the resulting problem and compare with the original problem.

(g) If your NJIT ID is odd, consider P_0 as a population scale, rescale the system (both t and P), solve the resulting problem and compare with the original problem.

(h) What are the advantages/disadvantages, if any, of using each rescaling? Compare your results with the results of your fellow students using a rescaling different from yours.

2. Write a code to simulate the logistic equation problem. Use this code to compare the results of your simulations with the results of your analytical calculations for representative parameter values. (What do I mean by representative parameter values?) Plot the superimposed curves corresponding to the analytical and numerical computations (with different colors!) and plot the relative error.

3. What would you need to add to the logistic equation model in Item 1 to describe a situation where the population either decreases to zero or increases to $P = K$? $P = 0$