Math 473/573
Fall 2016
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Homework 4

1. Investigate the following equation

$$\frac{dx}{dt} = -2x^3 + 3x^2 + r \tag{1}$$

   (a) Sketch all the qualitatively different vector fields that occur as the real parameter r is varied.

   (b) Sketch the bifurcation diagram of the fixed-points $\bar{x}$ versus r.

   (c) Show that a saddle-node bifurcation occurs at a critical value r_c and compute the critical value(s).

   (d) Write a code to simulate the ODE.

   (e) Simulate the ODE for $r = 1$ and $r = -1.2$.

   (f) Assume $r = -0.5$ and $x(0) = 0.6$. What is the minimal perturbation you need to make to the solution $x(t)$ at $t = 10$ for it to reach negative values (if possible)? Provide an approximate value based on your simulations.

   (g) Assume $r = -0.5$ and $x(0) = 0.6$. What is the minimal perturbation you need to make the parameter r at $t = 10$ for the solution to reach negative values (if possible)? Provide an approximate value based on your simulations.

   (h) Explain all your results.

2. Investigate the following equation (textbook problem 3.2.4).

$$\frac{dx}{dt} = x\left(r - e^{x}\right) \tag{2}$$

(a) Sketch all the qualitatively different vector fields that occur as the real parameter r is varied.

(b) Identify the fixed-points and their stability for these values of r.

(c) Sketch the bifurcation diagram of the fixed-points $\bar{x}$ versus r.

(d) Write a code to simulate the ODE.

(e) Simulate the ODE for $r = -1$, $r = 0.5$ and $r = 2$.

(f) Assume $r = 0$ and $x(0) = 0.1$. What is the minimal perturbation you need to make to the solution $x(t)$ at $t = 20$ for it to reach stationary negative values (if possible)? Provide an approximate value based on your simulations and on the bifurcation diagrams.

(g) Assume $r = 0$ and $x(0) = 0.1$. What is the minimal perturbation you need to make to make to the parameter r at $t = 20$ for the solution to reach stationary negative values (if possible)? Provide an approximate value based on your simulations and on the bifurcation diagrams.

(h) Assume $r = 0$ and $x(0) = 0.1$. What is the minimal perturbation you need to make to make to the parameter r at $t = 20$ for the solution to reach stationary zero values (if possible)? Provide an approximate value based on your simulations and on the bifurcation diagrams.

(i) Explain all your results.