Math 337 – Sample Exam

Instructions. *This sample exam is provided as an aid for preparing for the first midterm exam. Additionally, preparation for the midterm exam should include review of lectures, quizzes and all assigned homework problems.*

Problem 1. Suppose that $\mathbf{u} = (1,2)$, $\mathbf{v} = (3,4)$ and $\mathbf{w} = (1,4,0)$. Where possible compute the following:

$$(a)\ \mathbf{u}\cdot\mathbf{v} \qquad (b)\ \mathbf{u}\cdot\mathbf{w} \qquad (c)(\mathbf{u}+\mathbf{v})\cdot(\mathbf{u}-\mathbf{v}) - ||\mathbf{u}||^2 + ||\mathbf{v}||^2$$

Problem 2. Suppose that $\mathbf{u}$ and $\mathbf{v}$ are unit vectors and that $\mathbf{u}\cdot\mathbf{v} < 0$. Explain why $||\mathbf{u}+\mathbf{v}|| < \sqrt{2}$.

Problem 3. Let $\mathbf{u} = (1,1)$ and $\mathbf{v} = (1,-1)$. Sketch the set of all the linear combinations of $\mathbf{u}$ and $\mathbf{v}$ for which the coefficients in the linear combination are greater or equal to zero but less than or equal to one. That is, sketch the set

$$\{\alpha\mathbf{u} + \beta\mathbf{v} \mid 0 \le \alpha \le 1 \text{ and } 0 \le \beta \le 1\}.$$

Problem 4. Compute the following matrix products (if possible):

$$(a)\ \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad (b)\ \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$(c)\ \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix} \qquad (d)\ \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}$$

Problem 5. Find the coefficients that show that $\mathbf{x} = (1,2,3,4)$ is a linear combination of $\mathbf{y} = (1,0,0,0)$, $\mathbf{z} = (1,1,0,0)$, $\mathbf{w} = (1,1,1,0)$ and $\mathbf{v} = (1,1,1,1)$.

Problem 6. For each equation find the matrix A that makes the equation true:

$$(a) \qquad A\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 4 \\ 1 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 1 & 2 & 5 \end{bmatrix}$$

$$(b) \qquad A\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 4 \\ 1 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

$$(c) \qquad A\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 4 \\ 1 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 5 \\ 1 & 2 & 4 \end{bmatrix}$$

Problem 7. For each matrix find the inverse matrix if possible:

$$(a) \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \qquad (b) \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \qquad (c) \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$(d) \begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix} \qquad (e) \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem 8. Give the LU factorization of each matrix in problem 7 (if possible). Also give the rank of each matrix.

Problem 9. For the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}$$

determine whether or not each of the following vectors are in the null space of A, in the column space of A or neither: $\mathbf{a} = (1, 3, 6)$, $\mathbf{b} = (1, -2, 1)$, $\mathbf{c} = (1, 1, 1)$, $\mathbf{d} = (1, 0, 0)$

Problem 10. For which vectors (b_1, b_2, b_3) do these systems have a solution?

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Problem 11. Without reference to the determinant, show each of the following matrices is not invertible.

$$(a) \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \qquad (b) \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (c) \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Problem 12. Find all solutions of each of the following systems:

$$(a) \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \qquad (b) \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$