

Math 712 Homework Assignment 3

Due: Monday Sep. 29

- Instructions:** 1. *Everything must be returned in report form and must be type-written.*
2. *You must submit your code.*
3. *Late assignments are NOT accepted.*

Problem 1.

Consider the following initial value problem

$$\begin{aligned} u_t &= u_x, & 0 < x < 2\pi, & \quad t > 0, \\ u(x, 0) &= \sin(x), \end{aligned} \tag{1}$$

subject to periodic boundary conditions.

- (a) Implement the following three schemes: (i) Leap Frog, (ii) Upwind, and (iii) Lax-Wendroff.
- (b) Chose different ratios $\lambda = \frac{\Delta t}{\Delta x}$ to demonstrate stability and instability of each scheme.
- (c) Plot the exact and numerical solutions at $t = 0.1, 0.5, 1.0$ and $t = 2\pi$. Discuss and explain your results.
- (d) Perform a numerical convergence study and find the order of accuracy of each scheme, that is, compute solutions for successively decreasing values of the grid spacing k and plot the relative l^2 error of the solution (as compared with the exact solution) at final time against the grid spacing $k = \Delta t$.