Math 337 – Fall 2004 Midterm Examination 1

Instructions. *Show your work. Calculators are not permitted. Scoring: 12 points per problem, except problem 8 which is 16 points (parts of problems are equally weighted unless noted otherwise). This examination has eight problems; problem 5 through 8 are on the back of this page.*

Problem 1. (12 points) Where possible perform the following matrix calculations:

$$(a)\ \begin{bmatrix}1\\2\\3\end{bmatrix}+\begin{bmatrix}1\\0\\-2\end{bmatrix} \qquad (b)\ \begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}1\\-1\end{bmatrix} \qquad (c)\ \begin{bmatrix}1\\-1\end{bmatrix}\begin{bmatrix}1 & 2\end{bmatrix}$$

$$(d)\ \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}+\begin{bmatrix}1\\0\\0\\1\end{bmatrix} \qquad (e)\ \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix} \qquad (f)\ \begin{bmatrix}0&1&0&0&0\\1&0&0&0&0\\0&0&0&0&1\\0&0&0&1&0\\0&0&1&0&0\end{bmatrix}\begin{bmatrix}1&1&1&1&1\\2&2&2&2&2\\3&3&3&3&3\\4&4&4&4&4\\5&5&5&5&7\end{bmatrix}$$

Problem 2. (12 points) For each part, determine whether or not the given vector $\mathbf{x}$ is a linear combination of the vectors in the given set S. Explain your reasoning.

(a) $\mathbf{x} = (1, 2, 3)$, $S = \{(1, 0, 0),\ (0, 1, 0),\ (0, 0, 1)\}$

(b) $\mathbf{x} = (0, 0, 0)$, $S = \{(1, 2, 3),\ (4, 5, 6),\ (7, 8, 9)\}$

(c) $\mathbf{x} = (2, 3)$, $S = \{(1, 2),\ (2, 1),\ (3, 2)\}$

Problem 3. (12 points) Showing your work, find all 2×2 matrices A that satisfy

$$A\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix} = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}A.$$

Problem 4. (12 points; 3 points per part) (a) Which three matrices E_{21}, E_{31}, E_{32} put A into upper triangular form U?

$$A = \begin{bmatrix}1 & 0 & 1\\1 & 1 & 0\\1 & 1 & 1\end{bmatrix} \qquad \text{and} \qquad E_{32}E_{31}E_{21}A = U.$$

(b) Multiply those E's to get one matrix M that does elimination: $MA = U$. (c) If possible, find M^{-1}. (d) If possible, give the LU factorization of A.

(CONTINUED ON BACK)

Problem 5. (12 points) Compute the inverses of the following matrices (if possible):

$$(a) \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \qquad (b) \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix} \qquad (c) \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \qquad (d) \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \end{bmatrix}$$

Problem 6. (12 points) For each matrix A below, show (if possible) that there is a nonzero vector $\mathbf{x}$ in the null space of A; also determine if the matrix A is invertible.

$$(a) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad (b) \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Problem 7. (12 points) Suppose that $\mathbf{x}$ and $\mathbf{y}$ are vectors in $\mathbf{R}^2$ with $\mathbf{y} \neq 0$. Consider the linear combinations of $\mathbf{x}$ and $\mathbf{y}$ of the form $\mathbf{x} + t\mathbf{y}$ where t is a real number.

(a) Find the linear combination of this form having the shortest length; call it $\mathbf{w}$. Hint: Minimize $||\mathbf{x} + t\mathbf{y}||^2$.

(b) Show that $\mathbf{y}$ and $\mathbf{w}$ are perpendicular.

Problem 8. (16 points) Suppose

$$A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix} \qquad \text{and} \qquad \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.$$

(a) (12 points) Find all solutions $\mathbf{x}$ of $A\mathbf{x} = \mathbf{b}$. (b) (2 points) Prove or disprove that $\mathbf{b} \in C(A)$ where $C(A)$ is the column space of A. (c) (2 points) What is the rank of A?

END OF QUESTION SHEET