Math 473/573
Fall 2016
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Midterm Project

- The answer to each of the following items (and everything else you decide to say) **must be clearly, concisely and precisely written**.
- All answers **must be justified** (briefly and concisely).
- Graphs need not be colored, but **the information provided in the graphs should be clearly presented**. Use legends, axes, and different types of curves (solid, dashed, etc). Plot the graphs using appropriate scales for each variable.
- In the event you do not know the answer to one or more of the items, you **must clearly explain what are the difficulties.**
- **Provide all your codes**. Include a description of the talks for each code (as a comment in the code's file).

1. Investigate the following equation (textbook problem 3.1.4).

$$\frac{dx}{dt} = r + \frac{x}{2} - \frac{x}{1+x} \tag{1}$$

   (a) Sketch all the qualitatively different vector fields that occur as the real parameter r is varied.

   (b) Sketch the bifurcation diagram of the fixed-points $\bar{x}$ versus r. Provide explicit formulas for the fixed-points as a function of the control parameter r. You may choose to write a code to compute the bifurcation diagram.

(c) Show that a saddle-node bifurcation occurs at critical value r_c. Compute r_c. If there are more than one saddle-node bifurcation, compute all the values of r_c.

(d) Write a code to simulate the ODE for $r = 4$ and $r = -4$. Use initial conditions close to an unstable fixed-point (if it exists) such that the solutions approach a stable fixed-point (if it exists) in each case.

(e) Assume $r = -1$ and $x(0) = 5$. What is the minimal perturbation you need to make to the solution $x(t)$ at $t = 1$ for it to reach stationary values (if possible)? Provide an approximate value based on your simulations and on the bifurcation diagrams.

(f) Assume $r = -1$ and $x(0) = 5$. What is the minimal perturbation you need to make to the parameter r at $t = 1$. for the solution to reach stationary values (if possible)? Provide an approximate value based on your simulations and on the bifurcation diagrams.

(g) Assume $r = -2$ and $x(0) = 0$. What is the minimal perturbation you need to make to make to parameter r at $t = 1$ for the solution to reach stationary values (if possible)? Provide an approximate value based on your simulations and on the bifurcation diagrams.

2. Investigate the following consumer-producer equation.

$$\frac{dx}{dt} = r\,x\,(1 - x) - p\,x \tag{2}$$

where r is the growth rate (as in the logistic equation) and p is the consumption rate. Assume $r \geq 0$.

(a) Sketch all the qualitatively different vector fields that occur as the real parameter p is varied (e.g., $p = 0, 1, 2, 3, 4$) for representative values of the parameter r (e.g., $r = 1$, $r = 2$).

(b) Sketch all the qualitatively different vector fields that occur as the real parameter r is varied (e.g., $r = 1, 2, 3$) for representative values of the parameter p (e.g., $p = 0, 2, -2$.

(c) Identify the fixed-points and their stability in terms of the parameters r and p. For what values of r and p does the problem admit only one fixed-point?

(d) Sketch the bifurcation diagram of the fixed-points $\bar{x}$ versus r (for fixed values of p).

(e) Write a code to simulate the ODE for $p = 0$, $p = 2$, $p = -2$ and representative values of r in each case.

(f) Assume $r = 1$, $p = 2$ and $x(0) = -1.5$. What is the minimal perturbation you need to make to the solution $x(t)$ at $t = 0.5$ for it to reach stationary values (if possible)? Provide an approximate value based on your simulations and on the bifurcation diagrams.

(g) Assume $r = 1$, $p = 2$ and $x(0) = -1.5$. What is the minimal perturbation you need to make to the parameter p at $t = 0.5$ for it to reach stationary values (if possible)? Provide an approximate value based on your simulations and on the bifurcation diagrams.