

Math 614 Homework Assignment 1

Due: Jan. 31

Instructions: *Write neatly for textbook problems. Projects must be returned in report form and must be type-written. For projects, you must submit your code, output, and description of your algorithm (most important). Late assignments are NOT accepted.*

Readings:

Read the following two papers (both available on my course website: <http://web.njit.edu/~jiang/math614.html>):

1. Lloyd N. Trefethen, "The Definition of Numerical Analysis".
2. George E. Forsythe, "How do you solve a quadratic equation?"

Textbook problems:

Chapter 1 (p. 44-49) Problem 5(a); Problem 19(a); Problem 25(a), (d), (e); Problem 23; Problem 26(b).

Project:

Consider the following problem:

A photon moving at speed 1 in the x-y plane starts at $t = 0$ at $(x, y) = (0.6, 0.1)$ heading due east. Around every integer lattice point (i, j) in the plane, a circular mirror of radius $1/3$ has been erected. How far from the origin is the photon at $t = 10$?

1. Write a compute code to solve the above problem and plot out the trajectory of the photon and involved mirrors. (hint 1: the algorithm roughly consists of three iterative steps: 1. find the mirror upon which the photon is reflected; 2. find the reflection point on that mirror by solving a quadratic equation; 3. find the reflected velocity of the photon after reflection.) (hint 2: be careful in implementing the quadratic root finder!)
2. How accurate is your answer? Is this problem ill-conditioned or well-conditioned? Estimate the condition number of this problem either numerically or theoretically.