Game theory and market power

Josh Taylor

Section 6.1.3, 6.3 in *Convex Optimization of Power Systems*.

1 Market weaknesses

Recall

- Optimal power flow:

$$\begin{aligned} \underset{p,\theta}{\text{minimize}} \quad & \sum_i f_i(p_i) \\ \text{subject to} \quad & \\ \lambda_i : \quad & p_i = \sum_j b_{ij}(\theta_i - \theta_j) \\ \chi_{ij} \geq 0 : \quad & b_{ij}(\theta_i - \theta_j) \leq \overline{s}_{ij} \\ & \underline{p}_i \leq p_i \leq \overline{p}_i \end{aligned}$$

- Prices: λ_i: the price at node i. Agent i solves:

$$\underset{p_i}{\text{minimize}} \quad f_i(p_i) - \lambda_i p_i$$

When does nodal pricing / microeconomics fail?

- Nonconvexity - the power flow equations, unit commitment.
- Bounded rationality - agent i has limited time, information, computing power - can't find optimal p_i.
- **Price-taker** assumption: agent i oblivious to their influence on λ_i.

2 Real-world examples

2.1 Enron Scandal, late 1990's to 2001

Overview:

- Energy trading, building power plants, natural gas
- Posterchild for electricity/energy markets
- Very shady accounting practices - see Wikipedia
- Gov. Gray Davis' ruined political career

California Electricity Crisis:

- Making power seem to be from out of state (where does your power come from?)
- Blocking transmission lines (over scheduling) to raise nodal prices
- Overall bad planning/market design
- Rolling blackouts in 2000, 2001, prices increase by factor of 20.

2.2 JPMorgan, 2010-2012

- Manipulative bidding strategies ...
- JPMorgan pays $410 million in FERC settlement, 2013

2.3 Why?

- Lot's of markets have problems (healthcare, computer OS, diamonds)
- Failures of price-taker assumption in power
- Should we have power markets? Probably, but cautiously ...
- Were Enron, JPMorgan too smart?

Strategy:

- Markets need physically rigorous design
- Game theory helps identify vulnerabilities mathematically.

3 Game theory

Regular optimization:

$$\min_{x \in X} f(x).$$

Game theory:

$$\min_{x} f(x,y), \quad \min_{y} g(x,y)$$

Two players, know all about each other.

3.1 Example: prisoner's dilemma

Setup

- Two players, caught criminals
- Two actions: silence, or betray partner
- Made simultaneously (like rock paper scissors)

Payoffs:

- Both silent: both serve 1 year
- Both betray: both serve 3 years
- 1 silent, 1 betrays: silent 4 years, betrayer 0 years.

Anticipatory decisions:

- Both silent ... either improves by betraying
- 1 silent, 1 betrays ... silent improves by betraying
- Both betray ... no improvement for either

Nash equilibrium:

- Both players betray
- Stable under unilateral actions
- Worse than both silent

Strategic form games

- Players, $i = 1, ..., n$
- Pure player strategies, $\mathcal{S}_i$.
- Player utility function $u_i(s)$, $s \in \mathcal{S} = \times_i \mathcal{S}_i$
- *Ordinary optimization with just one player*

s is a Pure Nash Eq. (PNE) if

$$u_i(s) \leq u_i(t, s_{-i}) \quad \text{for all } t \in \mathcal{S}_i.$$

PNE guaranteed to exist if

- $u_i(s)$ convex in s_i, continuous in s_{-i}
- $\mathcal{S}_i$ convex and compact

Discussion

- PNE often don't exist.
- Uniqueness not guaranteed when it does exist.
- MNE describe real situations like sales.
- MNE almost always exist.
- Game theory PPAD complete - easier than NP-complete, still bad.

3.2 Bertrand competition

- Demand: d
- Prices: λ_i
- All demand goes to lowest price.

Equilibrium:

- If $\lambda_1 = \lambda_2 > 0$, $\lambda_1 - \epsilon$ is profitable for λ_1.
- If $\lambda_1 > \lambda_2$, $\lambda_1 = \lambda_2 - \epsilon$ is profitable for λ_1.
- Nash Eq: $\lambda_1 = \lambda_2 = 0$ (silly)

4 Load shifting with storage

- Time-varying, inelastic load $\delta(t)$, $t = 1, ..., T$
- Generation cost $f(p) = \frac{a}{2}p^2 + bp$
- Market clearing price:

$$\begin{aligned} \lambda &= \frac{df(p)}{dp} \\ &= ap + b \end{aligned}$$

- N storages inject/extract $s_i(t)$ - arbitrage
- Net load: $\delta(t) - \sum_{i=1}^{N} s_i(t)$

Centralized problem,

$$\begin{aligned} \underset{s}{\text{minimize}} \quad & \sum_{t=1}^{T} f\left(\delta(t) - \sum_{i=1}^{N} s_i(t)\right) \\ \text{subject to} \quad & \sum_{t=1}^{T} s_i(t) = 0 \end{aligned}$$

Optimal solutions:

$$\begin{aligned} s_i^c(t) &= \gamma_i \left(\delta(t) - \overline{\delta}\right) \\ \sum_{i=1}^{N} \gamma_i &= 1 \end{aligned}$$

where the average demand is

$$\overline{\delta} = \frac{1}{T} \sum_{t=1}^{T} \delta(t).$$

- Net load curve: $\overline{\delta}$
- γ which storage allocation

Remove price-taker assumption. Market price:

$$\lambda(t) = a\left(\delta(t) - \sum_{i=1}^{N} s_i(t)\right) + b.$$

Storage payoffs:

$$\begin{aligned} \underset{s_i}{\text{maximize}} \quad & \sum_{t=1}^{T} \left(a \left(\delta(t) - \sum_{i=1}^{N} s_i(t) \right) + b \right) s_i(t) \\ \text{subject to} \quad & \sum_{t=1}^{T} s_i(t) = 0 \end{aligned}$$

- Coupling ... N-player game
- Quantity competition - Cournot
- PNE:

$$s_i^g(t) = \frac{1}{N+1} \left(\delta(t) - \overline{\delta} \right)$$

- Flatter, but less so than centralized.

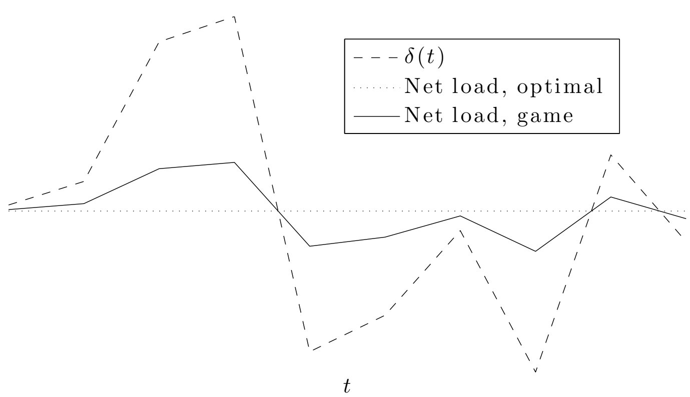

Figure 1: The nominal load without storage, δ, and the net load in the centralized and game outcomes with $N = 3$.

- Efficiency loss:

$$\begin{aligned} \Phi &= \frac{\sum_{t=1}^{T} f\left(\delta(t) - \sum_{i=1}^{N} s_i^c(t) \right)}{\sum_{t=1}^{T} f\left(\delta(t) - \sum_{i=1}^{N} s_i^g(t) \right)} \\ &= \frac{T\left(\frac{a}{2} \overline{\delta}^2 + b\overline{\delta} \right)}{\sum_{t=1}^{T} \frac{a}{2} \left(\frac{1}{N+1}\delta(t) + \frac{N}{N+1}\overline{\delta} \right)^2 + b\left(\frac{1}{N+1}\delta(t) + \frac{N}{N+1}\overline{\delta} \right)}. \end{aligned}$$

Letting $N \to \infty$, the efficiency loss vanishes, i.e. $\Phi \to 1$.

- "Price of anarchy"

- Worst case - duopoly (only monopoly worse)
- Game shows variations allowed to persist to preserve arbitrage
- More participants flattens net load, approaches true optimum.

References