

Name\_\_\_\_\_

337-F2006

Write your name above. Materials not needed for the exam, such as books and backpacks, must be placed at the front of the room during the exam. Electronic devices such as pagers and cell phones must be turned off and put away for the duration of the exam.

**Math 337 – Fall 2006 First Common Exam**

**Instructions** *Show your work and mark your answers clearly. All work must be done in the examination booklets provided. No books, notes, calculators or scratch paper are allowed. This question sheet must be submitted with your exam booklet. Put your name on all exam booklets. Sign the honor code pledge. Check your work; partial credits will be limited. You must remain in the classroom until the exam has ended.*

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**Problem 1 (26 points)** (a) (16 pts) True or false:

- (i) If  $\mathbf{u}$  and  $\mathbf{v}$  are unit vectors, then  $|\mathbf{u} \cdot \mathbf{v}| \leq 1$ .
- (ii) If  $A$  has row 1 + row 2 = row 3, then  $Ax = (1, 0, 0)$  has infinitely many solutions.
- (iii) A 5 by 5 matrix with a row of zeros is not invertible.
- (iv) If  $L$  is lower triangular, then  $L^T$  is upper triangular.
- (v) If  $A$  and  $B$  are symmetric, then their product  $AB$  is symmetric.
- (vi) All vectors  $(x, y, z)$  with  $x \leq y \leq z$  form a subspace of  $\mathbf{R}^3$ .
- (vii) All vectors  $(x, y, z)$  with  $x + y + z = 1$  form a subspace of  $\mathbf{R}^3$ .
- (viii) The column space of  $2A$  equals the column space of  $A$ .

(b) (10 pts) Let  $\mathbf{u} = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$ ,  $\mathbf{v} = \begin{bmatrix} 2 \\ -4 \\ 3 \end{bmatrix}$ ,  $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & -2 \end{bmatrix}$ ,  $B = \begin{bmatrix} 0 & 1 & 2 \\ 1 & -1 & 0 \end{bmatrix}$ .

Compute and simplify each expression below (if possible).

- (i)  $\mathbf{u} \cdot \mathbf{v}$ ,      (ii)  $\|\mathbf{u}\|$ ,      (iii)  $A + B$ ,      (iv)  $AB$ ,      (v)  $BA$ .

(CONTINUED ON THE BACK)

**Problem 2 (24 points)** Consider the matrix  $A$  and the vector  $\mathbf{b}$ .

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & -1 & 4 \\ 6 & 4 & 4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}.$$

- (a) (10 pts) Use elimination to solve  $A\mathbf{x} = \mathbf{b}$ .
- (b) (6 pts) Write down three elimination matrices  $E_{21}$ ,  $E_{31}$ , and  $E_{32}$  associated with your elimination steps in part (a).
- (c) (8 pts) Find the LU factorization and the LDU factorization of  $A$ .

**Problem 3 (24 points)** (a) (12 pts) Compute and simplify the inverses of the following matrices (if possible).

$$(i) \begin{bmatrix} 2 & 2 & 2 \\ 1 & 1 & 1 \\ 3 & 0 & 2 \end{bmatrix}, \quad (ii) \begin{bmatrix} 2 & 3 \\ 4 & 1 \\ 3 & 5 \end{bmatrix}, \quad (iii) \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}.$$

- (b) (12 pts) Given that  $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix} A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ , find  $A^{-1}$ .

(*Hint:* Best if you don't work too hard!)

**Problem 4 (26 points)** Consider the matrix  $A$  and the vector  $\mathbf{b}$ .

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ 2 & 6 & c \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ 3 \\ t \end{bmatrix}.$$

- (a) (10 pts) Find the number  $c$  that makes the matrix  $A$  singular (i.e., it has less than three pivots)
- (b) (6 pts) With  $c$  found in part (a), find the number  $t$  in  $\mathbf{b}$  so that the linear system  $A\mathbf{x} = \mathbf{b}$  has infinitely many solutions.
- (c) (10 pts) If  $c = 20$ , what are the column space  $C(A)$  and the nullspace  $N(A)$ ? Describe them in this specific case (not just repeat definitions).

END OF QUESTION SHEET

Carefully check all your answers.