Name____ 337-F2007

Write your name above. Materials not needed for the exam, such as books and backpacks, must be placed at the front of the room during the exam. Electronic devices such as pagers and cell phones must be turned off and put away for the duration of the exam.

Math 337 – Fall 2007 Second Common Exam

Instructions *Show your work and mark your answers clearly. All work must be done in the examination booklets provided. No books, notes, calculators or scratch paper are allowed. This question sheet must be submitted with your exam booklet. Put your name on all exam booklets. Sign the honor code pledge. Check your work; partial credits will be limited. You must remain in the classroom until the exam has ended.*

Problem 1 (28 points) (a) (12 pts) True or false:

(i) All vectors (b_1, b_2, b_3) satisfying $b_1 b_2 b_3 = 0$ form a vector subspace in R^3.

(ii) If R is the reduced row echelon form of A, then R^T is the reduced row echelon form of A^T.

(iii) If the columns of a matrix are dependent, so are the rows.

(iv) If the row space equals the column space, then $A^T = A$.

(v) If $Ax = b$ has a solution and $A^T y = 0$, then y is orthogonal to b.

(vi) Since $(1, 1, 1)$ and $(2, 3, 4)$ both have positive components, the vectors in the subspace spanned by these two vectors all have positive components.

(CONTINUED ON THE BACK)

(b) (16 pts) Construct a matrix with the required property or explain why that is impossible: (Show your work and state your reasoning clearly.)

(i) Column space contains $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, nullspace contains $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

(ii) Column space contains $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, row space contains $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$.

(iii) First column is $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, first row is $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, and the rank of the matrix is 1.

(iv) $Ax = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ has exactly two distinct solutions.

Problem 2 (24 points) (a) (12 pts) Determine the complete solution for each of the following systems of linear equations:

(i) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

(b) (12 pts) Consider the following linear system:

$$\begin{bmatrix} 1 & 3 & 1 & 2 \\ 2 & 6 & 4 & 8 \\ 0 & 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 - \gamma \\ \gamma \end{bmatrix}$$

(i) Find the value of the parameter γ so that the system is solvable.

(ii) For such value of γ, find the complete solution of the system.

Problem 3 (24 points)

(a) (6 pts) Find the nullspace of A:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}.$$

(b) (6 pts) Find the rank of B:

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}.$$

(c) (12 pts) Find a basis for each of the four fundamental subspaces of C:

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 & a & 1 \\ 0 & 0 & 1 & 0 & b & 2 \\ 0 & 0 & 0 & 1 & c & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Problem 4 (24 points)

(a) (12 pts) Find the largest possible number of independent vectors among $v_1 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, v_4 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, v_5 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}.$ (Show your work and state your reasoning clearly).

(b) (12 pts) Suppose that the vector subspace S is spanned by $(1, 2, 3)$. Find a basis for its orthogonal complement $S^{\perp}$. (Reminder: The orthogonal complement $S^{\perp}$ contains every vector that is perpendicular to S.)

END OF QUESTION SHEET

Carefully check all your answers.