Name____ 337-F2007

Write your name above. Materials not needed for the exam, such as books and backpacks, must be placed at the front of the room during the exam. Electronic devices such as pagers and cell phones must be turned off and put away for the duration of the exam.

Math 337 – Fall 2007 First Common Exam

Instructions *Show your work and mark your answers clearly. All work must be done in the examination booklets provided. No books, notes, calculators or scratch paper are allowed. This question sheet must be submitted with your exam booklet. Put your name on all exam booklets. Sign the honor code pledge. Check your work; partial credits will be limited. You must remain in the classroom until the exam has ended.*

Problem 1 (26 points) (a) (12 pts) True or false:

(i) If $\mathbf{u} \cdot \mathbf{v} = 0$, then $||\mathbf{u} + \mathbf{v}||^2 = ||\mathbf{u}||^2 + ||\mathbf{v}||^2$.

(ii) If rows 1 and 3 of B are the same, so are rows 1 and 3 of AB.

(iii) The bolck matrix $\begin{bmatrix} 0 & A \\ A & 0 \end{bmatrix}$ is always symmetric.

(iv) If A is invertible and $AB = BA$, then $A^{-1}B = BA^{-1}$.

(v) If A is invertible and symmetric, then A^{-1} is also symmetric.

(vi) If U is upper triangular, then $(U^{-1})^T$ is also upper triangular.

(b) (14 pts) Let $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, $A = \begin{bmatrix} 0 & -2 & 1 \\ -1 & 1 & 0 \end{bmatrix}$,
$B = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$, $C = \begin{bmatrix} 0.5 & 0.5 \\ -0.5 & -0.5 \end{bmatrix}$.

Compute and simplify each expression below if possible, state your reason if not possible.

(i) $\mathbf{u}\mathbf{v}^T$, (ii) $\mathbf{u}^T\mathbf{v}$, (iii) $||\mathbf{u}||$, (iv) $A + B$, (v) AB,
(vi) B^TA^T, (vi) C^{100}.

(CONTINUED ON THE NEXT PAGE)

Problem 2 (25 points) (a) (15 points) Find matrices A, B, and C that satisfy the equations below. Show your work, check your answer, and state your reasons clearly.

(i) $$A\begin{bmatrix} 2 & -1 & 1 \\ 4 & 1 & -4 \\ 6 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 3 & 4 \\ 2 & -1 & 1 \\ 4 & 1 & -4 \end{bmatrix}.$$

(ii) $$B\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}.$$

(iii) $$C\begin{bmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 4 & 0 \end{bmatrix}.$$

(b) (10 points) Find the linear combination of the vectors $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix}$ which gives the vector $\begin{bmatrix} 5 \\ 7 \\ 11 \end{bmatrix}$.

Problem 3 (24 points) (a) Consider the matrix A below:

$$A = \begin{bmatrix} a & a & a \\ a & b & b \\ a & b & c \end{bmatrix},$$

where $a \neq 0$, $b \neq a$, and $c \neq b$.

(i) (8 points) Apply elimination to reduce A into an upper triangular matrix. Write down three elimination matrices E_{21}, E_{31}, and E_{32} associated with your elimination steps.

(ii) (8 points) Find the LU factorization and the LDL^T factorization of A. Write down L, U, and D explicitly.

(b) (8 points) Consider the matrix A below:

$$A = \begin{bmatrix} 0 & 3 \\ 4 & 5 \end{bmatrix}.$$

Compute the $PA = LU$ factorization of A. Write down P, L, and U explicitly.

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Problem 4 (25 points) (a) (15 pts) Compute (and simplify) the inverses of following matrices if possible, state your reasons clearly if not possible.

$$\text{(i)} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}, \qquad \text{(ii)} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 4 & 0 \end{bmatrix}, \qquad \text{(iii)} \begin{bmatrix} 100 & 100 & 100 \\ 101 & 102 & 103 \\ 201 & 202 & 203 \end{bmatrix}.$$

(b) (10 pts) Consider the following linear system:

$$\begin{aligned} x + 2y \quad & -3z = 4 \\ 3x - y \quad & +5z = 2 \\ 4x + y \quad & +(a^2 - 14)z = a + 2 \end{aligned}$$

For what values of a does the system have (i) a unique solution, (ii) infinitely many solutions, (iii) no solutions? Show your work and state your reasons clearly.

END OF QUESTION SHEET

Carefully check all your answers.