FIR Filters

Lecture #7

Chapter 5

What Is this Course All About ?

- To Gain an Appreciation of the Various Types of Signals and Systems
- To Analyze The Various Types of Systems
- To Learn the Skills and Tools needed to Perform These Analyses.
- To Understand How Computers Process Signals and Systems

Discrete-time Systems: Filters

- Study Discrete-time systems
- Discrete-time Filters
- Finite Impulse Response (FIR) Filters
- Infinite Impulse Response (IIR) Filters
- Recall: Discrete-time system.

$$y[n]=\mathcal{T}\{x[n]\}$$

Moving Average Filter

- Moving average or running average filter:
- Choose a 3-point averaging method:

For example:

$$y[0]= 1/3\ (x[0]+x[1]+x[2])$$

$$y[1]= 1/3\ (x[1]+x[2]+x[3])$$

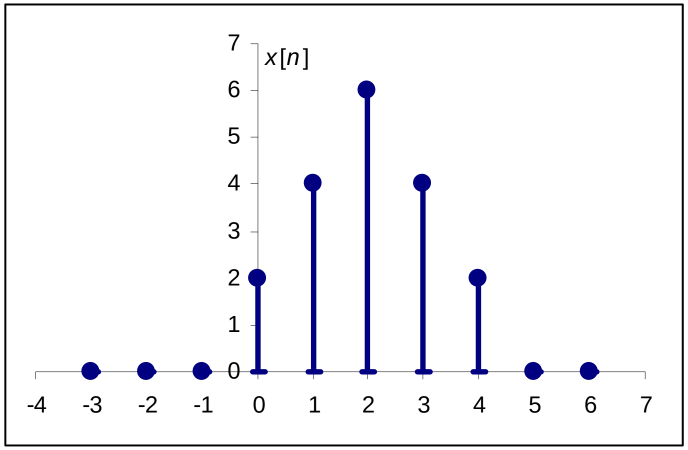
Or

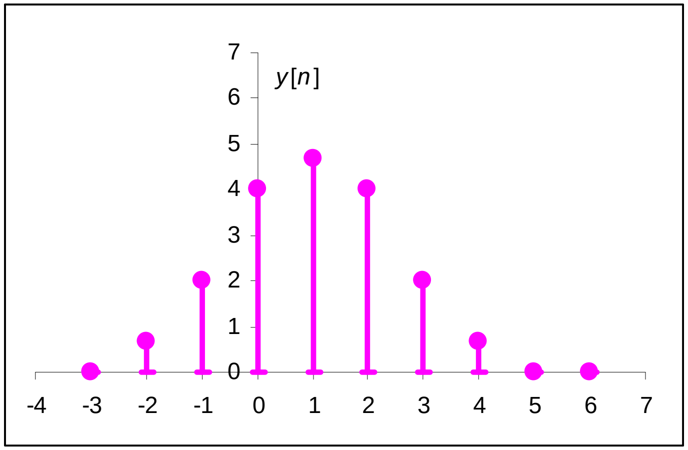
$$y[n]= 1/3\ (x[n]+x[n+1]+x[n+2])$$

- This is called a difference equation
  - Used to completely describe the FIR for $-\infty < n < \infty$

Example

- The following sequence, $x[n]$, is a finite length sequence since it only has values with a finite range $0 \le n \le 4$.
- This range is called the support of the sequence
- If we apply the difference equation we have the follow sequence for $y[n]$:





A closer Look

- If we tabulate the $x[n]$ and $y[n]$

n	-3	-2	-1	0	1	2	3	4	5	6
$x[n]$	0	0	0	2	4	6	4	2	0	0
$y[n]$	0	0.6666667	2	4	4.6666667	4	2	0.6666667	0	0

- The highlighted numbers show how $y[n]$ is calculated.
- Note that the output is longer than the input.
- And if n stands for time, then $y[n]$ is a prediction of the future.
  - For example, $y[0]$ depends not only on $x[0]$ but also on $x[1]$ and $x[2]$.

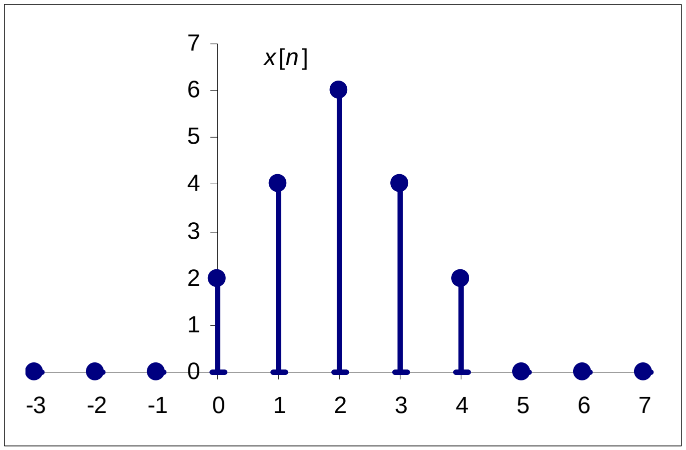
Causality

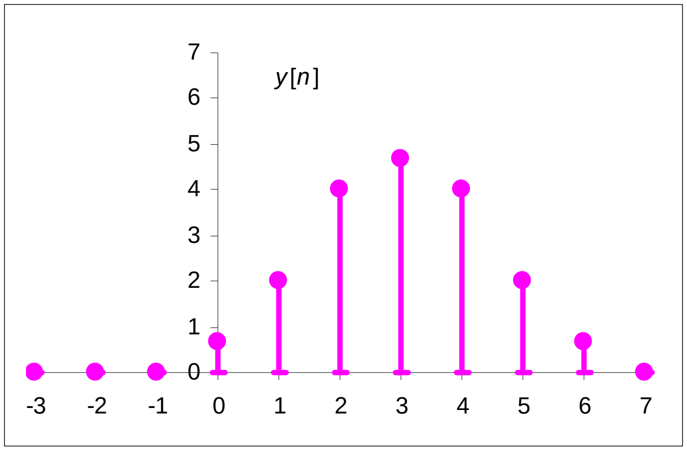
- A system whose present values depend only on the present and the past is called a Causal system
- A system whose present values depend on the the future is called a Non-causal system
- Let's rewrite our moving average filter to be causal:

$$y[n]= 1/3\ (x[n]+x[n-1]+x[n-2])$$

Causal Moving Average filter

- The new sequences:





- The new table:

n	-3	-2	-1	0	1	2	3	4	5	6	7
$x[n]$	0	0	0	2	4	6	4	2	0	0	0
$y[n]$	0	0	0	0.6666667	2	4	4.6666667	4	2	0.6666667	0

The General FIR Filter

- We can extend our moving average difference equation to this general form:

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

- Note that:
  - this system is causal
  - depends on a finite sequence of past values of $x[n]$
  - The coefficients, b_k's, can viewed as a weights and therefore our filter becomes a weighted running average
- We call M the order of the FIR and number of coefficients is called the filter length and is equal to M+1

Another Example

- Let's see how the following sequence is affected by a 3-point FIR and a 7-point FIR

$$x[n] = 1.02^n + \frac{1}{2}\cos(\frac{2\pi n}{8} + \frac{\pi}{4}) \text{ for } 0 \le n \le 40$$

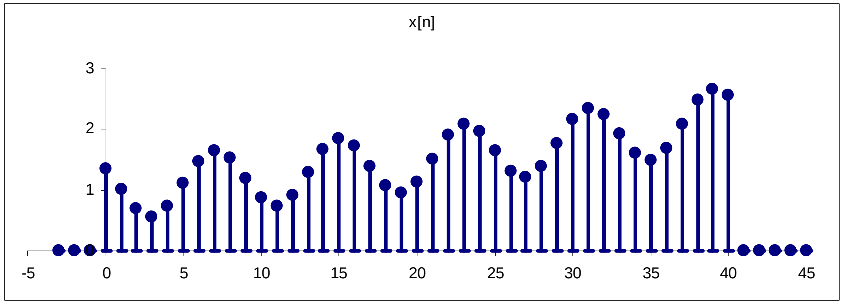
$$= 0 \text{ elsewhere}$$

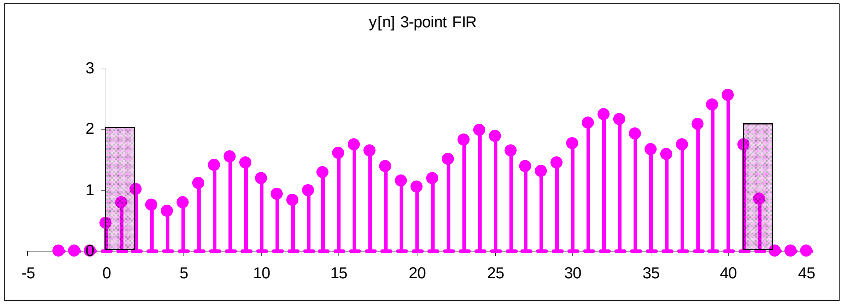
3-point FIR

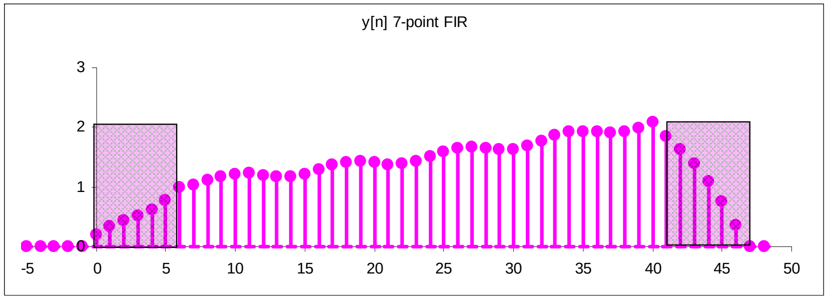
$$y[n] = \frac{1}{3}\sum_{k=0}^{2} x[n-k]$$

7-point FIR

$$y[n] = \frac{1}{7}\sum_{k=0}^{6} x[n-k]$$







Unit Impulse Sequence

- The unit impulse only has a value at n=0. The notation used to represent the unit impulse is called the (Kronecker) delta function:

$$\delta [n] = 1 \text{ for } n=0, 0 \text{ elsewhere}$$

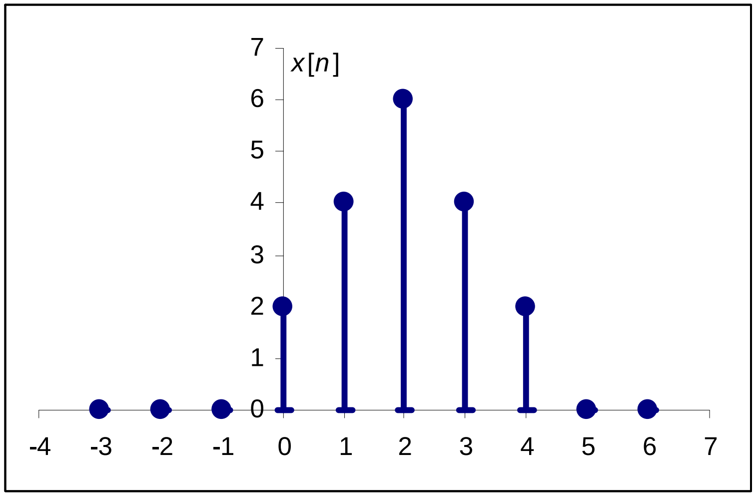
- Therefore, shifted impulses are:

$$\delta[n-2] = 1 \text{ for } n=2, 0 \text{ elsewhere}$$

$$\delta[n-k] = 1 \text{ for } n=k, 0 \text{ elsewhere}$$

Application of the Unit Impulse

- One may use the unit impulse to represent our first sequence as:



$$x[n] = 2\delta[n] + 4\delta[n-1] + 6\delta[n-2] + 4\delta[n-3] + 2\delta[n-4]$$

n	-1	0	1	2	3	4	5
2δ[n]	0	2	0	0	0	0	0
$4\delta[n\text{-}1]$	0	0	4	0	0	0	0
$6\delta[n\text{-}2]$	0	0	0	6	0	0	0
$4\delta[n\text{-}3]$	0	0	0	0	4	0	0
$2\delta[n\text{-}4]$	0	0	0	0	0	2	0
0	0	0	0	0	0	0	0
$x[n]$	0	2	4	6	4	2	0

Unit Impulse Representation of a Sequence

- In fact, any sequence can be represented as sum of unit impulse functions.

$$x[n]=\sum_{k} x[k]\delta[n-k]$$

$$=\cdots+x[-1]\delta[n+1]+x[0]\delta[n]$$

$$+x[1]\delta[n-1]+x[2]\delta[n-2]+\cdots$$

Unit Impulse Response Sequence

- When the input to a FIR is a unit impulse sequence $x[n]=\delta[n]$, the output is defined as the unit impulse response, $h[n]$.

$$h[n] = \sum_{k=0}^{M} b_k \delta[n-k] = \begin{cases} b_n & n = 0, 1, 2, \cdots, M \\ 0 & \text{otherwise} \end{cases}$$

- In other words, the impulse response $h[n]$ of the FIR is the sequence of difference equation coefficients.
- Since $h[n]=0$ for $n < 0$ and $n > M$, the length of the $h[n]$ is finite.
- This is why the system is called a finite impulse response, FIR, system

Unit Impulse Response Sequence

$$h[n] = \sum_{k=0}^{M} b_k \delta[n-k] = b_0\delta[n-0] + b_1\delta[n-1] + b_2\delta[n-2] + \cdots + b_l\delta[n-l] + \cdots + b_M\delta[n-M]$$

$$\vdots$$

$$h[-1] = 0$$

$$h[0] = b_0$$

$$h[1] = b_1$$

$$\vdots$$

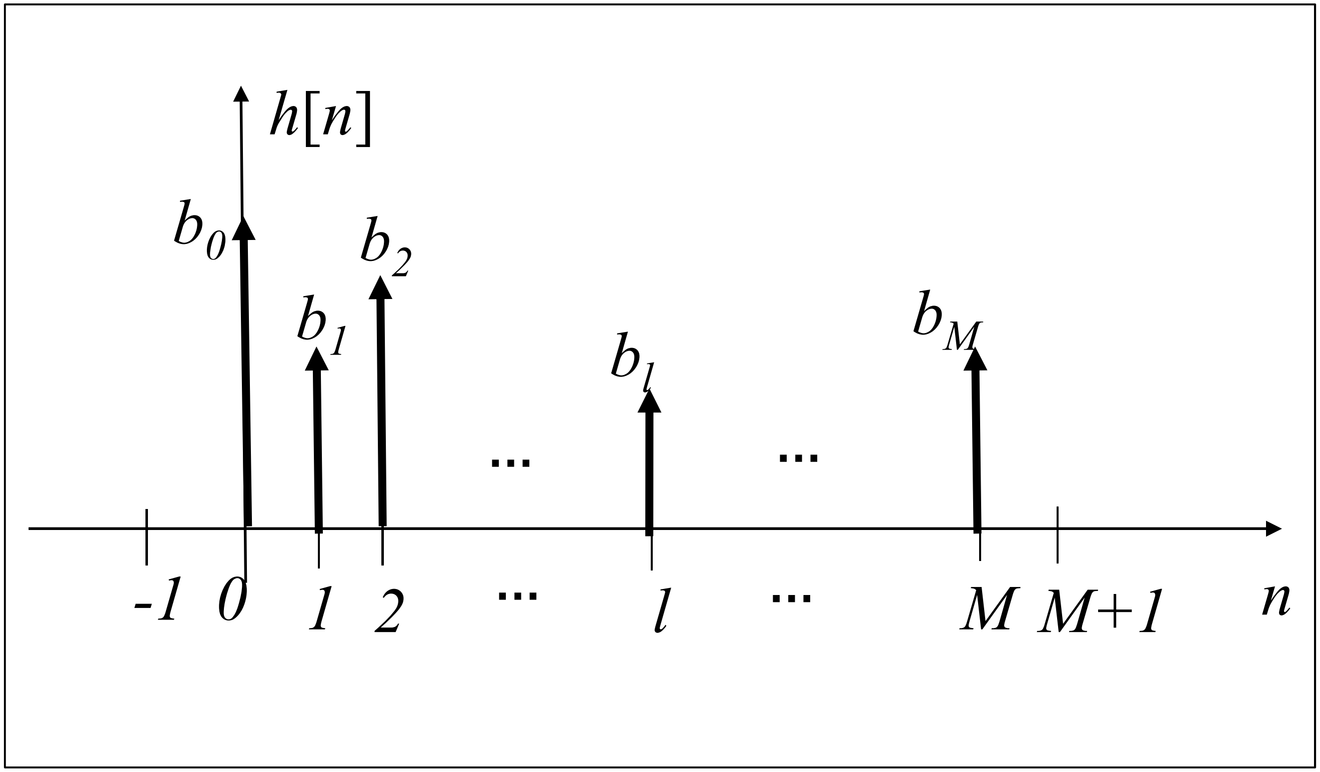
$$h[l] = b_l$$

$$\vdots$$

$$h[M] = b_M$$

$$h[M+1] = 0$$

$$\vdots$$



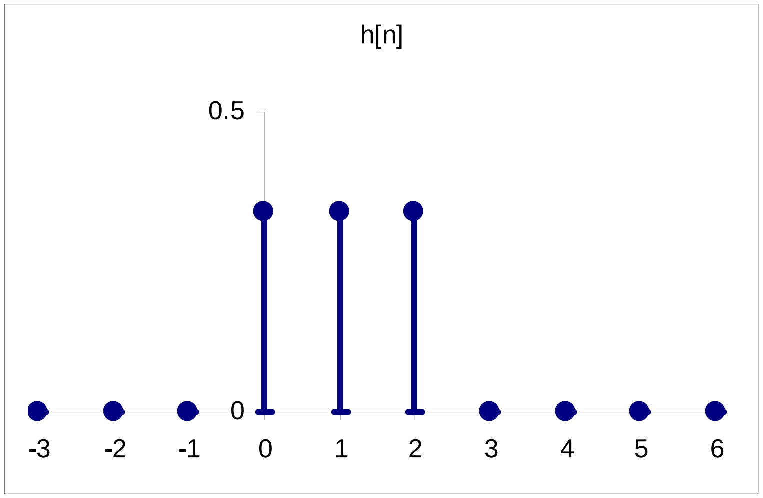
The Unit Impulse Response of a 3-point Average FIR

3 - point FIR

$$y[n] = \frac{1}{3}\sum_{k=0}^{2} x[n-k]$$

$$h[n] = \frac{1}{3}\sum_{k=0}^{2} \delta[n-k]$$

$$= \frac{1}{3}\{\delta[n-0] + \delta[n-1] + \delta[n-2]\}$$



Unit-Delay System

- A simple operator performs shift of the sequence by n_0 units.

$$y[n]=x[n - n_0]$$

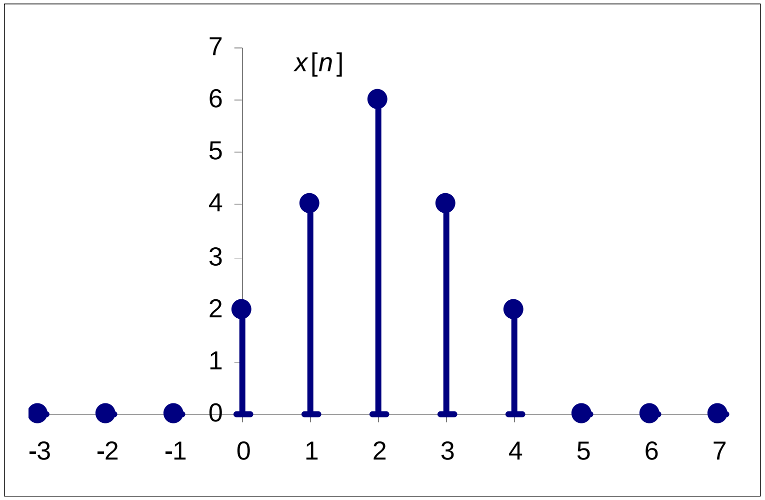
- When $n_0 = 1$, the system is called the unit delay.

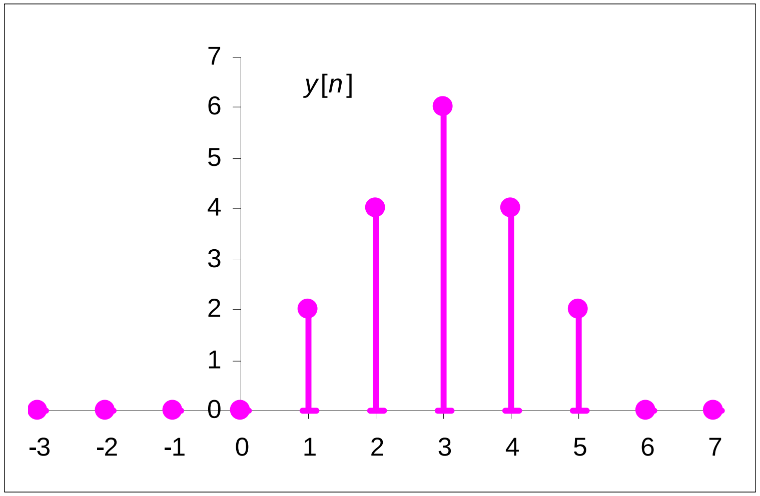
$$y[n]=x[n - 1]$$

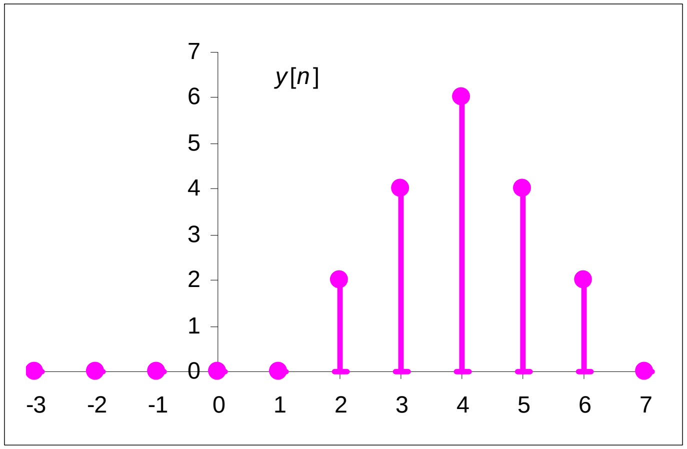
- The impulse response of this system becomes:

$$h[n]=\delta [n - n_0]$$

A Unit Delay System







- What is this one?

Homework

- Exercises:
  - 5.1 – 5.4
- Problems:
  - 5.1, 5.2, 5.3, 5.6