Math 337 – Fall 2004 Midterm Examination 2

Instructions. *Show your work. No books, notes, calculators or scratch paper are allowed. All work must be done in examination books provided. This question sheet must be submitted with your exam booklet. Checking your work is highly recommended.*

Problem 1. (16 points) Find the LU factorization of the matrix below:

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 4 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

Problem 2. (16 points) Find the general solution (that is, all solutions) of the following linear system:

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 2 & 2 & 0 \\ 3 & 2 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \\ 1 \\ 0 \end{bmatrix}.$$

Problem 3. (16 points) Find an orthonormal basis for each of the four fundamental subspaces (column space, null space, row space and left null space) of the symmetric matrix A below:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

Problem 4. (15 points) Compute the determinant of each of the following matrices.

$$(a) \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \qquad (b) \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 3 & 4 \\ 0 & 0 & 3 & 4 \\ 0 & 0 & 0 & 4 \end{bmatrix} \qquad (c) \begin{bmatrix} 13 & 22 & 17 & 13 \\ 13 & 26 & 18 & 13 \\ 13 & 56 & 77 & 13 \\ 13 & 32 & 78 & 13 \end{bmatrix}$$

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Problem 5. (14 points) Suppose that $\{\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_n\}$ is an orthonormal basis for $\mathbf{R}^n$. Suppose $\mathbf{x} \in \mathbf{R}^n$ and

$$\mathbf{x} \;=\; \alpha_1\mathbf{a}_1 \;+\; \alpha_2\mathbf{a}_2 \;+\; \ldots \;+\; \alpha_n\mathbf{a}_n$$

where $\alpha_1, \alpha_2, \ldots, \alpha_n$ are scalar values. Clearly showing your work, derive a formula for α_i in terms of $\mathbf{x}$ and as many of $\mathbf{a}_1$, $\mathbf{a}_2$, $\ldots$, $\mathbf{a}_n$ as are needed.

Problem 6. (14 points) Suppose that ϵ is a real number. Find the straight line best fitting the data $\{(-1,-1),\ (0,\epsilon),\ (1,1)\}$ in the least squares sense. That is, find the slope α and intercept β so that the line $b = \alpha t + \beta$ is the least squres best fit for the data points $(t_1, b_1) = (-1,-1)$, $(t_2, b_2) = (0, \epsilon)$ and $(t_3, b_3) = (1, 1)$.

Problem 7. (9 points) Suppose that $\mathbf{x}$ and $\mathbf{y}$ are vectors in $\mathbf{R}^2$ with $\mathbf{y} \neq 0$. Consider the linear combinations of $\mathbf{x}$ and $\mathbf{y}$ of the form $\mathbf{x} + t\mathbf{y}$ where t is a real number.

(a) Find the linear combination of this form having the shortest length; call it $\mathbf{w}$.
(b) Show that $\mathbf{y}$ and $\mathbf{w}$ are perpendicular.

END OF QUESTION SHEET