Renewables in markets

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1 Renewable in markets

What we saw earlier:

- Gens. submit supply fn. $f_i(p_i)$.
- Optimal power flow:

$$\begin{aligned}
\underset{p,\theta}{\text{minimize}} \quad & \sum_i f_i(p_i) \\
\text{subject to} \quad & \\
\lambda_i : \quad & p_i = \sum_j b_{ij}(\theta_i - \theta_j) \\
\chi_{ij} \geq 0 : \quad & b_{ij}(\theta_i - \theta_j) \leq \overline{s}_{ij} \\
& \underline{p}_i \leq p_i \leq \overline{p}_i
\end{aligned}$$

- Prices: Stationarity conditions:

$$\begin{aligned}
\frac{\partial f_i(p_i)}{\partial p_i} - \lambda_i &= 0 \\
\sum_j b_{ij}(\lambda_i - \lambda_j + \chi_{ij} - \chi_{ji}) &= 0
\end{aligned}$$

  λ_i: the price at node i. Agent i solves:

$$\underset{p_i}{\text{minimize}} \quad f_i(p_i) - \lambda_i p_i$$

How do renewables fit in?

- Currently: renewables treated as negative load: $\underline{p}_i = \overline{p}_i = p_{wind}$.
- Get paid nodal price, $\lambda_i p_{wind}$.

Problems with this?

- Wind is random - don't know p_{wind} well.
- Doesn't incentivize forecasting by wind producer.

Simple solution: imbalance fees.

- Producer forecasts $\hat{p}$, actually produces p.
- Payment:
$$\lambda p - \mu^-(\hat{p} - p)^+ - \mu^+(p - \hat{p})^+$$
- μ: imbalance fee.
- Terms: nodal payment, under production fee, over production fee.

2 Optimizing $\hat{p}$ based on λ, μ^+, and μ^-

Following [2].
Suppose we have PDF $f(p)$.

- $\int_{-\infty}^{\infty} f(x)dx = 1$
- $F(p) = \int_{-\infty}^{p} f(x)dx$

Recall expectation:
$$\mathbb{E}_p[g(p)] = \int_{-\infty}^{\infty} f(p)g(p)dp.$$
Don't know profits. Know expected profits:
$$\begin{aligned} J(\hat{p}) &= \mathbb{E}\left[\lambda p - \mu^-(\hat{p} - p)^+ - \mu^+(p - \hat{p})^+\right] \\ &= \lambda \int_{-\infty}^{\infty} pf(p)dp - \mu^- \int_{-\infty}^{\hat{p}} (\hat{p} - p)^+ f(p)dp - \mu^+ \int_{\hat{p}}^{\infty} (p - \hat{p})^+ f(p)dp \end{aligned}$$

- Can get rid of $()^+$ now.
- Maximize via
$$\frac{dJ(\hat{p})}{d\hat{p}} = 0.$$

Can show concave
$$\frac{d^2J(\hat{p})}{d\hat{p}^2} \leq 0.$$

2.1 Leibniz integral rule

$$\frac{d}{dx}\int_{a(x)}^{b(x)} g(x,y)dy = \int_{a(x)}^{b(x)} \frac{dg(x,y)}{dx}dy + g(x,b(x))\frac{db(x)}{dx} - g(x,a(x))\frac{da(x)}{dx}$$

Apply to each term of $\frac{dJ(\hat{p})}{d\hat{p}}$.

- 1st term ... no $\hat{p}$:
$$\frac{d}{d\hat{p}}\lambda\int_{-\infty}^{\infty} pf(p)dp = 0$$
- 2nd term (without $-\mu^-$):
$$\frac{d}{d\hat{p}}\int_{-\infty}^{\hat{p}} (\hat{p}-p)f(p)dp = \int_{-\infty}^{\hat{p}} f(p)dp + (\hat{p}-\hat{p})f(\hat{p}) - 0 = F(\hat{p})$$
- 3rd term (without $-\mu^+$):
$$\frac{d}{d\hat{p}}\int_{\hat{p}}^{\infty} (p-\hat{p})f(p)dp = \int_{\hat{p}}^{\infty} -f(p)dp + 0 - (\hat{p}-\hat{p})f(\hat{p}) = -(1-F(\hat{p}))$$

All together:
$$\frac{dJ(\hat{p})}{d\hat{p}} = -\mu^- F(\hat{p}^*) + \mu^+(1-F(\hat{p}^*)) = 0.$$

Arithmetic:
$$F(\hat{p}^*) = \frac{\mu^+}{\mu^- + \mu^+}$$

$F(\hat{p})$ is monotonic ... invertible (draw):
$$\hat{p}^* = F^{-1}\left(\frac{\mu^+}{\mu^- + \mu^+}\right)$$

... the optimal bid. Observe:

- No dependence on λ ... this part independent of p
- As $\mu^+ >> \mu^-$ (penalty for overproducing), $\hat{p}^* \to F^{-1}(1) = \infty$.
- As $\mu^+ << \mu^-$ (penalty for underproducing), $\hat{p}^* \to F^{-1}(0) = 0$ (assuming $f(p) = 0$ for $p < 0$).
- $\mu^+ = \mu^-$ ... $\hat{p}^* = F^{-1}(1/2)$ ... the median! Half the outcomes above, half below.

Historical background, newsvendor problem:

- Order newspapers day before
- Avoid waste (overproduction penalty)
- Avoid lost sales (underproduction penalty)
- Random demand ... identical setup.

2.2 Change of parameters

Since

$$p = \hat{p} - (\hat{p} - p)^+ + (p - \hat{p})^+,$$

equivalent payment

$$\begin{aligned}
\lambda p - \mu^-(\hat{p} - p)^+ - \mu^+(p - \hat{p})^+ &= \lambda\left(\hat{p} - (\hat{p} - p)^+ + (p - \hat{p})^+\right) \\
&\quad -\mu^-(\hat{p} - p)^+ - \mu^+(p - \hat{p})^+ \\
&= \lambda\hat{p} - (\mu^- + \lambda)(\hat{p} - p)^+ - (\mu^+ - \lambda)(p - \hat{p})^+ \\
&= \lambda\hat{p} - \gamma^-(\hat{p} - p)^+ - \gamma^+(p - \hat{p})^+
\end{aligned}$$

Set

$$\begin{aligned}
\gamma^+ &= \mu^+ - \lambda \\
\gamma^- &= \mu^- + \lambda
\end{aligned}$$

Substitution into optimal bid:

$$\hat{p}^* = F^{-1}\left(\frac{\gamma^+ + \lambda}{\gamma^- + \gamma^+}\right)$$

Forward part of contract, $\lambda\hat{p}$ can be paid ahead of time.

3 Aggregating renewable producers

- One producer - imbalance fees $\sim$ standard deviation.
- Multiple producers - negative correlations can reduce variation.
- Following [3]. Also see [1, 4]

Problem:

- Assume $\gamma^+ \geq 0$ (all deviations penalized)
- Producer i bids $\hat{p}_i$ to aggregator, $i = 1, ..., n$
- Aggregator bids $\hat{q} = \sum_i \hat{p}_i$ to SO.
- Produce $q = \sum_i p_i$ actual power
- Total payment:

$$\lambda\hat{q} - \gamma^- (\hat{q} - q)^+ - \gamma^+ (q - \hat{q})^+$$

- How to share this payment among $i = 1, ..., n$ producers?

Producer i:

- $e_i = p_i - \hat{p}_i$, $e \in \mathbb{R}^n$
- $D_i(e, \gamma^-, \gamma^+)$ penalty for i's deviation
- Payment

$$\lambda\hat{p}_i + D_i(e, \gamma^-, \gamma^+)$$

Goal: design D_i. Desirable properties:

- Budget balance:

$$\sum_i D_i(e, \gamma^-, \gamma^+) = -\gamma^- (\hat{q} - q)^+ - \gamma^+ (q - \hat{q})^+$$

- Ex-post rationality (better off in the group than alone):

$$D_i(e, \gamma^-, \gamma^+) \geq -\gamma^- (\hat{p}_i - p_i)^+ - \gamma^+ (p_i - \hat{p}_i)^+$$

- Fairness: $e_i = e_j \Longrightarrow D_i(e, \gamma^-, \gamma^+) = D_j(e, \gamma^-, \gamma^+)$

Definition (surplus and shortfalls)

$$W^+ = \{i \mid e_i \geq 0\}, \quad W^- = \{i \mid e_i < 0\}$$

The mechanism:

- If $\sum_i e_i = 0$, then $D_i(e, \gamma^-, \gamma^+) = 0$ for all i (contained by other cases)

- If $\sum_i e_i < 0$ (shortfall), define σ

$$\sum_{i \in W^-} \min(\sigma, |e_i|) = \sum_{i \in W^+} e_i$$

Then

$$\begin{aligned} D_i(e, \gamma^-, \gamma^+) &= 0, \quad i \in W^+ \\ D_i(e, \gamma^-, \gamma^+) &= -\gamma^-(|e_i| - \min\{\sigma, |e_i|\}), \quad i \in W^- \end{aligned}$$

Sum up over $i \in W^-$ to see budget balance.

- If $\sum_i e_i > 0$ (surplus), define τ

$$\sum_{i \in W^+} \min(\tau, e_i) = \sum_{i \in W^-} |e_i|$$

Then

$$\begin{aligned} D_i(e, \gamma^-, \gamma^+) &= -\gamma^+(e_i - \min\{\tau, e_i\}), \quad i \in W^+ \\ D_i(e, \gamma^-, \gamma^+) &= 0, \quad i \in W^- \end{aligned}$$

Intuition:

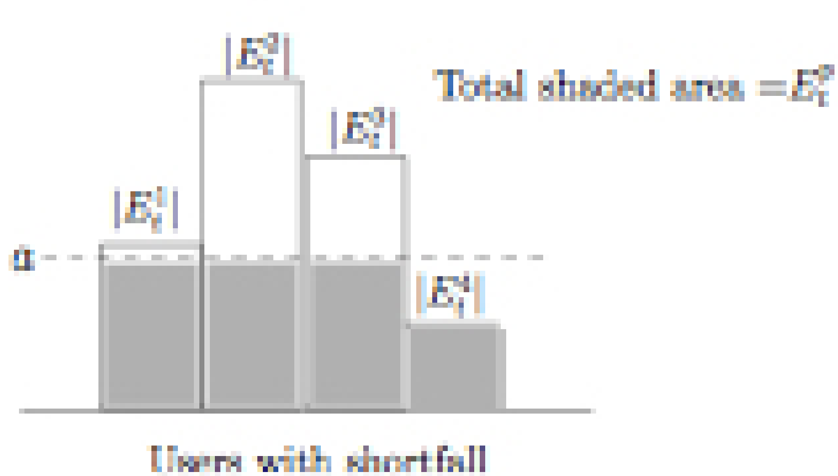

Figure 1: From [3]

Theorem. The mechanism satisfies the desirable properties. **Proof sketch**.

- Budget balance: proven by arithmetic (summing both sides).

- Rationality. By definition,

$$D_i(e, \gamma^-, \gamma^+) \geq -\gamma^- \left(\hat{p}_i - p_i\right)^+ - \gamma^+ \left(p_i - \hat{p}_i\right)^+$$

- Fairness: Implicit in symmetry of $D_i(e, \gamma^-, \gamma^+)$ for all i.

3.1 Contract game

- Each producers expected payoff is

$$u_i(\hat{p}) = \lambda \hat{p}_i + \mathbb{E}_p \left[D_i(e, \gamma^-, \gamma^+) \right]$$

- Producer i maximizes over $\hat{p}_i$.
- Since $D_i(e, \gamma^-, \gamma^+)$ depends on all other $\hat{p}_i$, this is a game.
- Nash Eq:

$$u_i(\hat{p}^*) \geq u_i(\hat{p}_i, \hat{p}^*_{-i}) \quad \forall\, \hat{p}_i, i$$

Theorems:

- PNE exists via continuity of each u_i and concavity in $\hat{p}_i$.
- At a Nash Eq., payoff $\geq$ payoff outside of aggregate.
- $\hat{p}_i$ at Nash Eq. is greater than outside of aggregate.

Implications:

- A single renewable producer should not bid their maximum because it heightens intermittency.
- An aggregation can leverage negative correlations without knowing statistics.
- The aggregation can bid more together than apart as a result - more renewables.

References

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[2] E.Y. Bitar, R. Rajagopal, P.P. Khargonekar, K. Poolla, and P. Varaiya. Bringing wind energy to market. *Power Systems, IEEE Transactions on*, 27(3):1225–1235, Aug 2012.

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[4] Y. Zhao, J. Qin, R. Rajagopal, A. Goldsmith, and H.V. Poor. Wind aggregation via risky power markets. *Power Systems, IEEE Transactions on*, 2014.