Nodal pricing and financial rights

Josh Taylor

Section 6.2.3 in *Convex Optimization of Power Systems.*

1 Nodal pricing with transmission

Linearized OPF:

$$\begin{aligned} \underset{p,\theta}{\text{minimize}} \quad & \sum_i f_i(p_i) \\ \text{subject to} \quad & \\ \lambda_i : \quad & p_i = \sum_j b_{ij}(\theta_i - \theta_j) \\ \chi_{ij} \geq 0 : \quad & b_{ij}(\theta_i - \theta_j) \leq \overline{s}_{ij} \end{aligned}$$

λ_i is the nodal price. Incentivizes centrally optimal behavior via:

$$\underset{p_i}{\text{minimize}} \quad f_i(p_i) - \lambda_i p_i$$

KKT conditions yield:

$$\sum_i \lambda_i p_i + \sum_{ij} \chi_{ij} \overline{s}_{ij} = 0.$$

- First term is SO budget.
- If line ij is congested, $\chi_{ij} > 0$ ... SO has extra money.
- Is this OK? No... what to do with $\sum_{ij} \chi_{ij} \overline{s}_{ij}$.
- How do lines make money?

Fed. Energy Reg. Comm.:

- Gen. assets buy and sell in wholesale markets at nodal prices
- Trans. assets (including capacitor banks, FACTS devices) cannot participate in wholesale markets, get rate payments

- Utility assets - utility domain.

2 Financial transmission rights

What if lines bought and sold at either end. Then SO budget:

$$\begin{aligned}
B &= \sum_i \lambda_i p_i - \sum_{ij} (\lambda_i - \lambda_j) p_{ij} \\
&= \sum_i \lambda_i \sum_j p_{ij} - \sum_{ij} (\lambda_i - \lambda_j) p_{ij} \\
&= \sum_{ij} \lambda_i p_{ij} + \sum_{ij} \lambda_j p_{ji} - \sum_{ij} (\lambda_i - \lambda_j) p_{ij} \\
&= \sum_{ij} \lambda_i p_{ij} - \sum_{ij} \lambda_j p_{ij} - \sum_{ij} (\lambda_i - \lambda_j) p_{ij} \\
&= 0
\end{aligned}$$

They don't. Idea:

- Hypothetical: lines arbitrage over space - buy at one end, sell at other
- Can they make those profits another way?

2.1 Flowgate rights

Idea: real-time electricity market

- Line ij
- Divided into N_{ij} rights
- Holder of right k gets paid $\chi_{ij}\overline{s}_{ij}^k$, $k = 1, ..., N_{ij}$.
- If $\sum_{k=1}^{N_{ij}} \overline{s}_{ij}$, SO budget balances.
- Load, gen., or trader can hold right
- Typically, rights sold in auctions, last for months to years

Functionality:

- Redistribute SO's budget surplus (good)

- Hedge against risk:

Recall 2-node example with congestion

- $\lambda_2 > \lambda_1$, SO has $(\lambda_2 - \lambda_1)P_{ij} > 0$ extra money.
- λ_2 is a price spike - financial risk to load
- Gen. wants a piece of extra profit, or couldn't sell as much power as planned
- FGRs counter both of these - insurance.

Originally proposed in [1], discussed in [2]

2.2 Point-to-point rights

- Holder is paid $(\lambda_i - \lambda_j)P_{ij}^k$, i, j, not necessarily adjacent.
- Two requirements:
  - Feasibility: P_{ij}^k, $ij \in E$, $k = 1, ..., N_{ij}$ is physically feasible
  - Revenue adequacy: $\sum_i \lambda_i P_i - \sum_{ij} \sum_k (\lambda_i - \lambda_j)P_{ij}^k \geq 0$.
- Obligation: get paid or pay in case of positive or negative.
- Option: get paid, ignore negative outcomes.

Comparison

- FGRs are simpler to implement and favored in literature, PTP-FTRs are more popular with traders and ISOs (PJM, NYSO, ISONE, more)
- FGRs have guaranteed properties, PTP-FTRs may not lead to adequacy in practice

PTP-FTRs originally proposed in [3].

2.3 Other formulations

- Contract paths: payment based on path through network. Bad since power doesn't flow in paths.
- Physical rights: holder has physical control. Contradicts real-time operation, also garbage.

3 Auctions

Used for:

- Allocating transmission rights
- Procuring reserves

Basic single item auction:

- Each bidder has a private valuation, α_i, bids $\beta_i \leq \alpha_i$
- Highest bid, $\text{argmax}_i \beta_i$ receives the item (e.g., a painting)
- Utility fn: $\alpha_i - \beta_i$
- Payments:
  - First price: pay as bid
  - Second price (AKA Vickrey): highest bidder pays second highest price ... generalization used by Google and Yahoo for advertising.
- Game theory:
  - First price: suppose valuations ordered $\alpha_1 \leq ... \leq \alpha_n$. k has incentive to underbid $\alpha_{k-1} + \epsilon$... just above next most expensive.
  - Second price auction leads to truthful bidding ... revenue equivalence. If k is winner, ends up paying α_{k-1}.
  - Both have same expected profits - revenue equivalence.
- Many properties of other auctions extrapolated from revenue equivalence of 1st and 2nd price auctions.

3.1 Reserve auctions

- SO needs R reserves
- Firms submit price+quantity bids (λ_i, q_i)
- Bids accepted in ascending order, k lowest price firms s.t. $\sum_{i=1}^{k} q_i \geq R$.
- Uniform auction (gen. of second price): all firms paid $\hat{\lambda} R_k$, where R_k is portion of demand received and $\hat{\lambda}$ is highest accepted bid.

- Discriminatory auction (gen. of first price): pay-as-bid

3.2 FTR auctions

- Firms bid price/unit: λ_i\$/MW FGR or PTP-FTR
- Auction:

$$\begin{aligned} \underset{p}{\text{maximize}} \quad & \sum_i \lambda_i p_i \\ \text{subject to} \quad & p_i \text{ is a feasible power flow} \end{aligned}$$

Firms can be paid uniform or discriminatory.

4 Higher level view

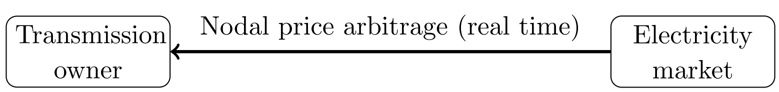

Figure 1: Hypothetical revenue path: Spatial arbitrage

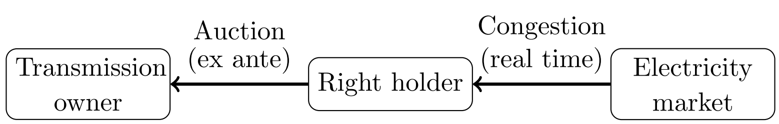

Figure 2: Actual revenue path: Transmission Rights

Why this way?

- Trans. lines are critical resources like highways, water pipes
- Slowly changes - no info to declare in real-time markets
- Insurance for market participants

5 Financial storage rights

How should storage be paid?

- Storage provides load shifting, power balancing, regulation

- Presently: arbitrages market prices, bids in regulation markets

Storage–transmission comparison:

- Expensive infrastructure
- Cheap to operate - no fuel
- Hard capacity limits

Can we do storage rights? Yes, see [4].
Simple MOPF:

$$\begin{aligned}
\underset{p,\theta,u,s}{\text{minimize}} \quad & \sum_{i,t} f_i^t(p_i^t) \\
\text{subject to} \quad & \\
\lambda_i^t : \quad & p_i^t = -u_i^t + \sum_j b_{ij}(\theta_i^t - \theta_j^t) \\
\chi_{ij}^t \geq 0 : \quad & b_{ij}(\theta_i^t - \theta_j^t) \leq \overline{s}_{ij} \\
\mu_i^t \geq 0 : \quad & 0 \leq s_i^t \leq \overline{c}_i \\
& s_i^{t+1} = s_i^t + u_i^t
\end{aligned}$$

Recall from one period case:

$$\sum_i \lambda_i p_i + \sum_{ij} \chi_{ij} \overline{s}_{ij} = 0.$$

Multiperiod case:

$$\sum_t \sum_i \lambda_i^t p_i^t + \sum_i \mu_i^t \overline{c}_i + \sum_{ij} \chi_{ij} \overline{s}_{ij} = 0.$$

- If storage arbitrages, middle term gets absorbed into SO budget. If not, middle term left out like in transmission.
- Redistribute via flowgate-like rights:

Definition

- Storage i
- Divided into N_i rights

- Holder of right k gets paid $\mu_i \overline{c}_i^k$, $k = 1, ..., N_i$.
- If $\sum_{k=1}^{N_i} \overline{c}_i$, SO budget balances.
- Same logistics as transmission
- Other constraints, details accommodable

References

[1] Hung-Po Chao and Stephen Peck. A market mechanism for electric power transmission. *Journal of Regulatory Economics*, 10(1):25–59, 1996.

[2] Hung-Po Chao, Stephen Peck, Shmuel Oren, and Robert Wilson. Flow-based transmission rights and congestion management. *The Electricity Journal*, 13(8):38 – 58, 2000.

[3] William W. Hogan. Contract networks for electric power transmission. *Journal of Regulatory Economics*, 4:211–242, 1992.

[4] J. A. Taylor. Financial storage rights. *Power Systems, IEEE Transactions on*, 30(2):997–1005, March 2015.