

# **QUANTUM ENGINEERING of Nanoelectronic Devices**

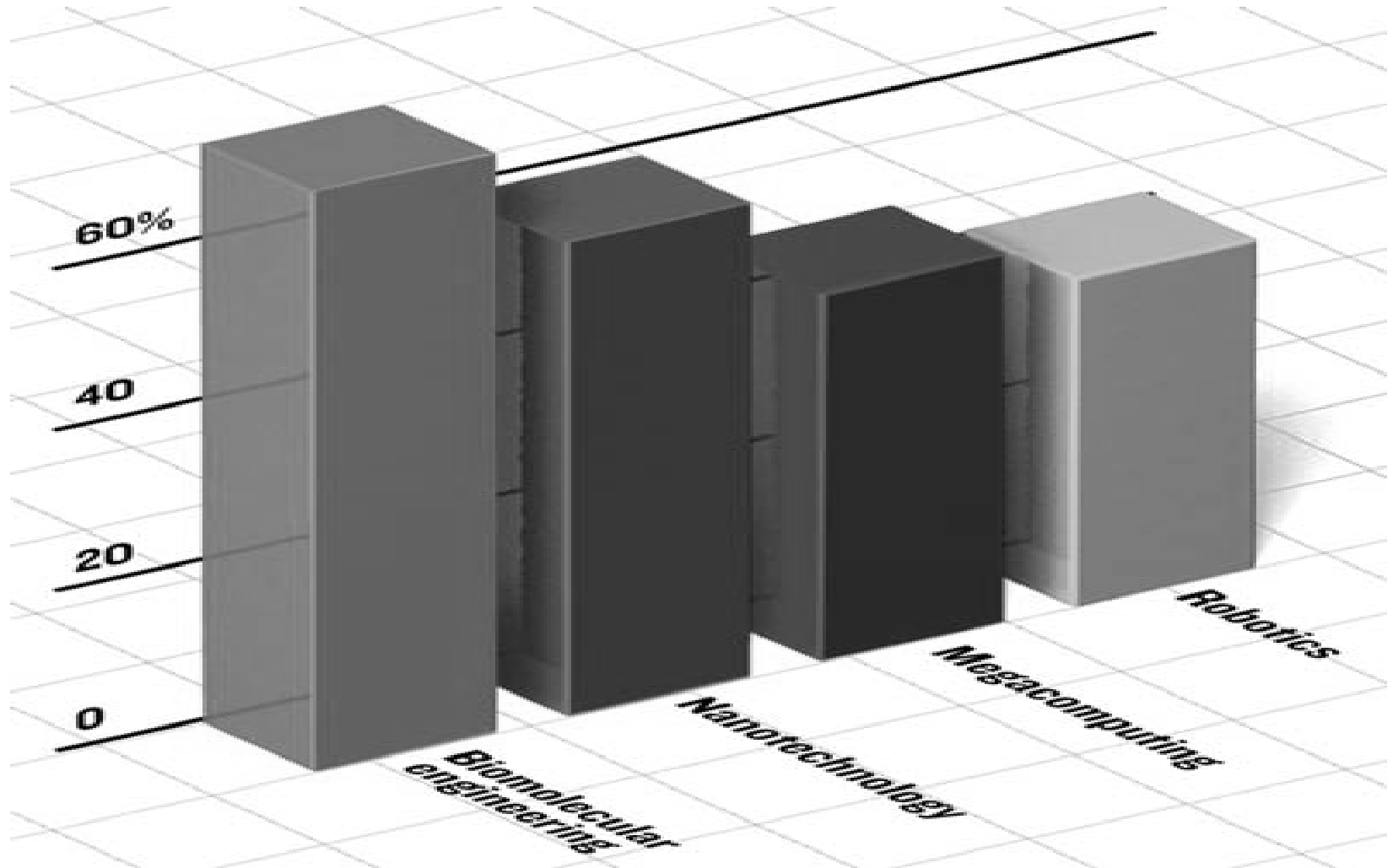
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# Emerging Technologies





# Our Motivation and Economics

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Adam Smith, “An Enquiry into Nature and Causes of the Wealth of Nations” (1776)

*The wealth is created by laissez-faire economy and free trade*

John Maynard Keynes, “The General Theory of Employment, Interest, and Money” (1936)

*The wealth is created by careful government planning and government stimulation of economy*

1990’s and Beyond

*The wealth is created by innovations and inventions*



# 20<sup>th</sup> Century Paradigm

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- Formulate a hypothesis or theory
- Accumulate data
- Do extensive experimentation and Check
- Publish if newsworthy
- Respect others' work helping them to grow in the profession
- Demonstrate character ethics that puts community interests above personal aggrandizement

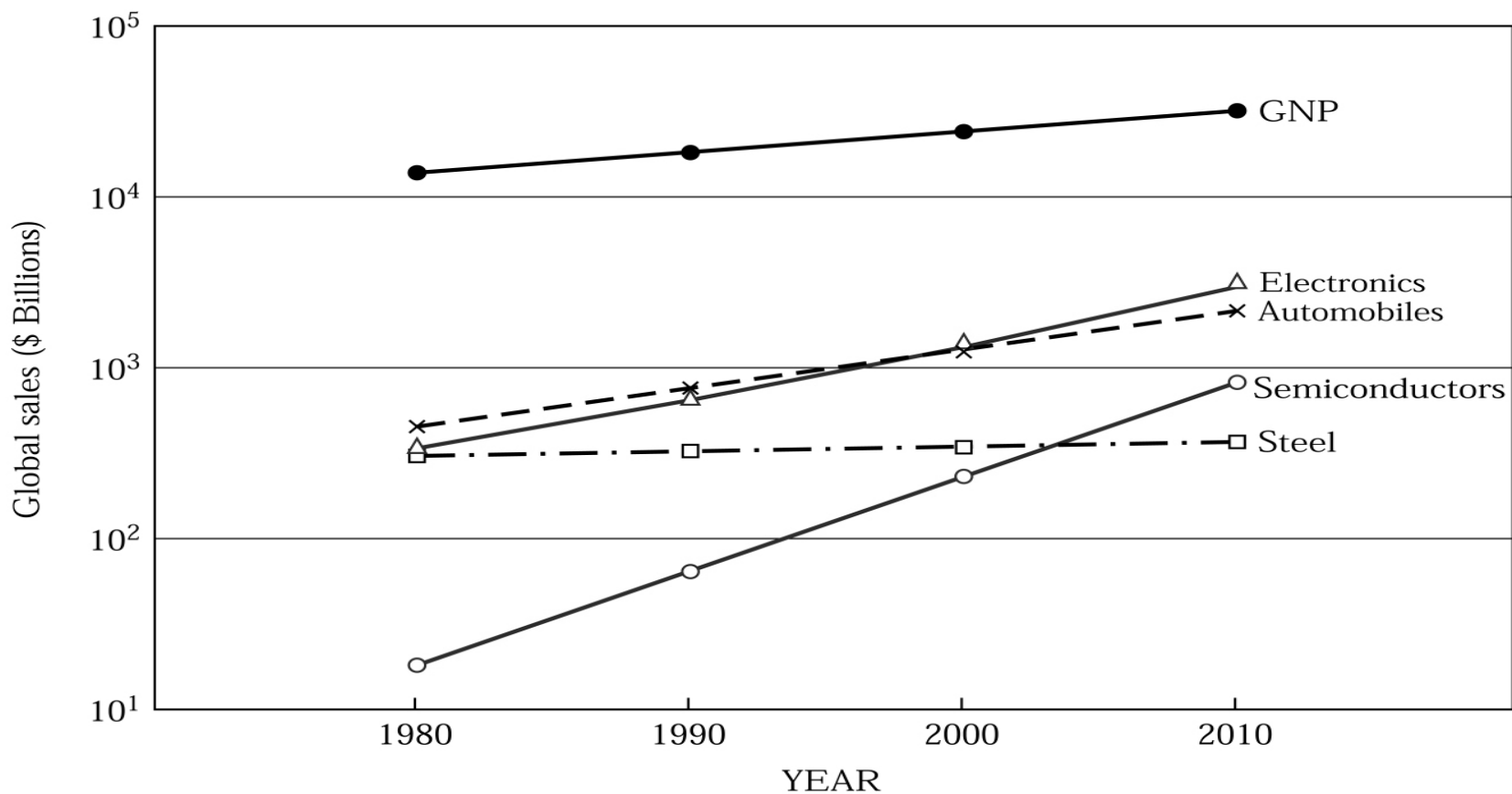


# 21<sup>st</sup> Century Paradigm

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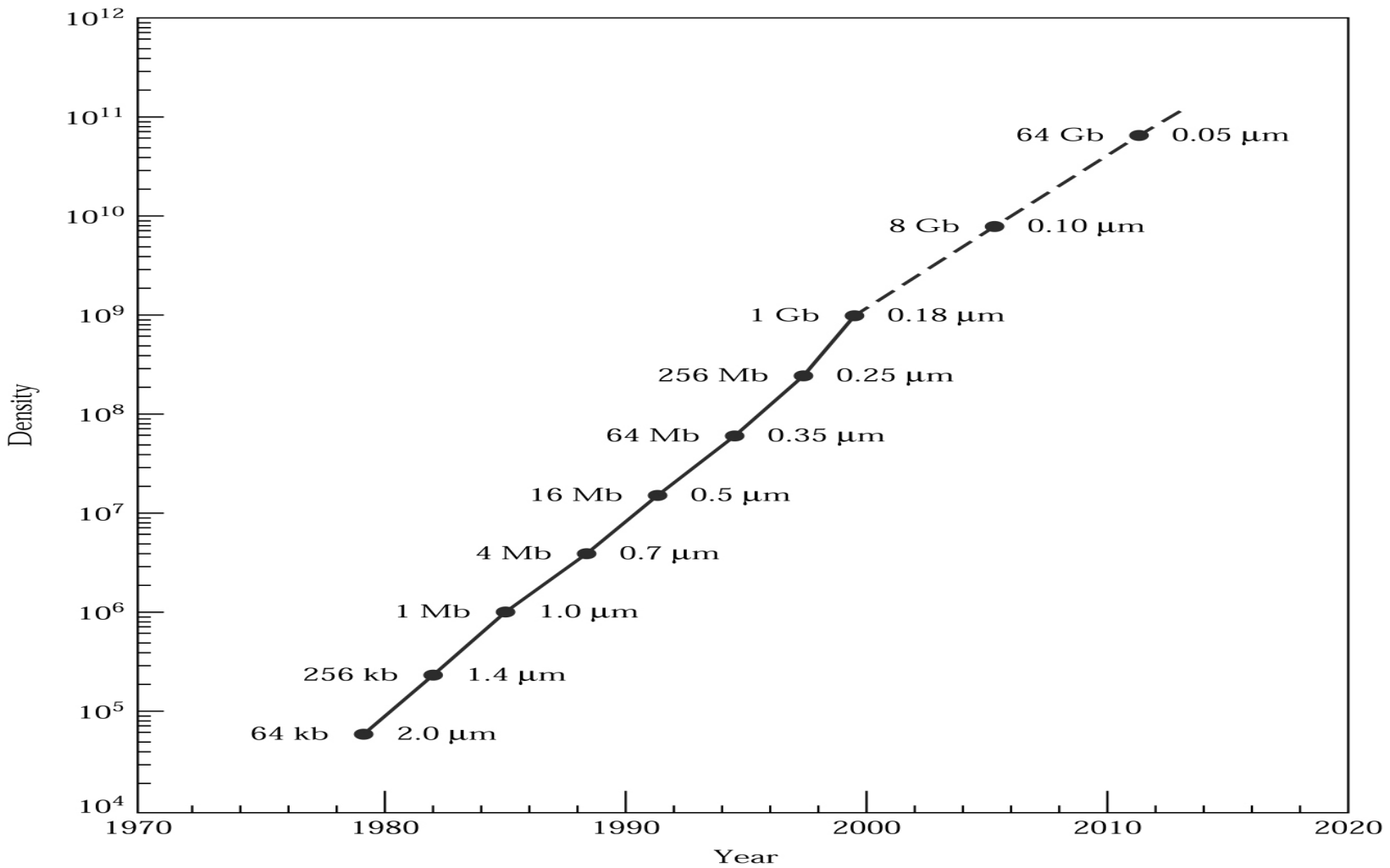
- Formulate a hypothesis or theory or design
- Make a prototype structure
- Patent it
- Raise 17 million dollars and start an IPO
- Sue your competitor for stealing your idea
- Demonstrate personality ethics that lubricates the process of human interaction for personal aggrandizement

# Gross world product and sales volumes



# Exponential Growth

## SIA roadmap





# Historical Trends

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- New Technology generation every three years
- For each generation, memory density increase by 4 times and logic density increases by 2.5 times
- Rule of Two: In every two generations (6 years), the feature size decreased by 2, transistor current density, circuit speed, chip area, chip current and maximum I/O pins increased by 2



# Research Scenario

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- **A comprehensive transport theory for quantum processes at nanoscale**
- **High-field distribution in quantum wells**
- **Optimization of the shape and size of quantum wells for high frequencies**
- **Quantum Computing: Multi-state logic by using quantum states**
- **Failure of Ohm's Law: Re-assessment of the circuit theory principles**



# Goals for High Speed Performance

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➤ **Large transistor current**

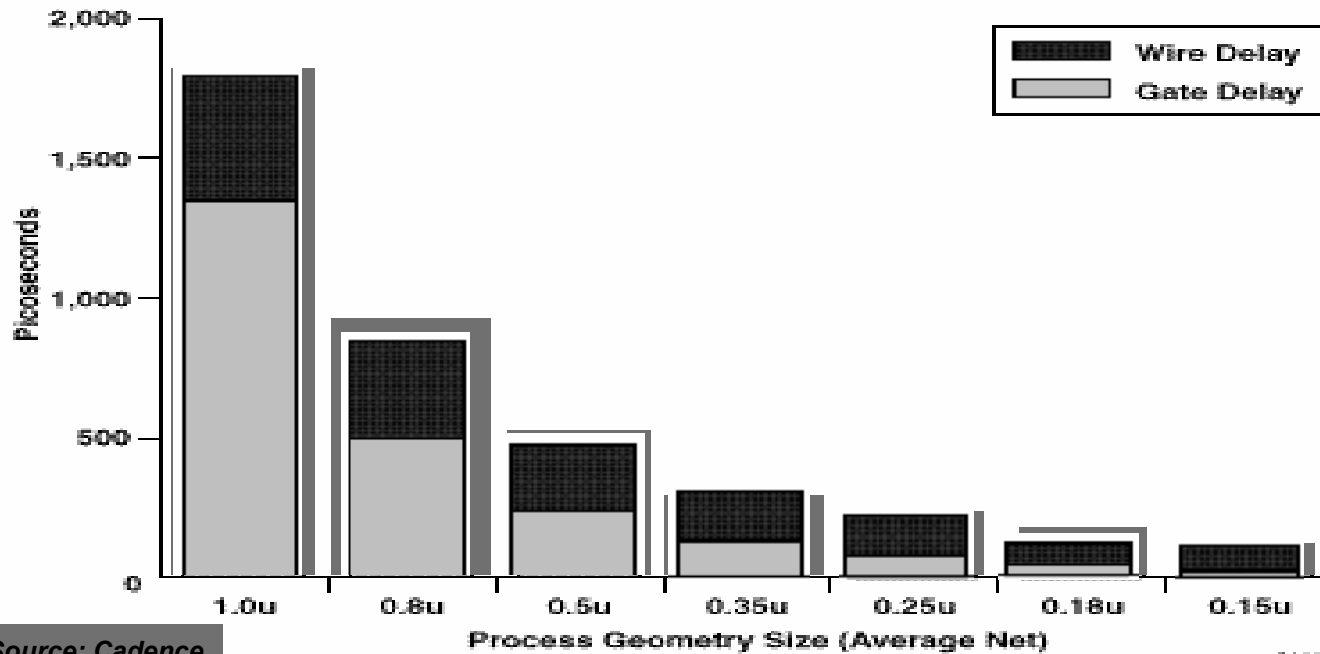
- **Time constants**
- **Interconnects**
- **Cross talk**

➤ **Reduced transit time**

- **Increased Mobility**
- **High Saturation Velocity**
- **Reduced Size**

# RC and Transit Time Delays

*GATE DELAY VS. WIRE DELAY  
AT DIFFERENT PROCESS GEOMETRIES*



Source: Cadence



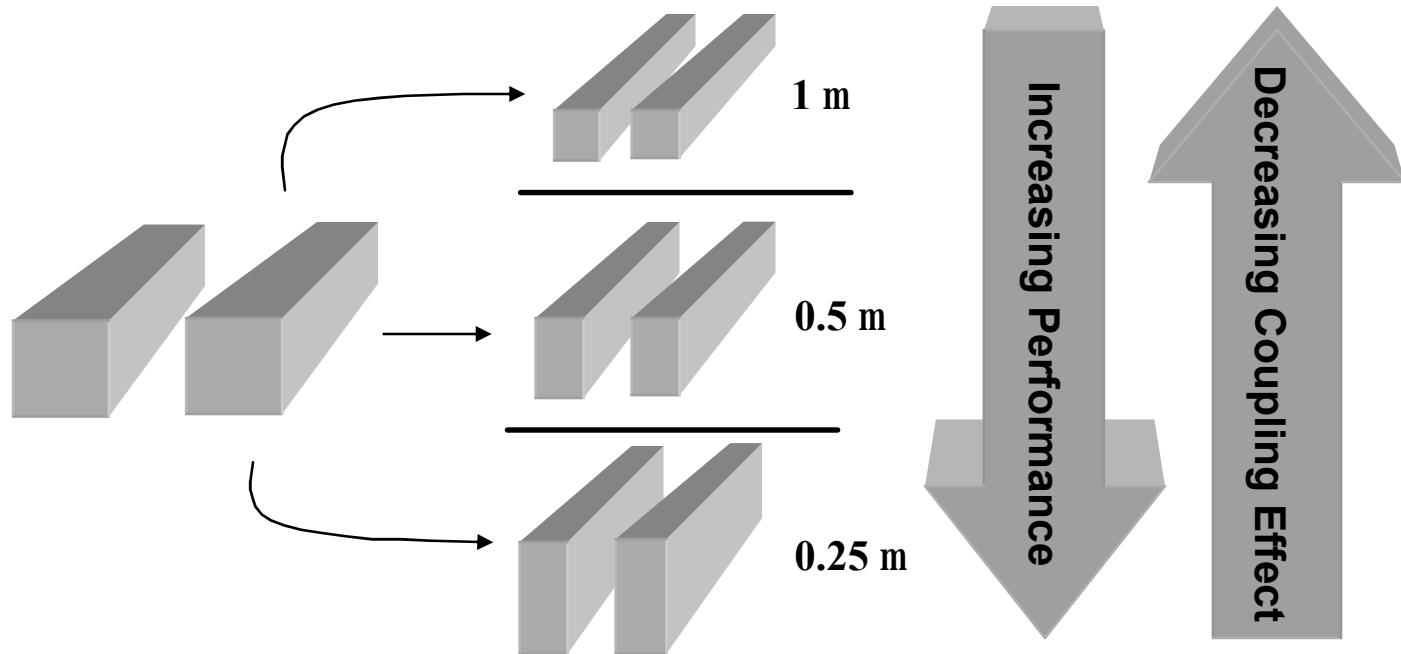
# Interconnect Problems

## RC Time Delays

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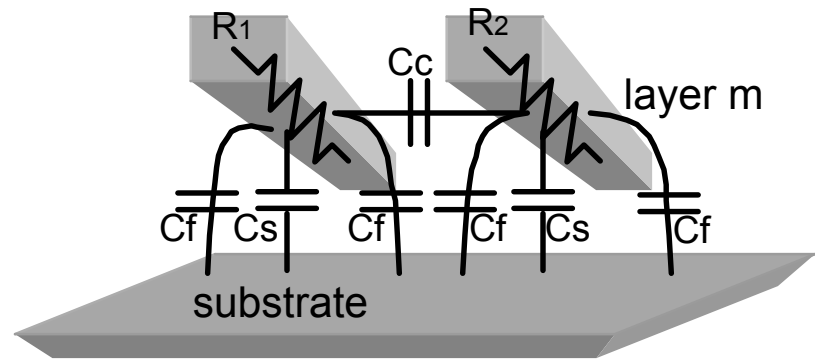
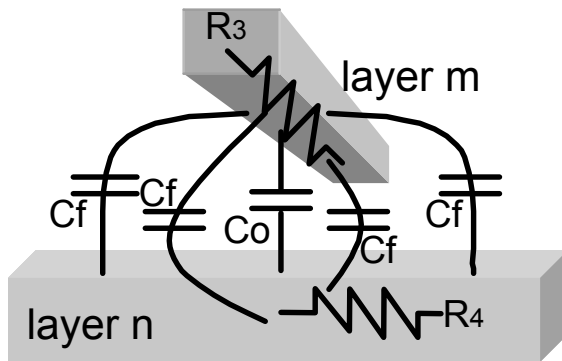
- RC time delay is increasing rapidly
- Wire resistance is rising
- Wires have larger cross-section ...  
introduce coupling
- Electromigration imposes current limits
- System performance, area and reliability  
are determined by interconnect  
quality, not devices!!!

# Interconnect Performance



Increased cross-section improves performance but also increases noise and capacitive and inductive coupling

# RC Delay Considerations



$$C_{int} = C_f + C_s + C_o + C_{load}$$

$$\tau = R_{int} * ( C_{int} + C_c / (C_{int} + C_c) )$$

$$\tau = R_{int} * ( C_{int}^2 + C_{int} * C_c + C_c ) / ( C_{int} + C_c )$$

- $C_c$  depends on dimensional shrink due to increased in cross-section
- In VLSI, make  $C_c$  becomes insignificant as possible, then

$$\tau = R_{int} * C_{int}$$



# Physical Effects

➤ **Quantum Effects**

$$L \leq l_D, \text{ a few nm}$$

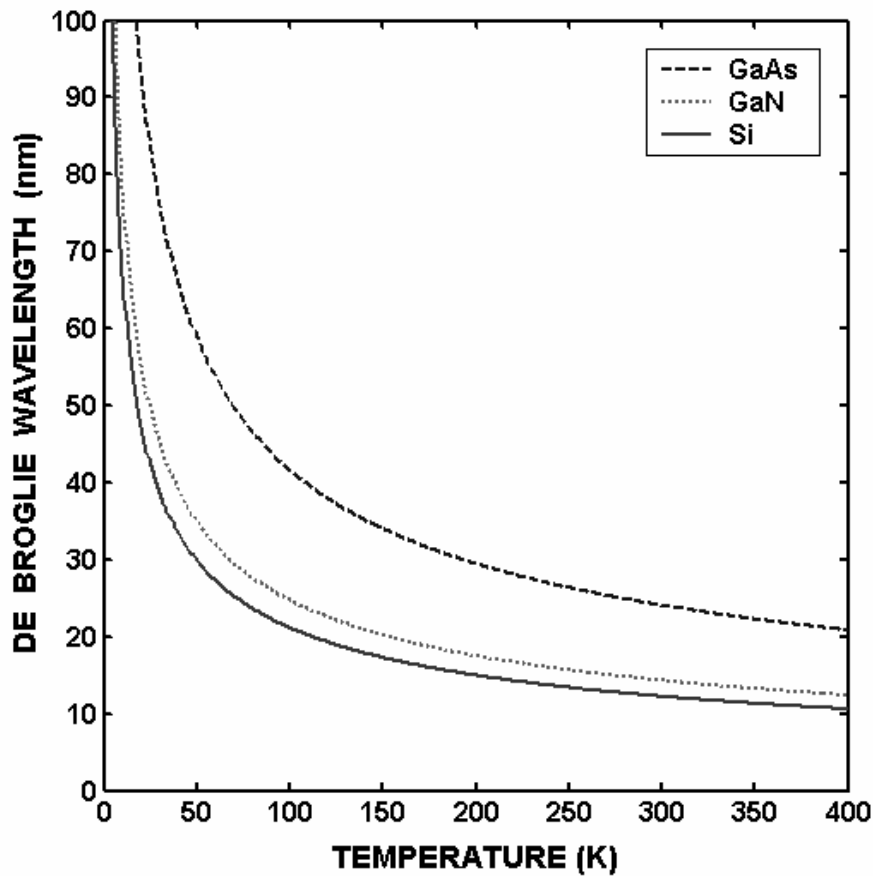
➤ **High-Field Effects**

$$E = \frac{V}{L} = \frac{5V}{1\text{mm}} = 50 \frac{\text{kV}}{\text{cm}}$$

➤ **Field Broadening**

$$qEl_D \geq \frac{\hbar}{t} \text{ or } qEl \geq k_B T$$

# Nano-Scale Quantum Engineering



$$l_D = \frac{h}{p}$$
$$= \frac{h}{\sqrt{3m * k_B T}}$$

# Bulk Semiconductors

$$L_{x,y,z} \gg L_D$$

All 3 cartesian directions analog-type

Density of States:

$$g_c(E) = \frac{1}{V} \frac{dN}{dE} = 4p \left( \frac{2m_e^*}{h^2} \right)^{\frac{3}{2}} (E - E_{co})^{\frac{1}{2}}$$

# Quasi-Two-Dimensional QW

$$L_z \leq l_D \quad L_{x,y} \gg l_D$$

z-direction digital-type

x,y-directions analog-type

$$E_{nk} = E_{co} + \frac{\hbar^2 (k_x^2 + k_y^2)}{2 m_e^*} + n^2 e_{oz}$$

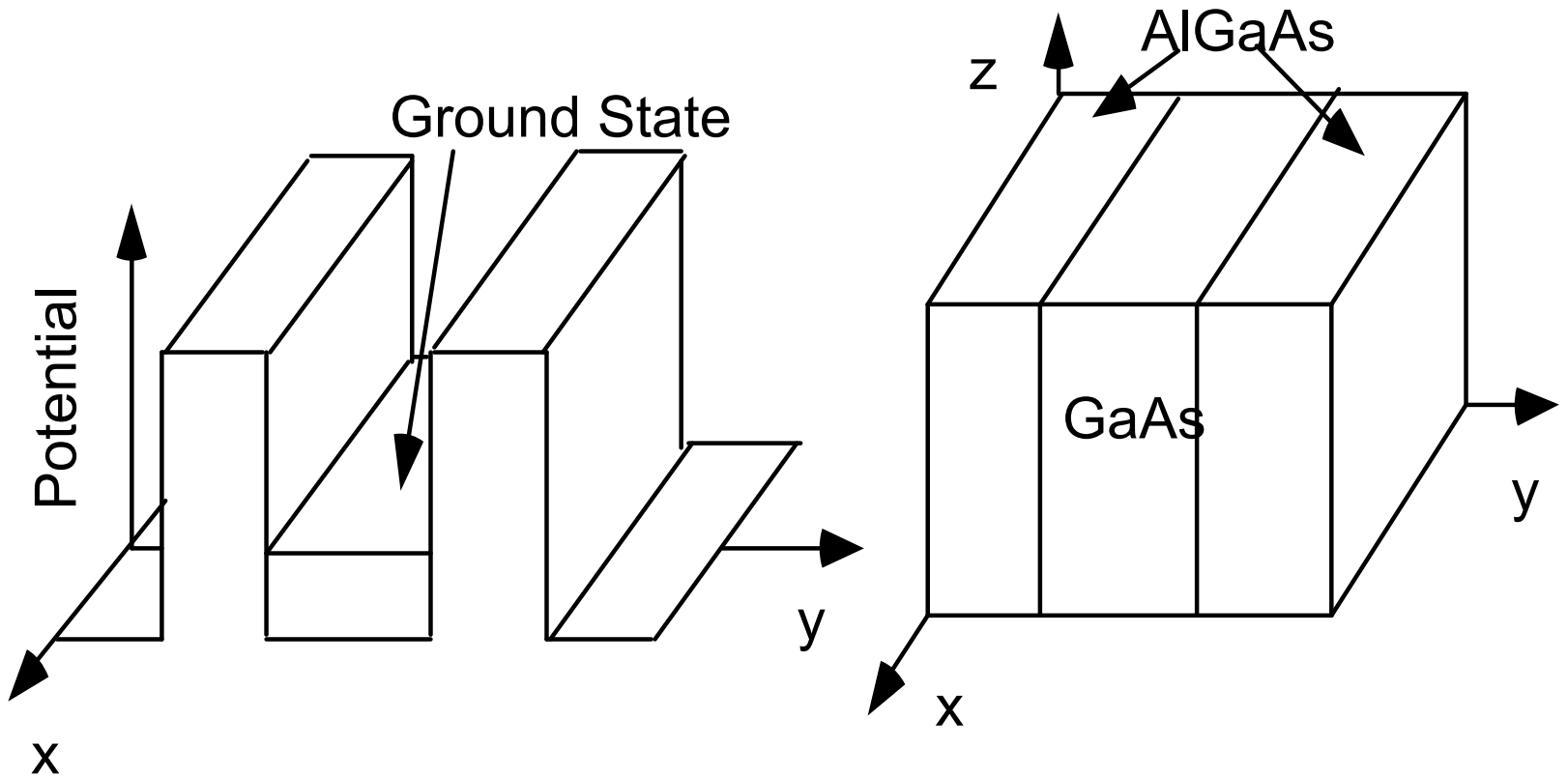
$$n = 1, 2, 3, \dots$$

$$e_{oz} = \frac{p^2 \hbar^2}{2 m_e^* L_z^2}$$

Density of States:

$$g_{c2}(E) = \frac{1}{A} \frac{dN}{dE} = \frac{m_e^*}{p \hbar^2} \text{Int} \left( \frac{E - E_{co}}{e_{oz}} \right)$$

# AlGaAs/GaAs/AlGaAs Prototype Quantum Well



# Quasi-One-Dimensional QW

$$L_{y,z} \leq L_D \quad L_x \gg L_D$$

y, z-direction digital-type

x-directions analog-type (QWW)

$$E_{nk} = E_{co} + \frac{\hbar^2 k_x^2}{2m_e^*} + m^2 e_{oy} + n^2 e_{oz}$$

$$m, n = 1, 2, 3, \dots$$

$$e_{o(y,z)} = \frac{p^2 \hbar^2}{2m_e^* L_{y,z}^2}$$

Density of States:

$$g_{cl}(E) = \frac{1}{L_x} \frac{dN}{dE} = \frac{(2m_e^*)^{1/2}}{p\hbar} \left( E - (E_{co} + m^2 e_{oy} + n^2 e_{oz}) \right)^{-1/2}$$

# Quasi-Zero-Dimensional Quantum Well

All 3 cartesian directions digital-type  
Quantum box (dot)

$$L_{x,y,z} \leq L_D$$

$$E_{nk} = E_c + \ell^2 e_{ox} + m^2 e_{oy} + n^2 e_{oz}$$

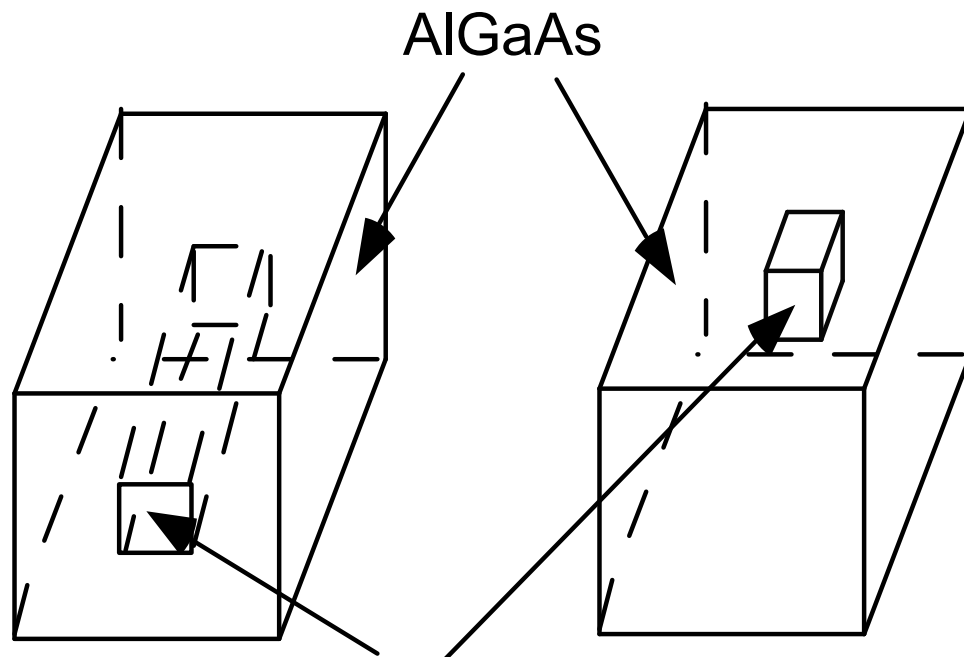
$$m, n, \ell = 1, 2, 3, \dots$$

$$e_{o(x,y,z)} = \frac{p^2 \hbar^2}{2m_e^* L_{x,y,z}^2}$$

# Quantum Well Wire

# Quantum Box (Dot)

Quantum wire

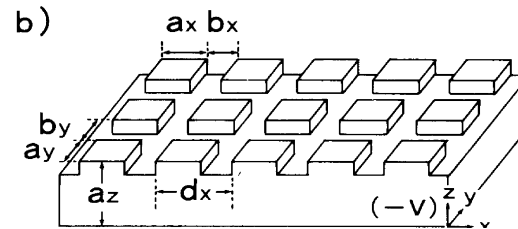
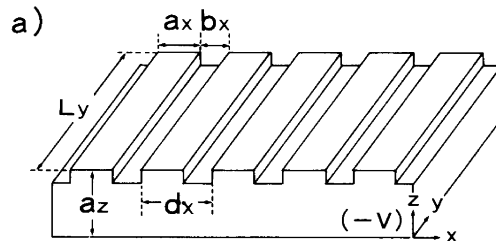
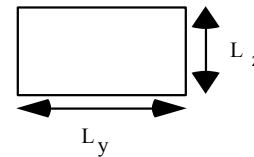
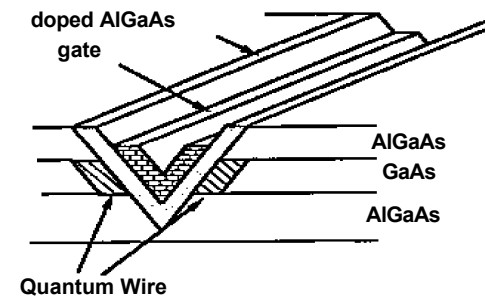
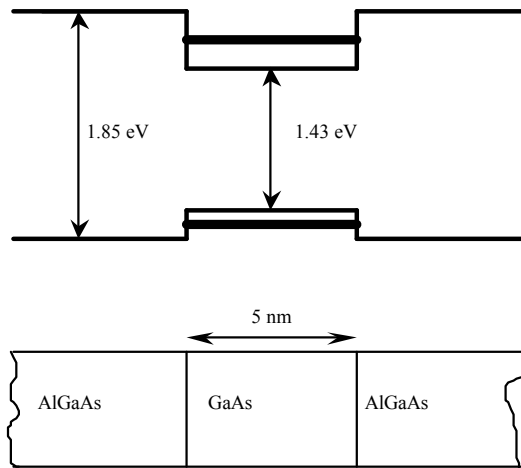


AlGaAs

GaAs inside

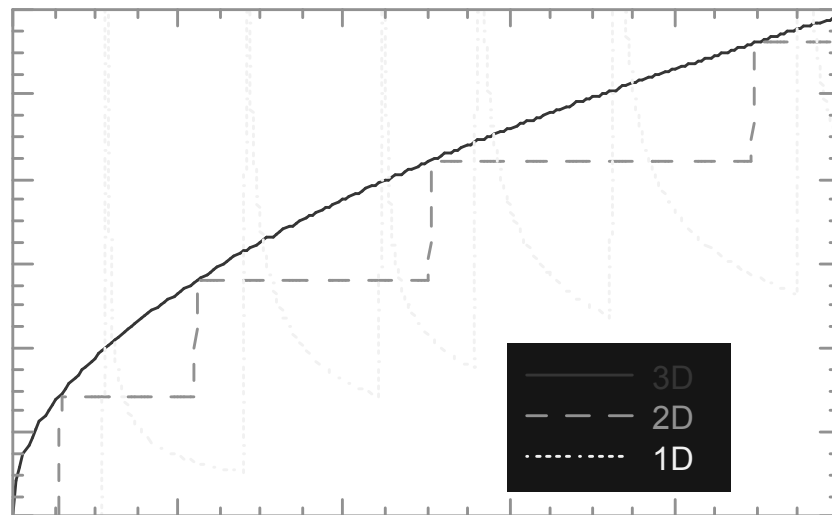
Quantum box

# Quantum Well Arrays

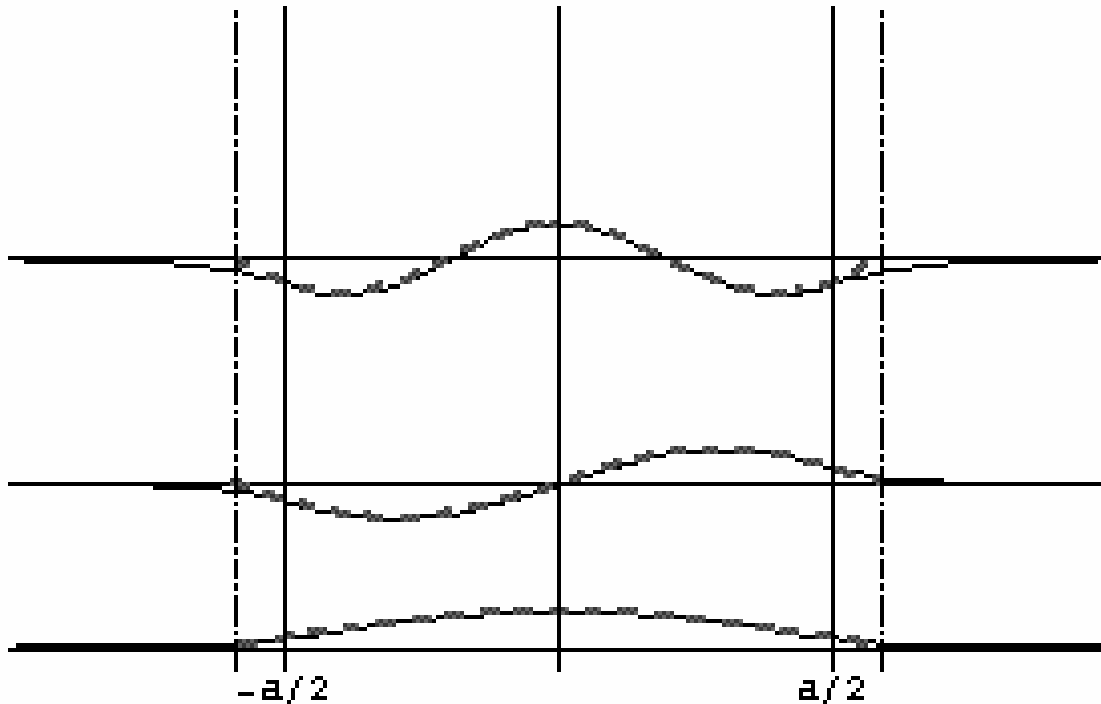


# Density of States

1



# Quantum Well with Finite Boundaries

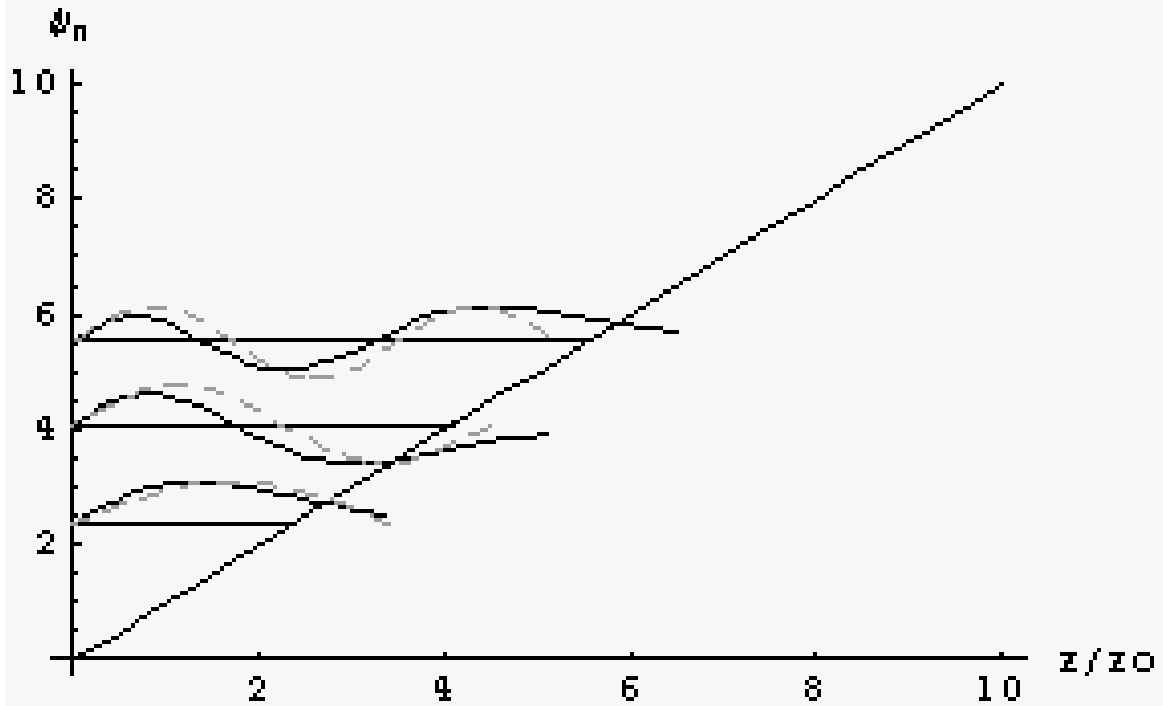


$$L_z = \left(1 + \frac{1}{P}\right)a$$

$$P = \left(\frac{2m^* \Delta E}{\hbar^2}\right)^{\frac{1}{2}} \frac{a}{2}$$

$$Z_n(z) = \sqrt{\frac{2}{L_z}} \sin\left(\frac{n\pi z}{L_z}\right)$$

# Triangular Quantum Well



Approximate:

$$Z_n(z) = \sqrt{\frac{2}{L_n}} \sin\left(\frac{n\pi z}{L_n}\right)$$

$$L_n = \frac{2}{a_n^2} z_0$$

$$a_n = \frac{0.53556}{\text{Ai}'(-\xi_n)}$$

Exact:

$$Z_n(z) = \frac{1}{\text{Ai}'(-\mathbf{x}_n) z_0^{1/2}} \text{Ai}\left(\frac{z}{z_0} - \mathbf{x}_n\right)$$

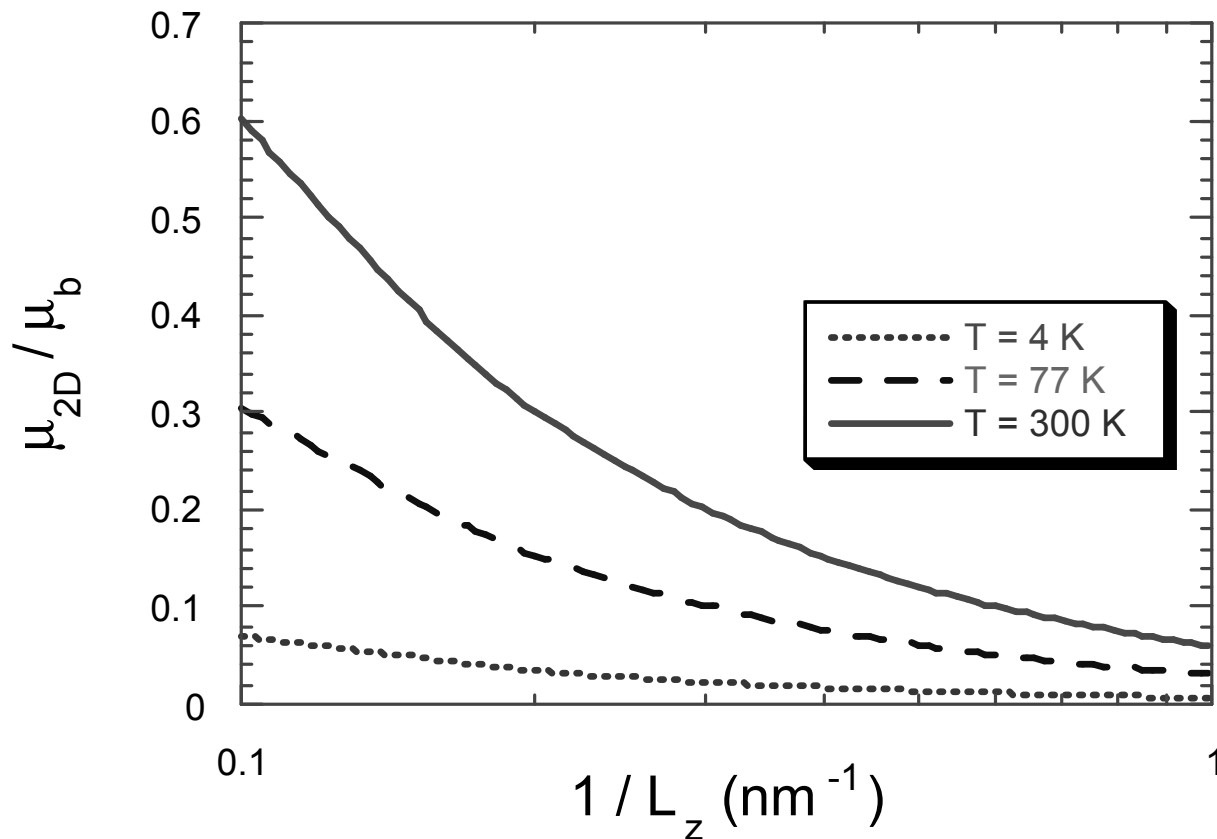
# Quantum-Confined Mobility Degradation

## ➤ Changes in the Density of States

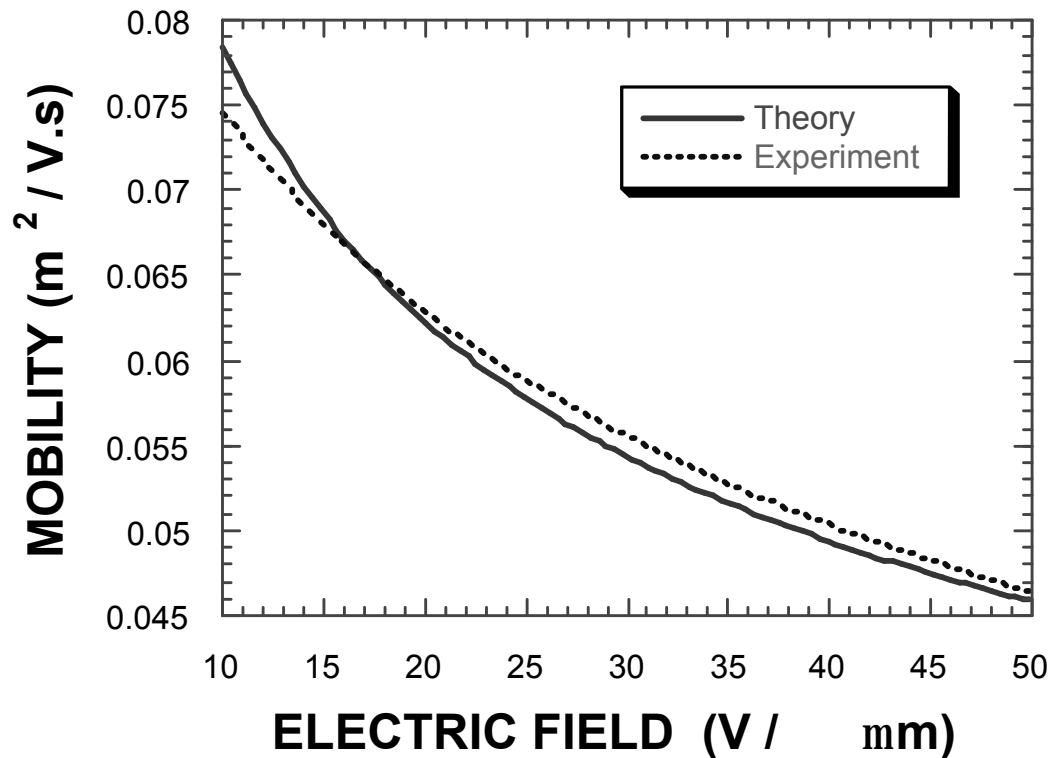
$$\frac{m_{QW}}{m_{bulk}} \Big|_{isotropic} = \frac{\sqrt{p} L_z}{l_D} \quad L_z \leq l_D$$

## ➤ Changes in the relative strength of each scattering interaction

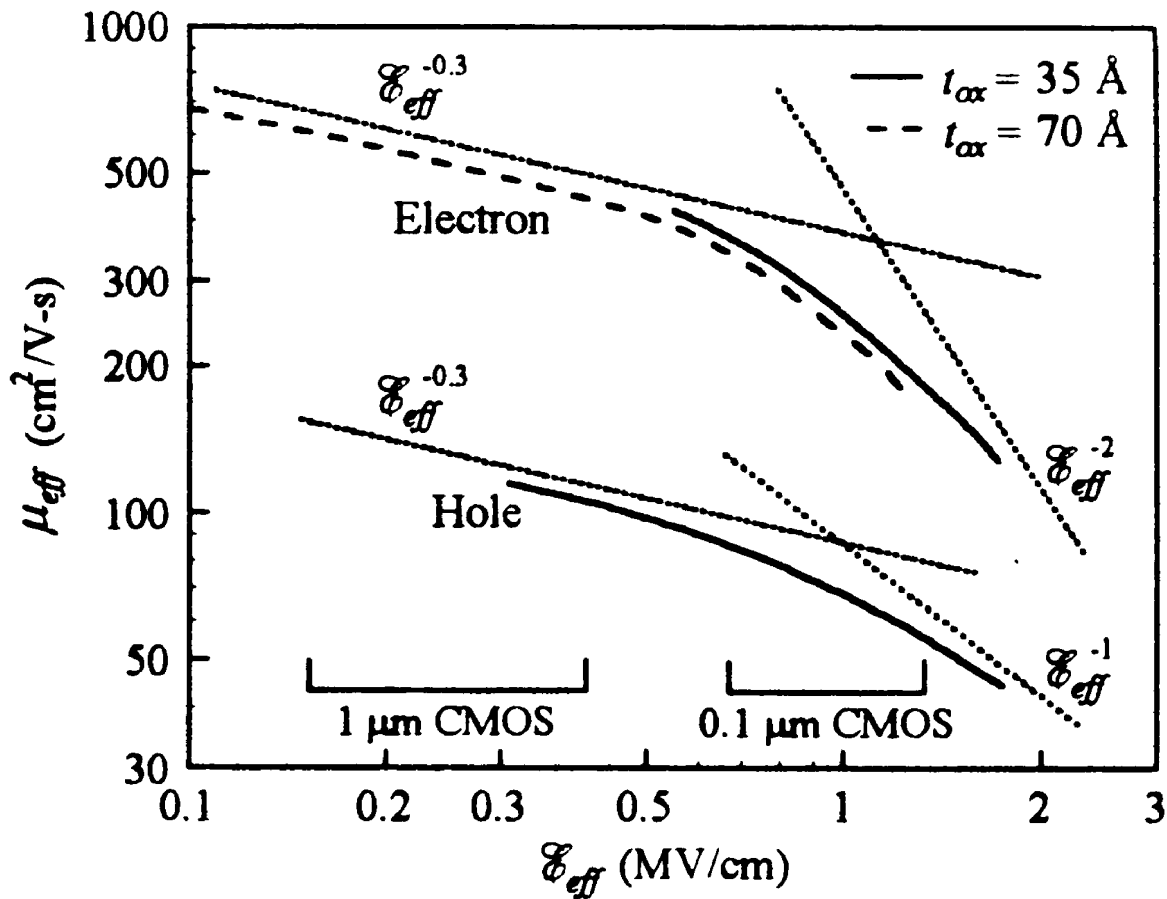
# Mobility Degradation Versus Quantum Confinement



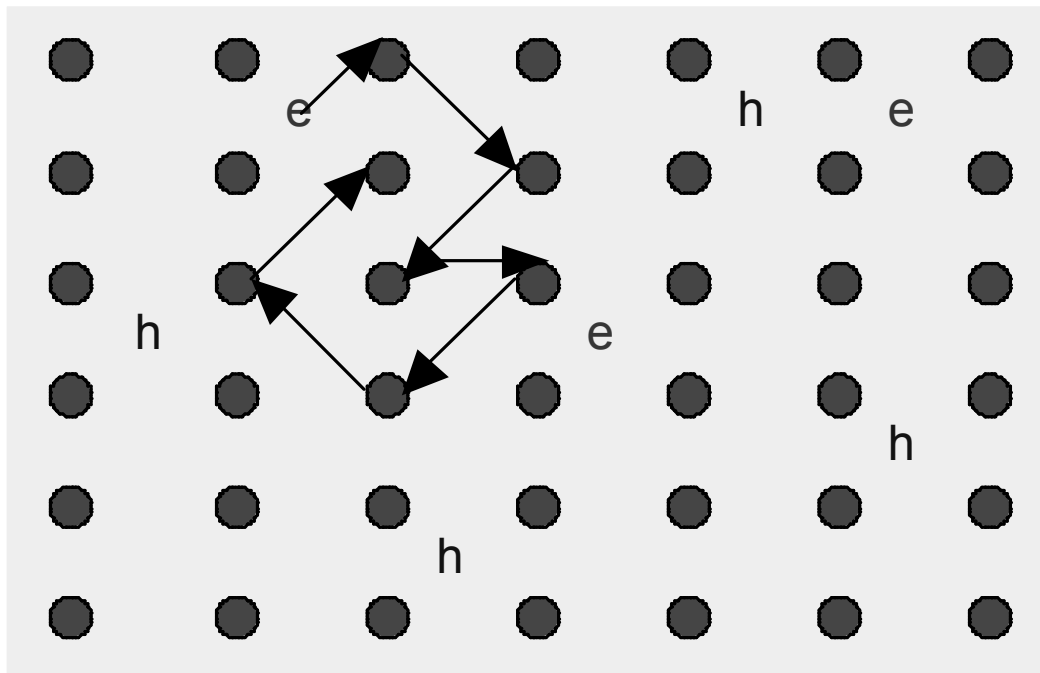
# Gate-Field Confinement Mobility Degradation in a TQW



# Electron and Hole Mobility in Submicron CMOS



# Random Thermal Motion

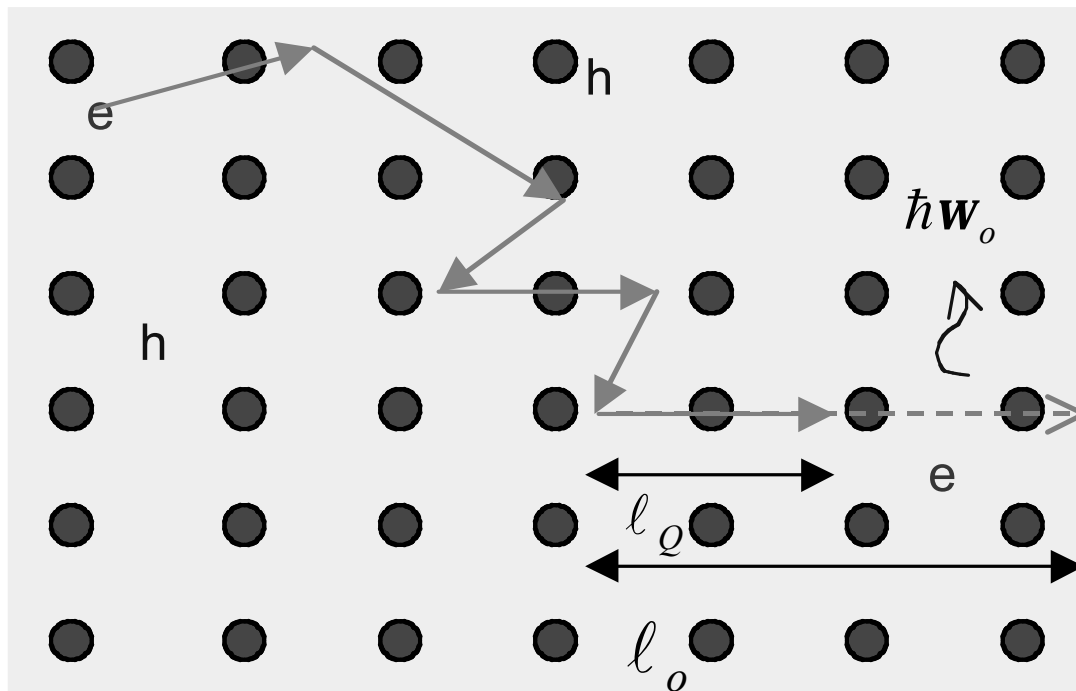


- Ions
- e Electrons
- h Holes

$$\langle \langle \vec{v}_{th} \rangle \rangle = 0$$

$$v_{th} = \sqrt{\frac{3k_B T}{m^*}} \approx 10^5 \text{ m/s}$$

# Quantum Emission



- Atoms
- e Electrons
- h Holes

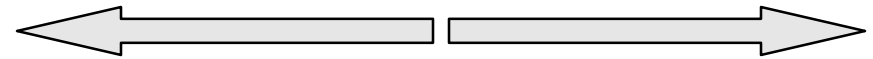
$$q\mathcal{E}l_Q = \hbar\omega_0 \quad \vec{\mathcal{E}}$$

$$l_Q = \frac{\hbar\omega_0}{q\mathcal{E}}$$

# Randomness to Streamlining

Velocity Vectors in Equilibrium

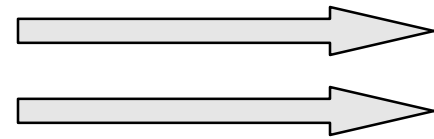
Randomness:



$$\vec{v}_d = \langle \vec{v}_{th} \rangle = 0$$

Velocity Vectors in a Very High Field

Streamlined:



$$\vec{v}_d = \langle \vec{v}_{th} \rangle = -v_{th} \hat{e}$$

# Ultimate Velocity-Bulk

$$\langle v \rangle_{3D} = \frac{2}{\sqrt{p}} v_{th} \frac{\mathcal{F}_1(\mathbf{h})}{\mathcal{F}_{1/2}(\mathbf{h})} \quad v_{th} = \sqrt{\frac{2k_B T}{m^*}}$$

$$\mathcal{F}_j(\mathbf{h}) = \frac{1}{\Gamma(j+1)} \int_0^\infty \frac{x^j}{1+e^{x-h}} \quad \text{Fermi Integral}$$

$$\mathbf{h} = \frac{V - E_c}{k_B T} \quad \text{Normalized Fermi Energy}$$

# Saturation Velocity Limits

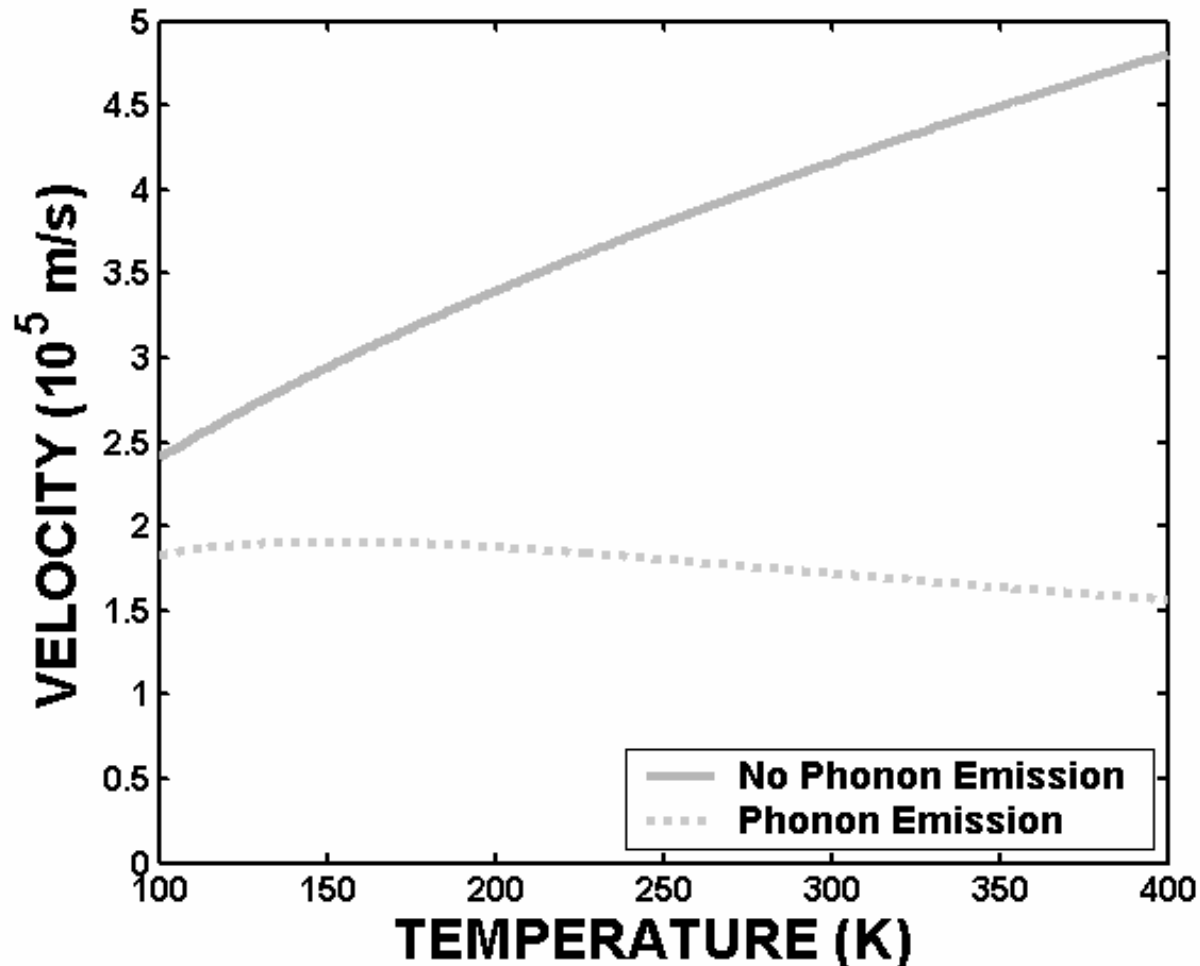
$$v_{sat} = \frac{2}{\sqrt{p}} v_{th} = \sqrt{\frac{8k_B T}{p m^*}}$$

Non-degenerate  
limit

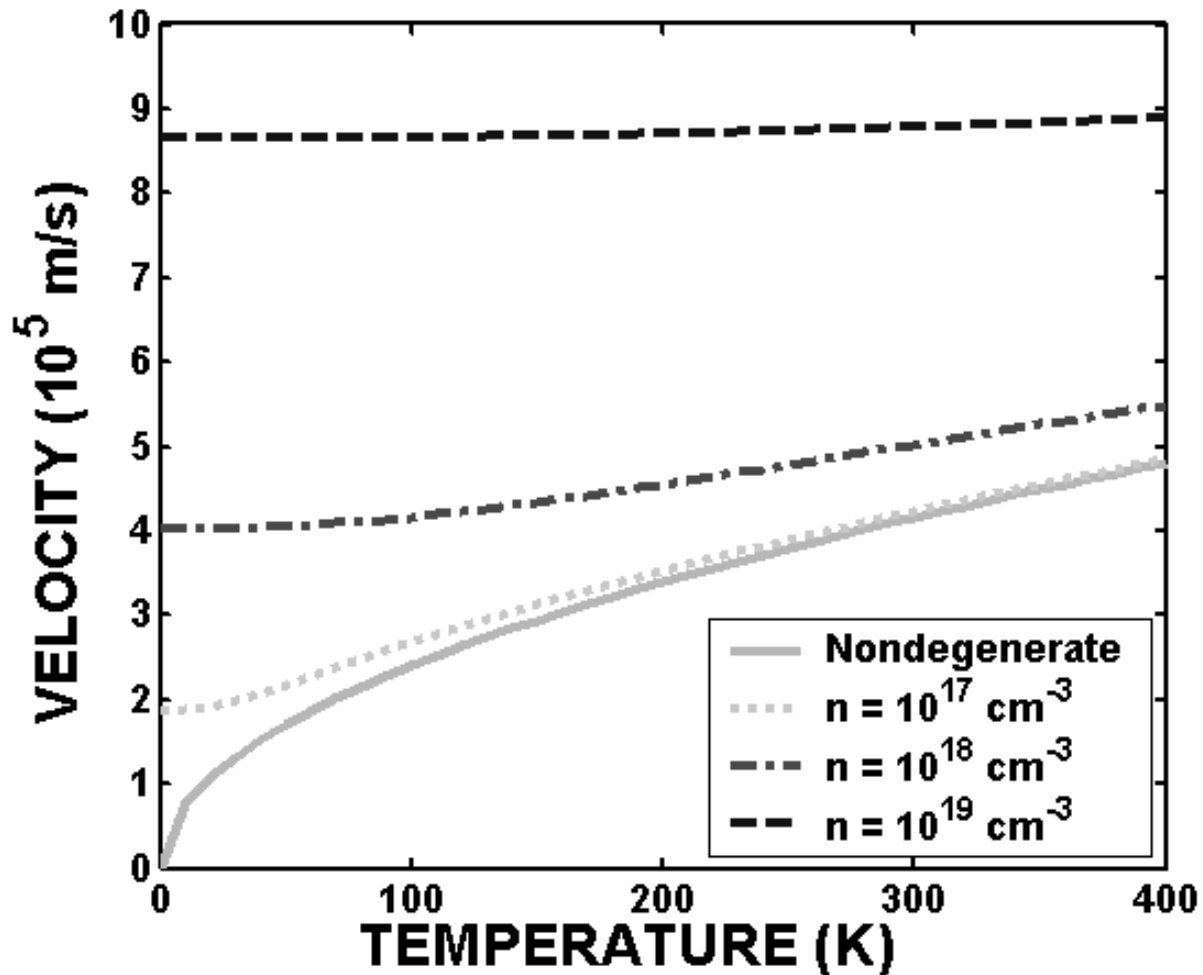
$$v_{sat} = \frac{3}{4} \frac{h}{m^*} \left( \frac{3n}{8p} \right)^{\frac{1}{3}}$$

Degenerate  
limit

# Velocity versus Temperature- Nondegenerate



# Velocity versus Temperature- Degenerate



# Saturation Velocity-Q2D

$$\langle v \rangle_{2D} = \frac{\sqrt{p}}{2} v_{th} \frac{\mathcal{F}_{1/2}(h)}{\mathcal{F}_0(h)}$$

$$\mathcal{F}_0(h) = \ln(1 + e^h)$$

$$h = \frac{V - E_c}{k_B T} \quad E_c = E_{co} + e_{oz}$$

# Saturation Velocity-Q1D

$$\langle v \rangle_{1D} = \frac{1}{\sqrt{p}} v_{th} \frac{\mathcal{F}_0(h)}{\mathcal{F}_{-1/2}(h)}$$

$$h = \frac{V - E_c}{k_B T} \quad E_c = E_{co} + e_{oy} + e_{oz}$$

# Modeling Transport

$$\frac{dv}{dt} = -\frac{q\mathcal{E}}{m^*} - \frac{v - \langle v_{th} \rangle}{t_c} = 0$$

*Transient Response:*  $v = -\frac{qt_c}{m^*} \mathcal{E} \left( 1 - e^{-\frac{t}{t_c}} \right)$

$$v_d = -\frac{qt_c}{m^*} \mathcal{E} = -m_o \mathcal{E}$$

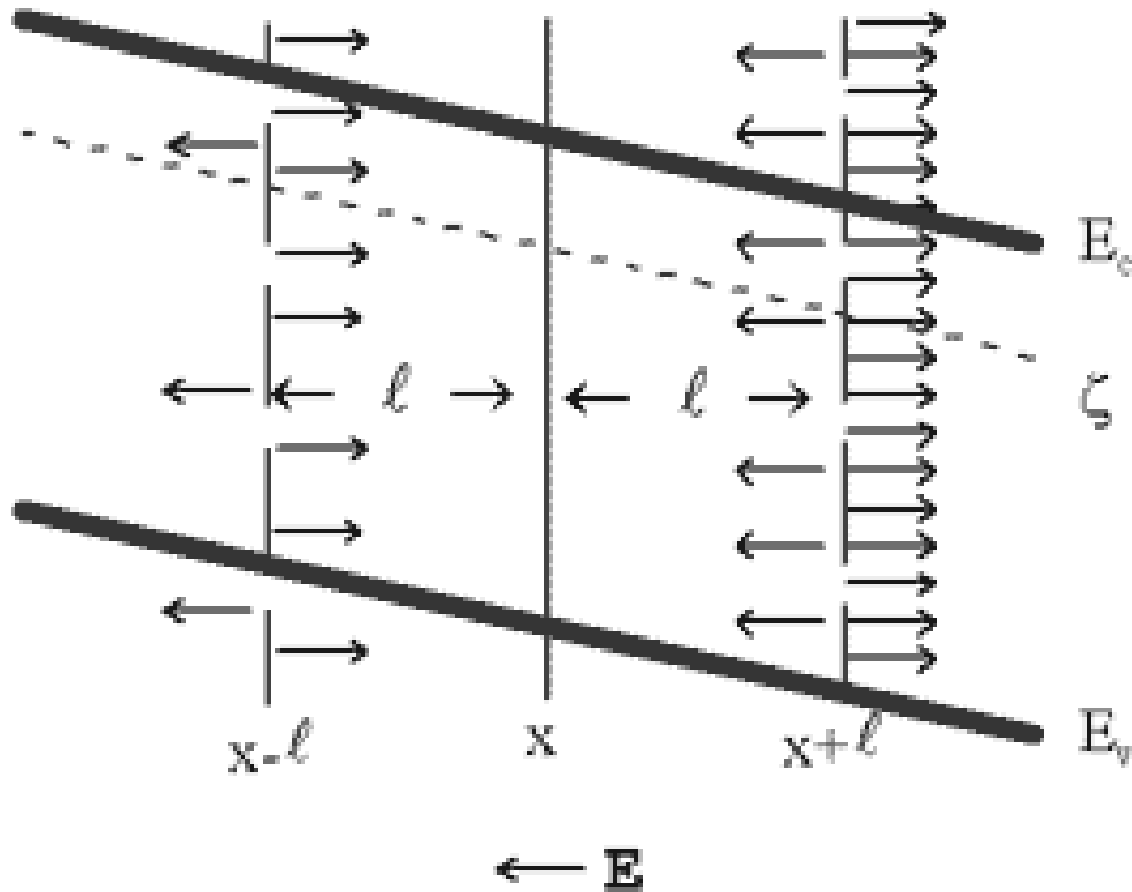
# Quantum Emission

$$q\mathcal{E}l_Q = \hbar\omega_o \longrightarrow l_Q = \frac{\hbar\omega_o}{q\mathcal{E}} \longrightarrow t_Q = \frac{\hbar\omega_o}{q\mathcal{E}v_{th}}$$

Effective Collision time:  $t_{eff} = t_c \left( 1 - e^{-\frac{t_Q}{t_c}} \right)$

Effective collision length:  $l = l_o \left( 1 - e^{-\frac{l_Q}{l_o}} \right)$

# 1-D Random Walk in a Bandgap semiconductor

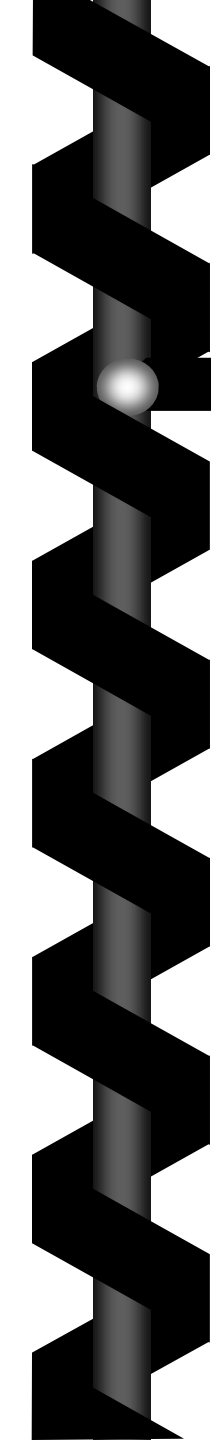


# Modeling the Distribution

$$f(\mathbf{e}_a, \vec{\mathcal{E}}) = \frac{1}{e^{\frac{\mathbf{e}_a - z + q\vec{\mathcal{E}} \cdot \vec{\ell}}{k_B T}} + 1} = \frac{1}{e^{x+d} + 1}$$

$$x = \frac{\mathbf{e}_a - V}{k_B T} \quad \delta = \delta_o \left( 1 - e^{-\frac{\delta_Q}{\delta_o}} \right)$$

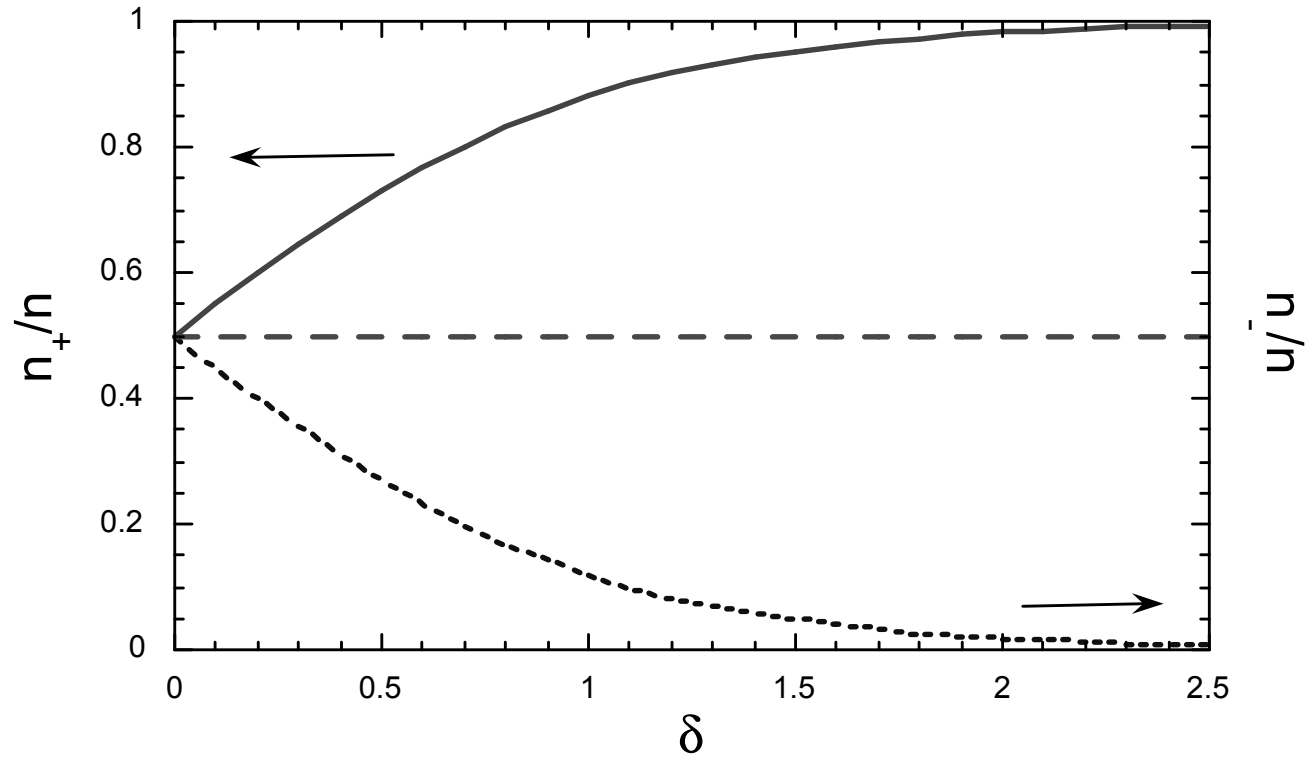
$$d_o = \frac{q\mathcal{E}\ell_o}{k_B T} = \frac{\mathcal{E}}{\mathcal{E}_{co}} = \frac{V}{V_{co}} \quad \delta_Q = \frac{\hbar\omega_o}{k_B T}$$



# Left-Right Asymmetry Itinerant Electron Population

$$\frac{n_{\pm}(x)}{n(x)} = \frac{e^{\pm d}}{e^{+d} + e^{-d}} = \frac{e^{\pm d}}{2 \cosh(d)}$$

# Streamlining the Randomness



# Drift-Diffusion

$$J(x) = n(x) q v_{th} \tanh(d)$$

Drift

$$+ q v_{th} \ell \frac{dn}{dx}$$

Diffusion

$$v_d = v_{th} \tanh(d)$$

Drift Velocity

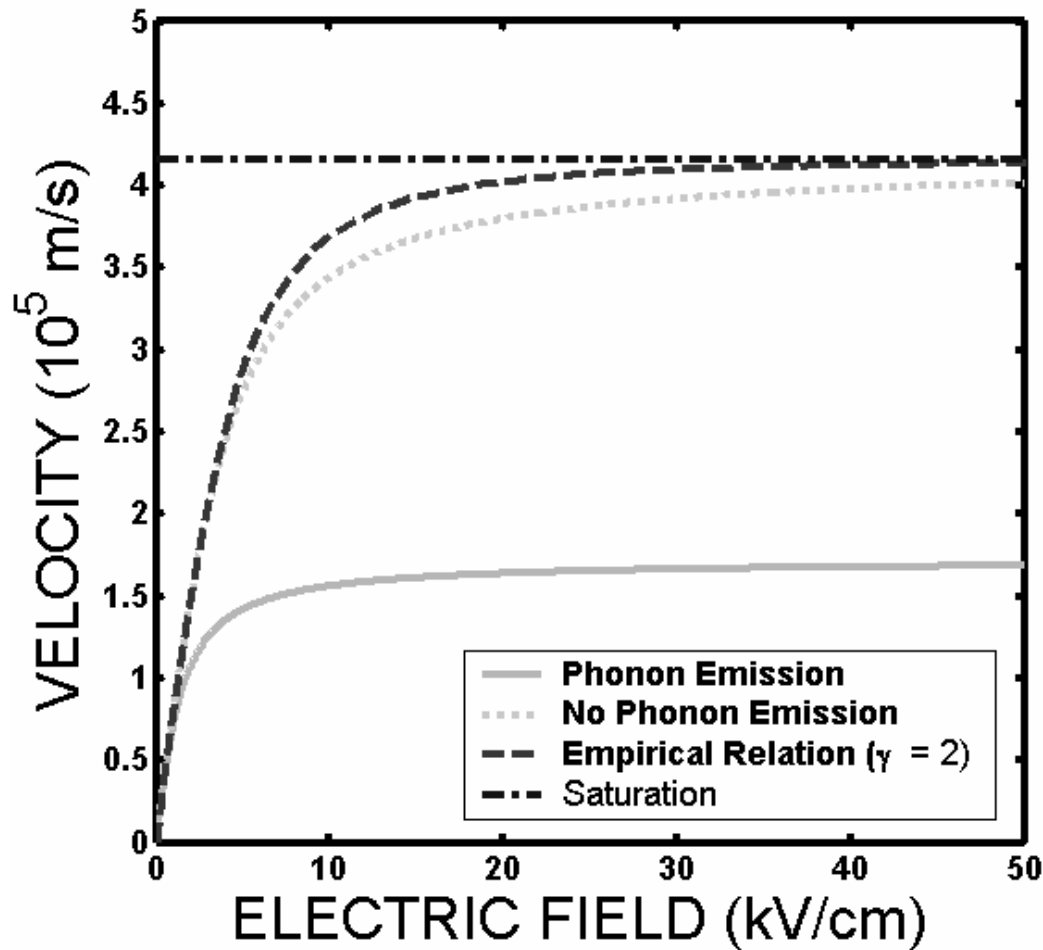
$$D_n = v_{th} \ell = m_{no} V_t \frac{d}{d_o}$$

Diffusion Coefficient

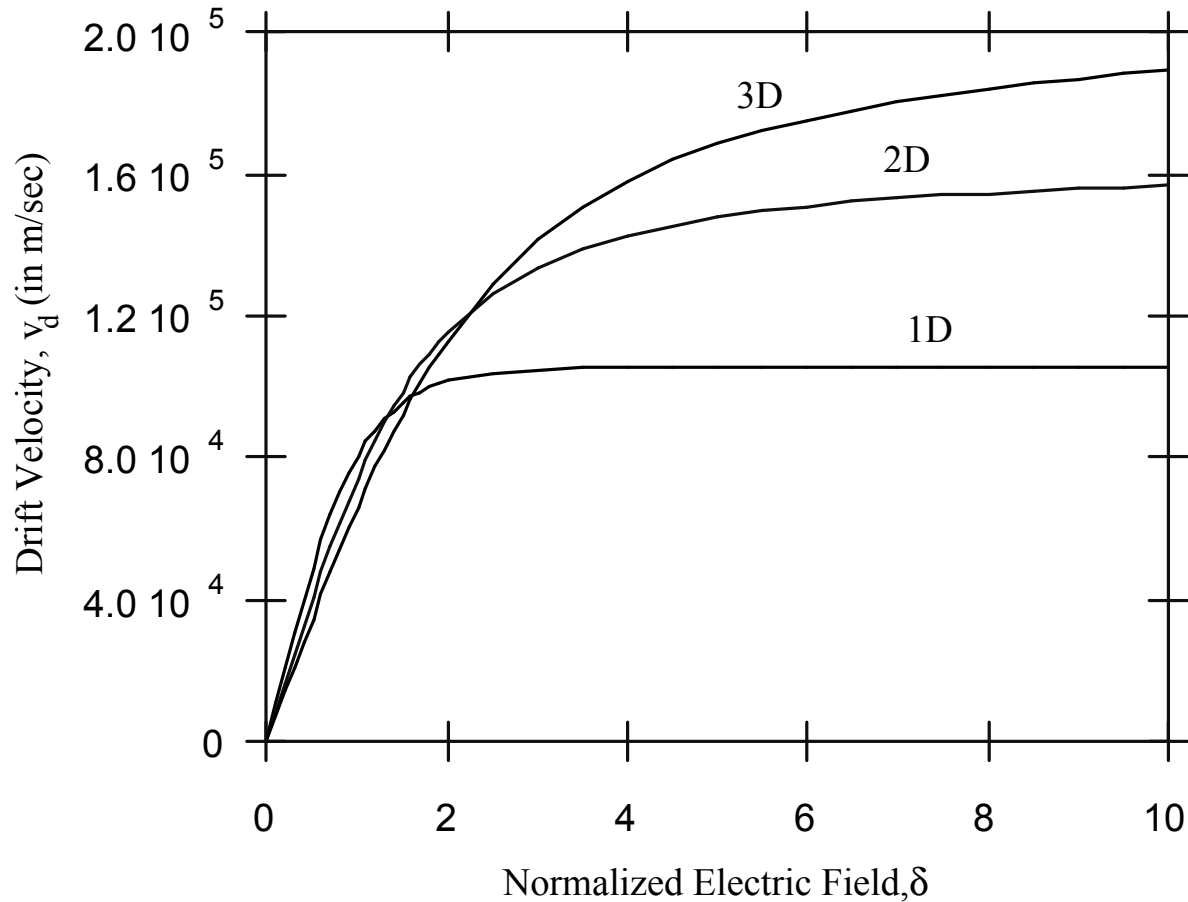
$$m_{no} = \frac{q \ell_o}{m_n^* v_{th}} = \frac{q t_{cn}}{m_n^*}$$

$$V_t = \frac{k_B T}{q}$$

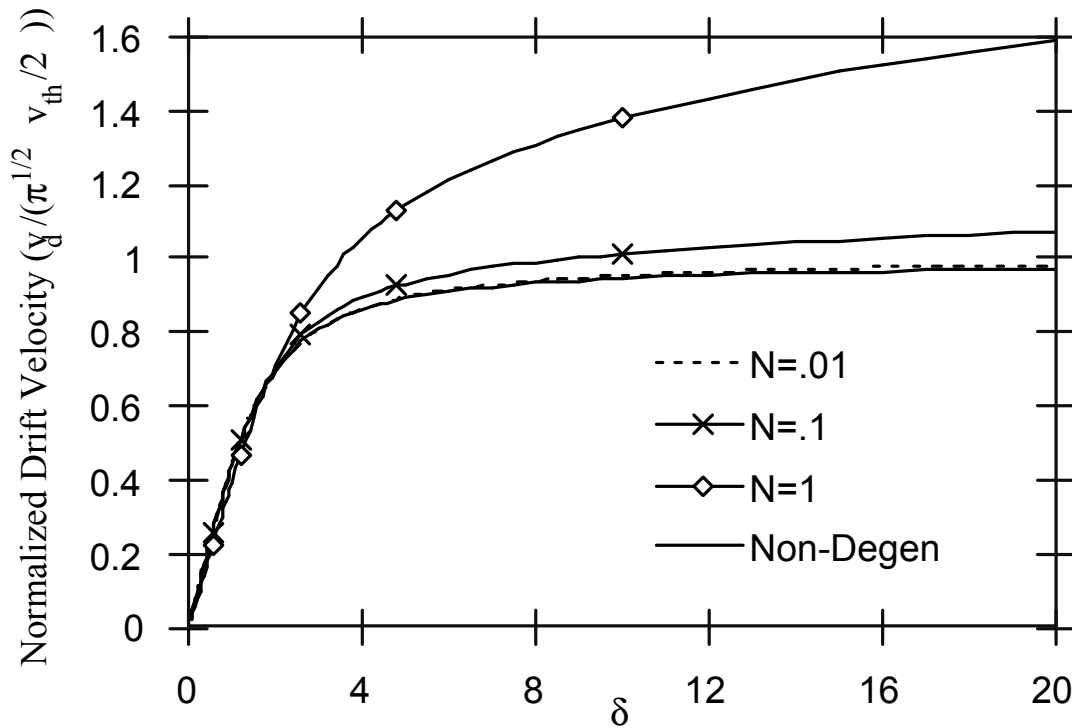
# Single-Valley v-E Characteristics



# Velocity-Field Characteristics



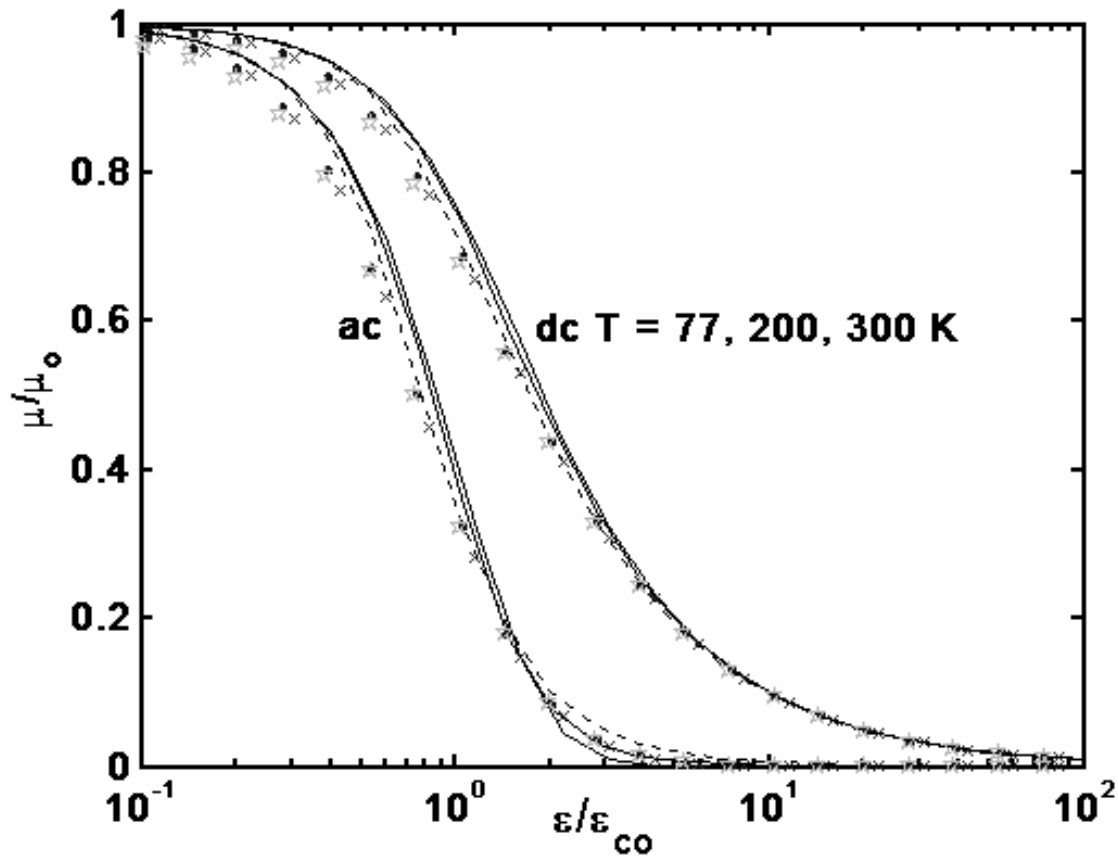
# Effect of Degeneracy (2-D)



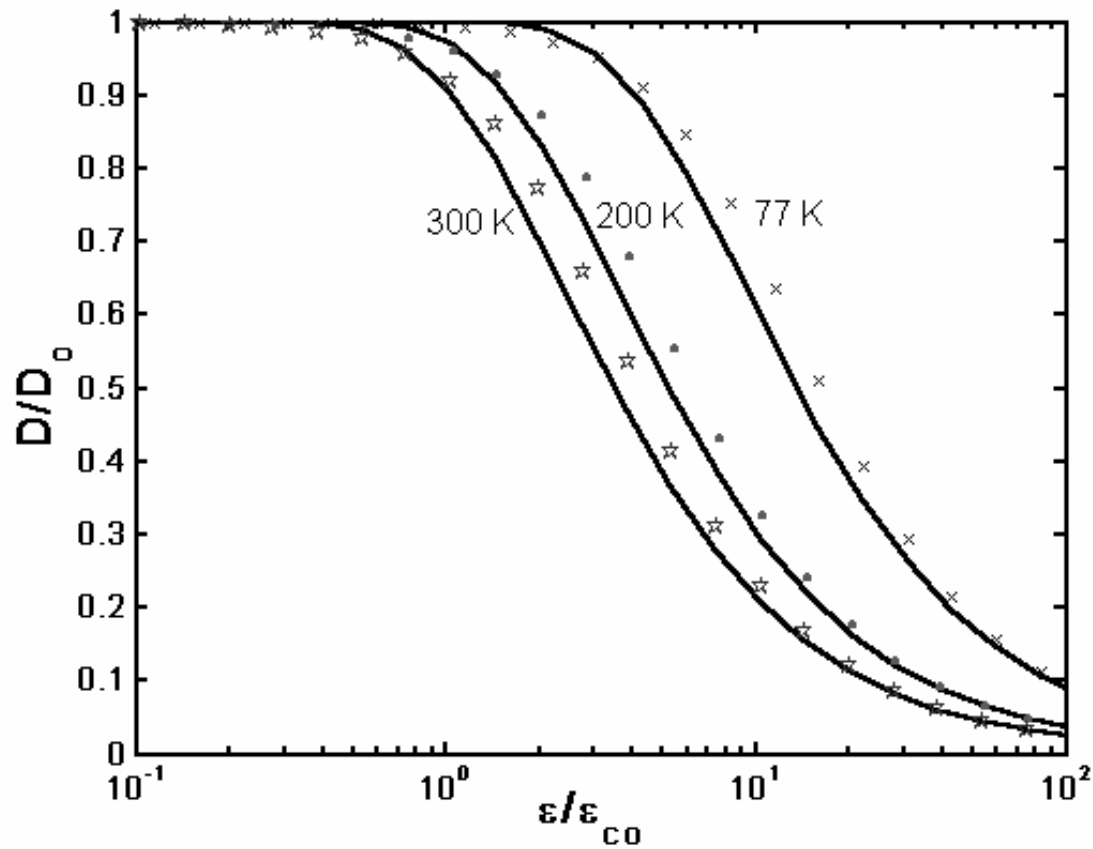
$$N = n_s l_D^2$$

$$l_D = \frac{h}{\sqrt{2m^* k_B T}}$$

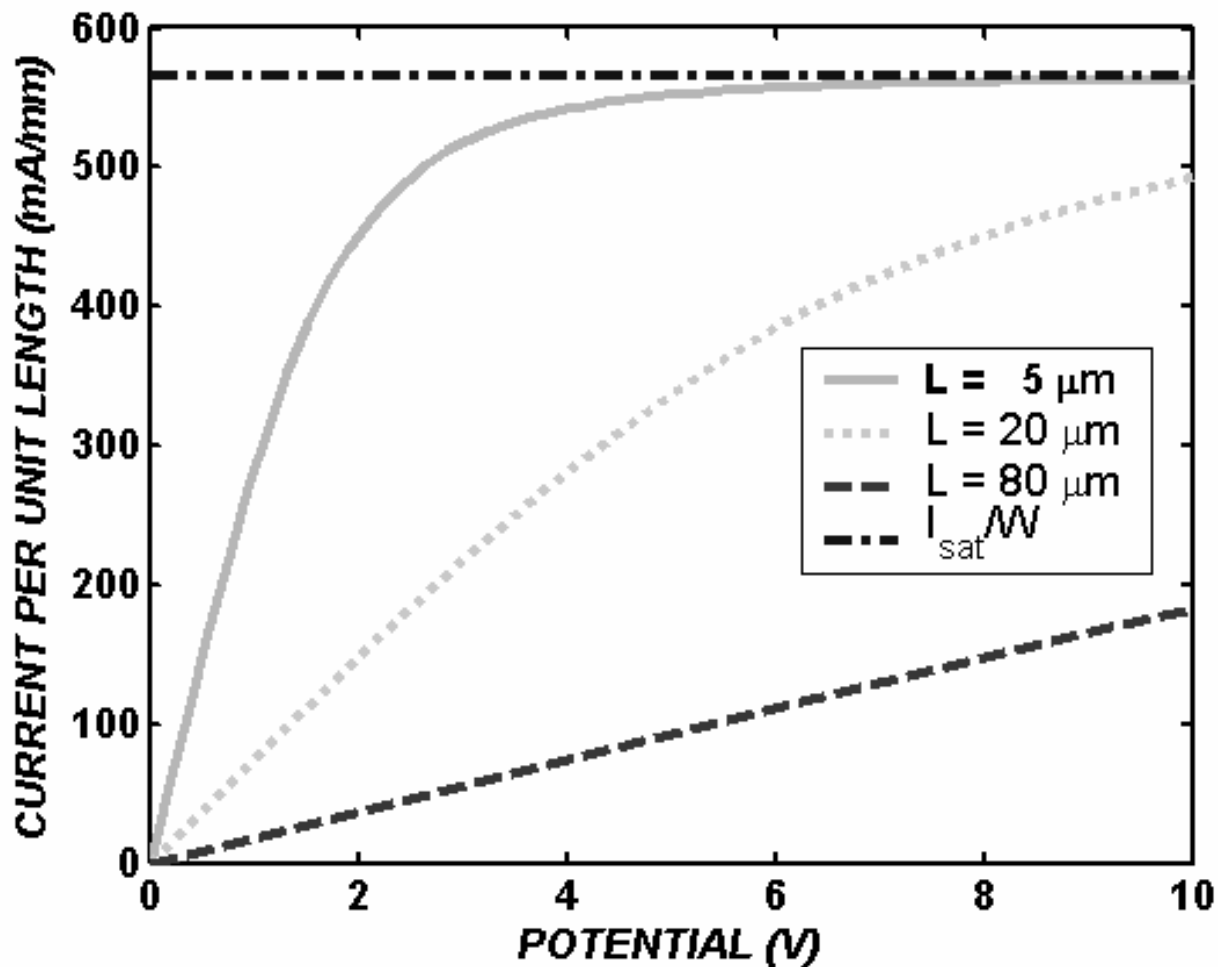
# Mobility Degradation



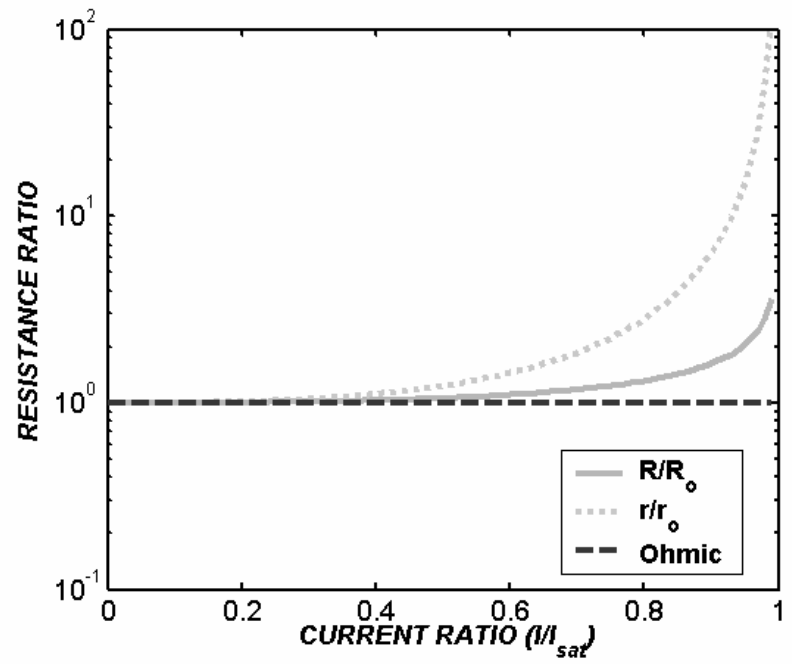
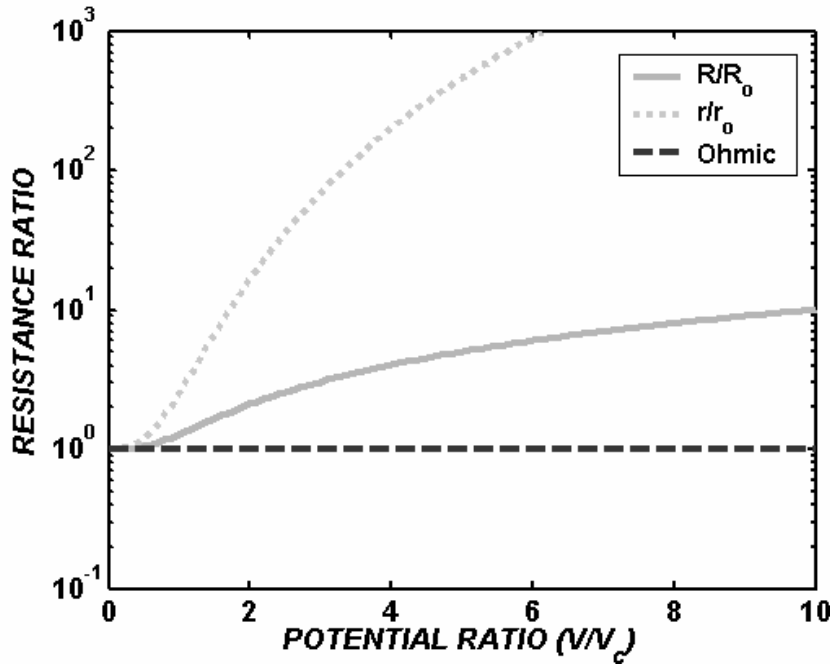
# Diffusion Coefficient Degradation



# I-V Characteristics Microresistors



# Resistance Blow-Up



$$V_c = E_c L = \frac{V_t}{\ell_0} L \approx \begin{cases} 2.6 \text{ kV} & \text{for } L = 1 \text{ cm} \\ 0.26 \text{ V} & \text{for } L = 1 \text{ mm} \end{cases}$$

# Power Law

$$P = VI = \frac{VV_c}{R_o} \tanh\left(\frac{V}{V_c}\right) = VI_{sat} \tanh\left(\frac{V}{V_c}\right)$$

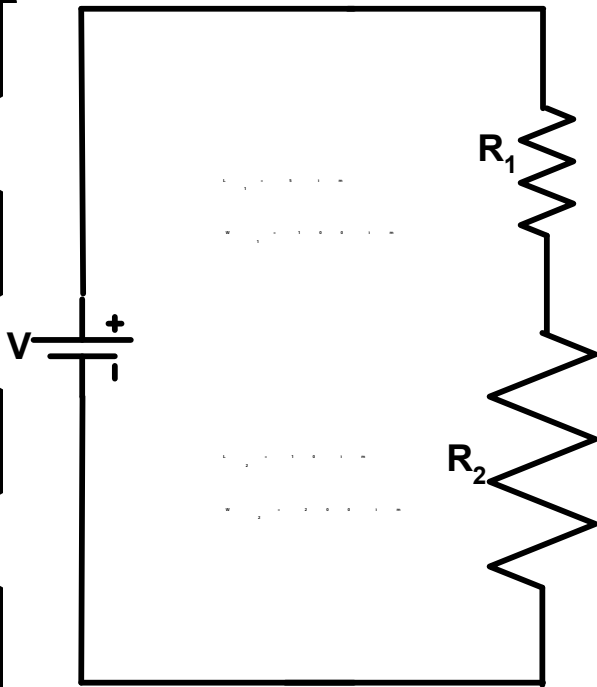
$$P = \frac{V^2}{R_o}$$

$$V < V_c \text{ (Ohmic)}$$

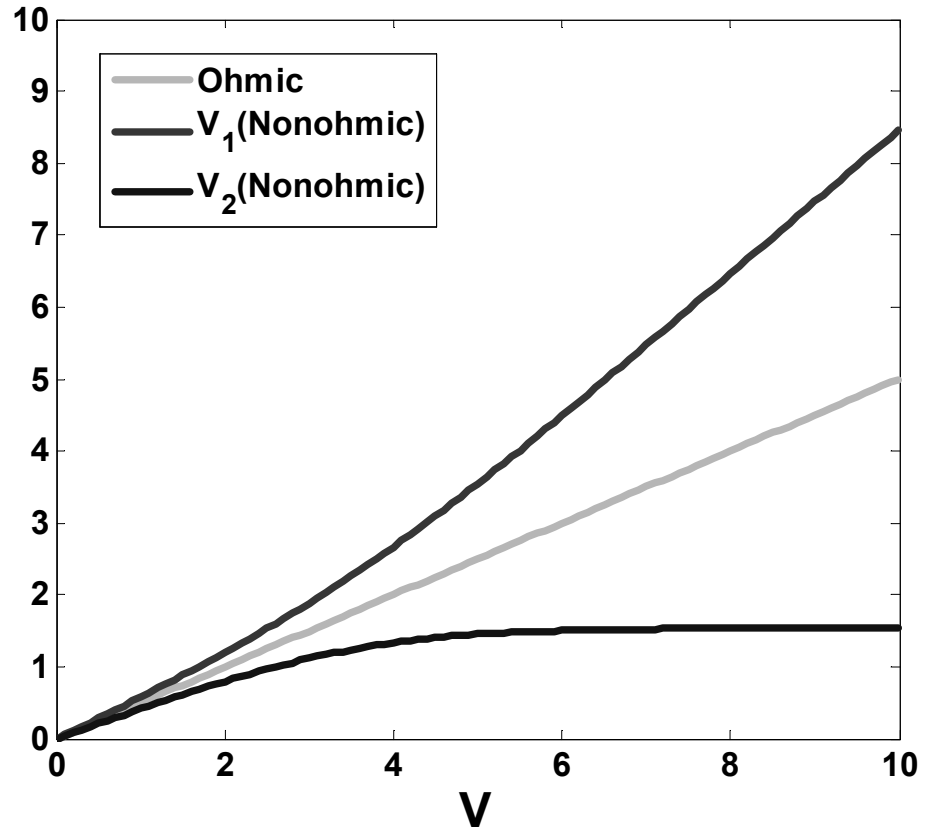
$$P = \frac{VV_c}{R_o}$$

$$V \gg V_c$$

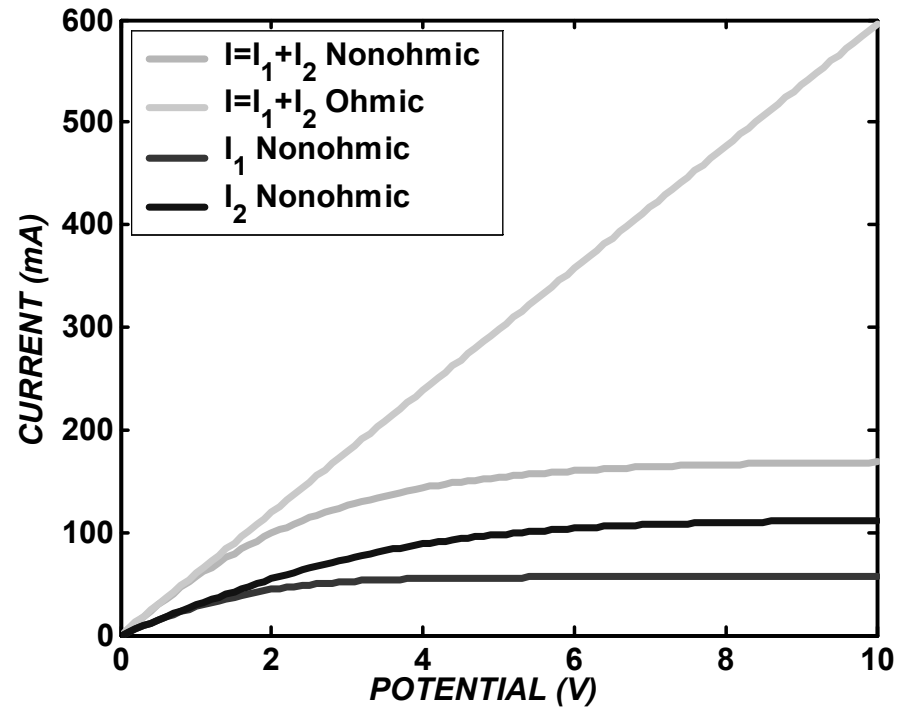
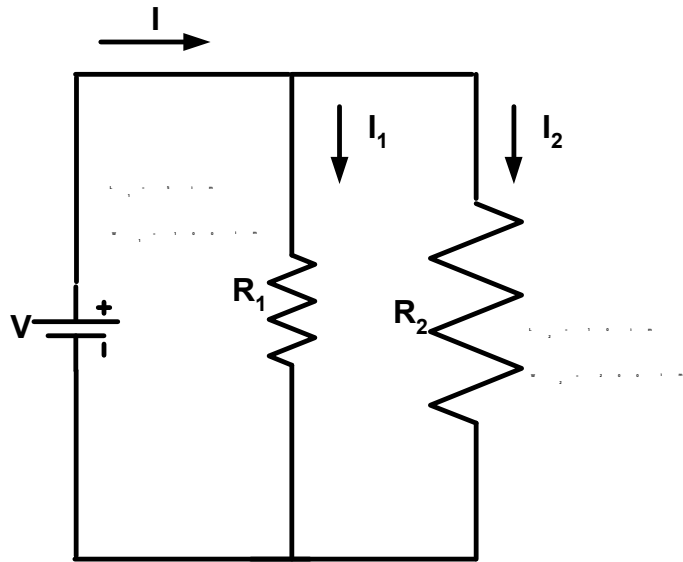
# Voltage Divider



$V_1$  OR  $V_2$

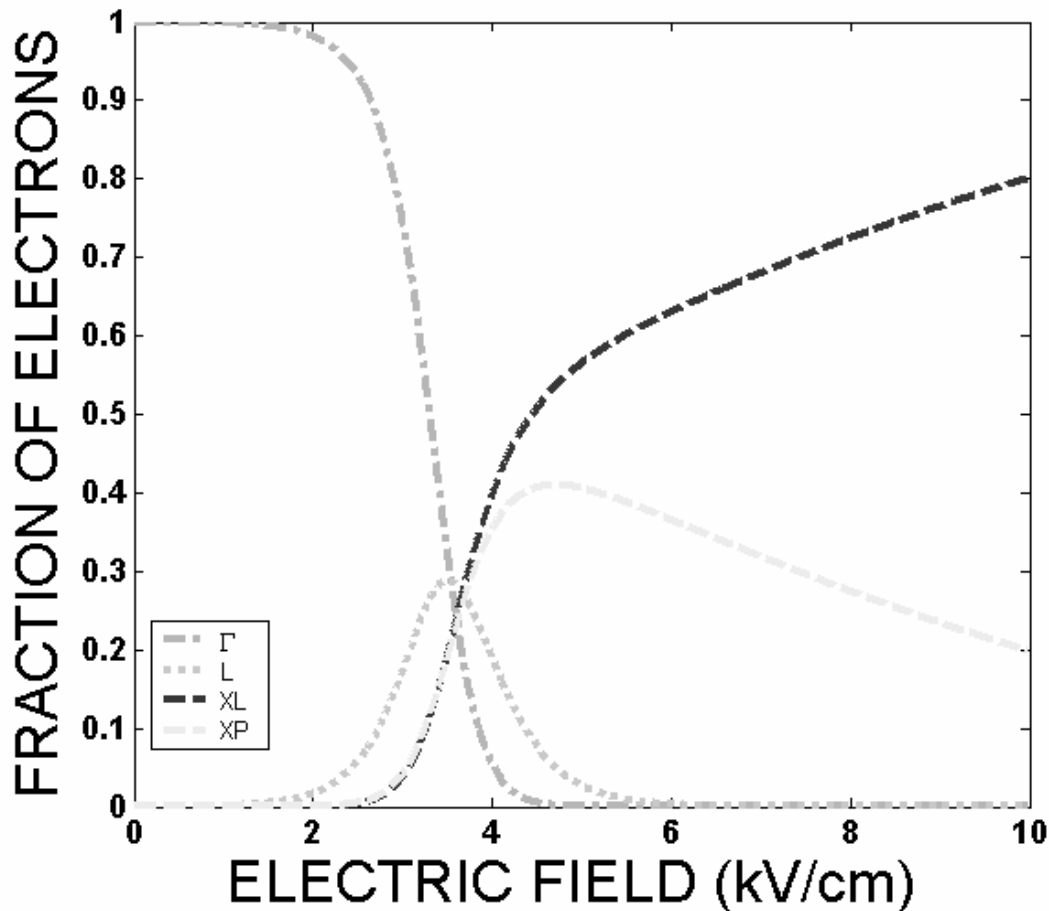


# Current Divider



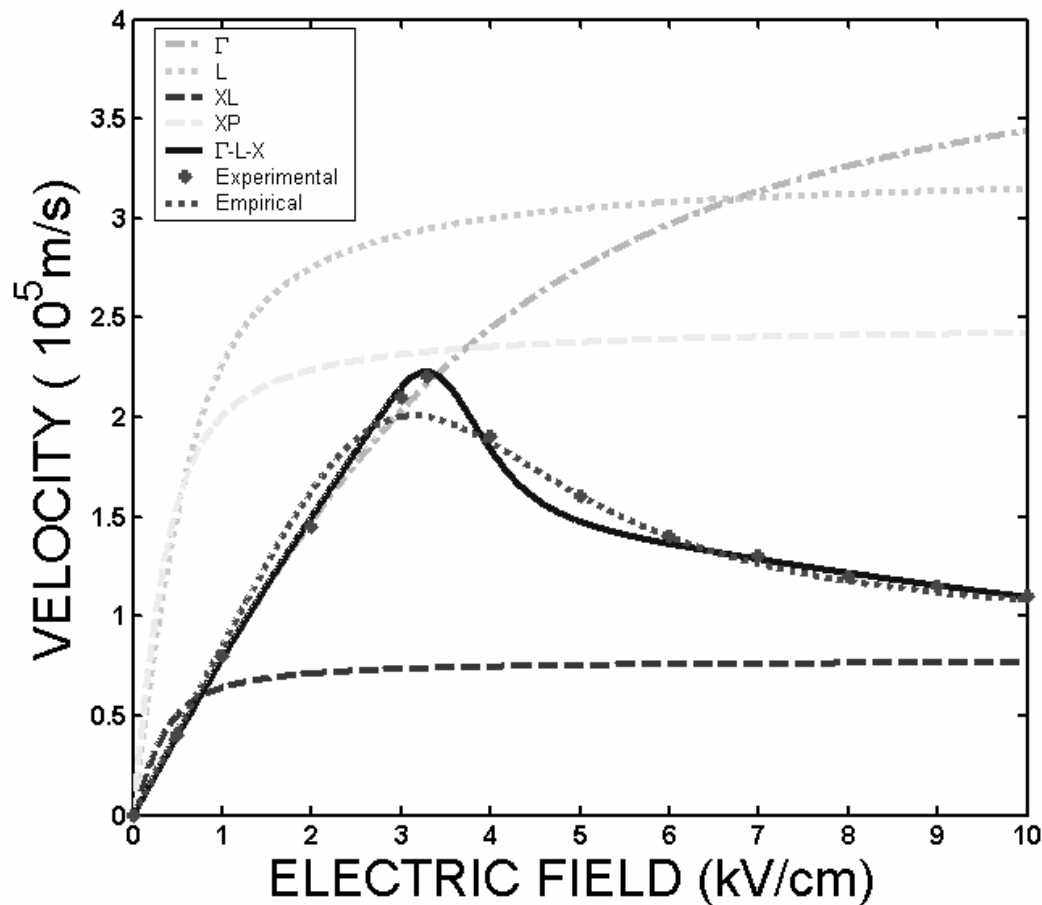
# Multi-Valley Transport in GaAs

## Intervalley Electron Transfer



# Multi-Valley Transport in GaAs

## Velocity-Field Characteristics





# High-Frequency Transport

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dc Conductivity Degradation

ac Conductivity Degradation



# Conclusions

## Quantum Confinement

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- **Transport properties function of confinement length in QW's because of the change in the Density of States**
- **Relative strength of each scattering different from bulk**
- **Electrons tend to stay away from the interface as wave function vanishes near the interface**



# **Conclusions**

## **High-Field Driven Transport**

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- **Electric field puts an order into otherwise completely random motion**
- **Higher mobility may not necessary lead to higher saturation velocity**
- **Saturation velocity is limited by Fermi /thermal velocity depending on degeneracy**
- **Saturation velocity is lowered by the quantum remission process**
- **RC time constants will dominate over transit time delay because of enhanced resistance**



# **Conclusions**

## **Failure of Ohm's Law**

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**Effective resistance may rise dramatically as current approaches saturation level**

**Familiar voltage divider and current divider rule may not be valid on submicron scales**



# Golden Rule

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- **No matter what the size, make it smaller**
- **No matter what the speed, make it faster**
- **No matter what the function, make it larger**
- **No matter what the cost, make it cheaper**
- **No matter how little it heats up, make it cooler**