Circuit Analysis

Lesson #1

Circuit Analysis

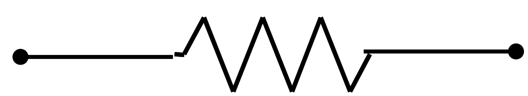
- Circuit Elements
  - Passive Devices
  - Active Devices
- Circuit Analysis Tools
  - Ohms Law
  - Kirchhoff's Law
  - Impedances
  - Mesh and Nodal Analysis
  - Superposition
- Examples

Characterize Circuit Elements

- Passive Devices: dissipates or stores energy
  - Linear
  - Non-linear
- Active Devices: Provider of energy or supports power gain
  - Linear
  - Non-linear

Circuit Elements – Linear Passive Devices

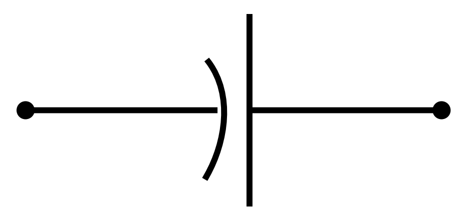
- **Linear**: supports a linear relationship between the voltage across the device and the current through it.
  - **Resistor**: supports a voltage and current which are proportional, device dissipates heat, and is governed by Ohm's Law, units: resistance or ohms Ω



$V_R(t) = I_R(t)R$ where R is the value of the resistance associated with the resistor

Circuit Elements – Linear Passive Devices

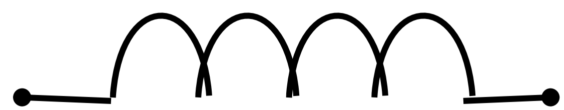
– **Capacitor**: supports a current which is proportional to its changing voltage, device stores an electric field between its plates, and is governed by Gauss' Law, units: capacitance or farads, *f*



$I_C(t) = C\,\frac{dV_C(t)}{dt}$ where C is the value of the capacitance associated with the capacitor

Circuit Elements – Linear Passive Devices

- **Inductor**: supports a voltage which is proportional to its changing current, device stores a magnetic field through its coils and is governed by Faraday's Law, units: inductance or henries, *h*



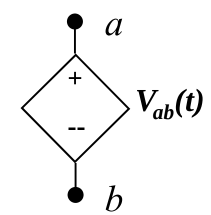
$$V_L(t) = L\frac{dI_L(t)}{dt} \text{ where L is the value of the inductance associated with the inductor}$$

Circuit Elements - Passive Devices Continued

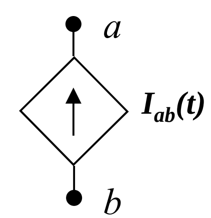
- **Non-linear**: supports a non-linear relationship among the currents and voltages associated with it
  - **Diodes**: supports current flowing through it in only one direction

Circuit Elements - Active Devices

- **Linear**
  - **Sources**
    - **Voltage Source**: a device which supplies a voltage as a function of time at its terminals which is independent of the current flowing through it, units: Volts
      - DC, AC, Pulse Trains, Square Waves, Triangular Waves



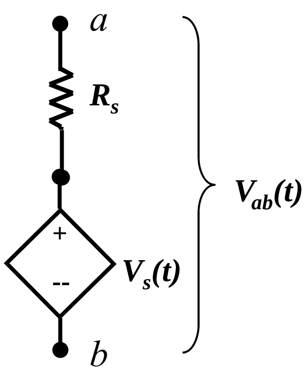
    - **Current Source**: a device which supplies a current as a function of time out of its terminals which is independent of the voltage across it, units: Amperes
      - DC, AC, Pulse Trains, Square Waves, Triangular Waves

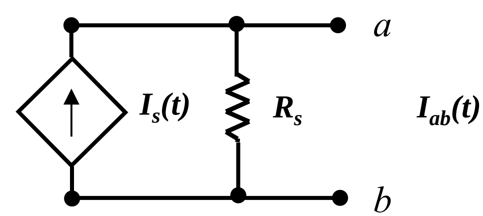


Circuit Elements - Active Devices Continued

– **Ideal Sources vs Practical Sources**

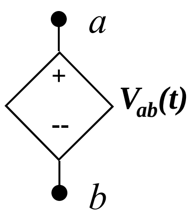
- An ideal source is one which only depends on the type of source (i.e., current or voltage)
- A practical source is one where other circuit elements are associated with it (e.g., resistance, inductance, etc.)
  – A practical voltage source consists of an ideal voltage source connected in series with passive circuit elements such as a resistor
  – A practical current source consists of an ideal current source connected in parallel with passive circuit elements such as a resistor



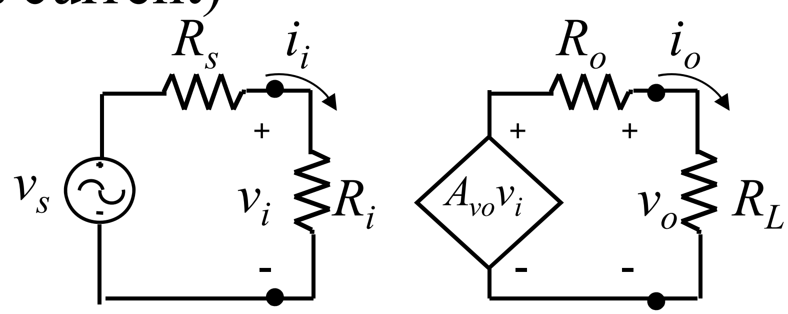


Circuit Elements - Active Devices Continued

– Independent vs Dependent Sources



- An independent source is one where the output voltage or current is not dependent on other voltages or currents in the device
- A dependent source is one where the output voltage or current is a function of another voltage or current in the device (e.g., a BJT transistor may be viewed as having an output current source which is dependent on the input current)



Circuit Elements - Active Devices Continued

- **Non-Linear**
  - **Transistors**: three or more terminal devices where its output voltage and current characteristics are a function on its input voltage and/or current characteristics, several types BJT, FETs, etc.

Circuits

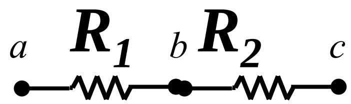
- A circuit is a grouping of passive and active elements
- Elements may be connecting is series, parallel or combinations of both

Circuits Continued

- Series Connection: Same current through the devices
  - The resultant resistance of two or more Resistors connected in series is the sum of the resistance
  - The resultant inductance of two or more Inductors connected in series is the sum of the inductances
  - The resultant capacitance of two or more Capacitors connected in series is the inverse of the sum of the inverse capacitances
  - The resultant voltage of two or more Ideal Voltage Sources connected in series is the sum of the voltages
  - Two of more Ideal Current sources can not be connected in series

Series Circuits

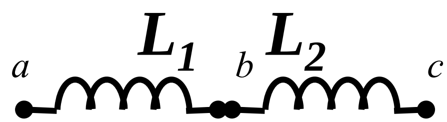
- Resistors



$$R_T = R_1 + R_2$$

$$V_{ac} = V_{ab} + V_{bc} = IR_1 + IR_2$$
$$= I(R_1 + R_2) = IR_T$$

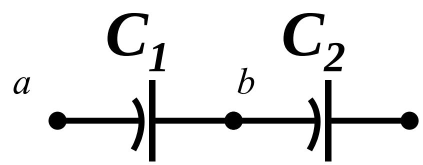
- Inductors



$$L_T = L_1 + L_2$$

$$V_{ac} = V_{ab} + V_{bc} = L_1 \frac{dI}{dt} + L_2 \frac{dI}{dt}$$
$$= (L_1 + L_2)\frac{dI}{dt} = L_T \frac{dI}{dt}$$

- Capacitors

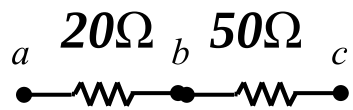


$$C_T = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2}$$

$$V_{ac} = V_{ab} + V_{bc} = \frac{1}{C_1}\int Idt + \frac{1}{C_2}\int Idt$$
$$= (\frac{1}{C_1} + \frac{1}{C_2})\int Idt = \frac{1}{C_T}\int Idt$$

Series Circuits

- Resistors

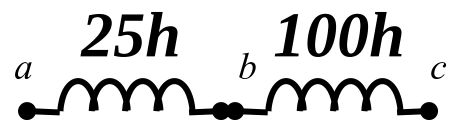


$$\boldsymbol{R_T = 20 + 50 = 70\Omega}$$

$$V_{ac} = V_{ab} + V_{bc} = I20 + I50$$
$$= I(20 + 50) = I70$$

Series Circuits

- Inductors



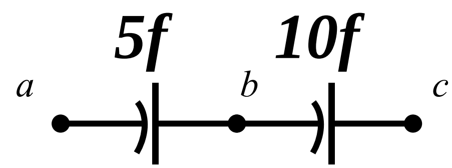
L_T =25+ 100 = 125h

$$V_{ac} = V_{ab} + V_{bc} = 25\frac{dI}{dt} + 100\frac{dI}{dt}$$

$$= (25+100)\frac{dI}{dt} = 125\frac{dI}{dt}$$

Series Circuits

- Capacitors

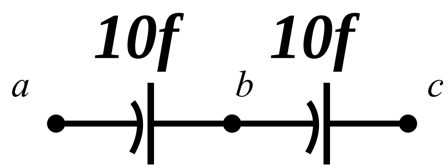


$$C_T = \frac{1}{\frac{1}{5}+\frac{1}{10}} = \frac{5\times 10}{5+10} = \frac{50}{15} = \frac{10}{3} = 3.33f$$

$$V_{ac} = V_{ab} + V_{bc} = \frac{1}{5}\int Idt + \frac{1}{10}\int Idt$$
$$= (\frac{1}{5}+\frac{1}{10})\int Idt = \frac{3}{10}\int Idt$$

Series Circuits

- Capacitors



$$C_T = \frac{1}{\frac{1}{10}+\frac{1}{10}} = \frac{10\times 10}{10+10} = \frac{100}{20} = \frac{10}{2} = 5f$$

$$V_{ac} = V_{ab} + V_{bc} = \frac{1}{10}\int Idt + \frac{1}{10}\int Idt$$

$$= (\frac{1}{10}+\frac{1}{10})\int Idt = \frac{2}{10}\int Idt = \frac{1}{5}\int Idt$$

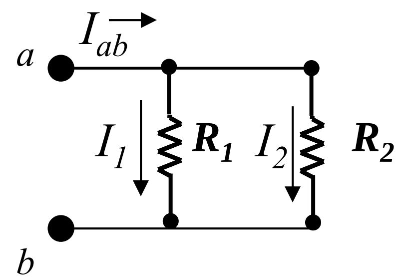
Circuits Continued

- Parallel Connection: Same Voltage across the devices
  - The resultant resistance of two or more Resistors connected in parallel is the inverse of the sum of the inverse resistances
  - The resultant inductance of two or more Inductors connected in parallel is the inverse of the sum of the inverse inductances
  - The resultant capacitance of two or more Capacitors connected in parallel is the sum of the capacitances
  - The resultant current of two or more Ideal Current Sources connected in parallel is the sum of the currents
  - Two of more Ideal Voltage sources can not be connected in parallel

Parallel Circuits

- Resistors

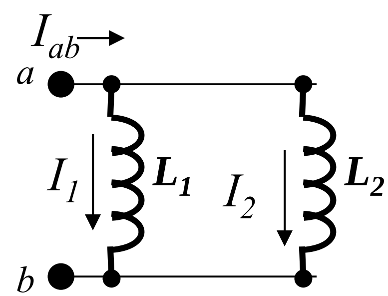
$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$



$$I_{ab} = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2}$$
$$= (\frac{1}{R_1} + \frac{1}{R_2})V = \frac{V}{R_T}$$

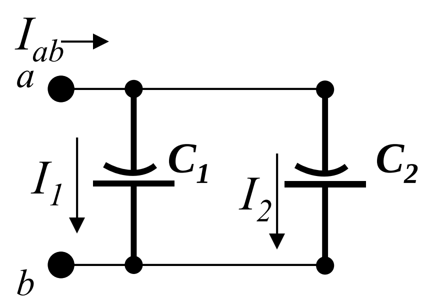
- Inductors

$$L_T = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2}} = \frac{L_1 L_2}{L_1 + L_2}$$



$$I_{ab} = I_1 + I_2 = \frac{1}{L_1}\int V dt + \frac{1}{L_2}\int V dt$$
$$= (\frac{1}{L_1} + \frac{1}{L_2})\int V dt = \frac{1}{L_T}\int V dt$$

- Capacitors $\boldsymbol{C_T = C_1 + C_2}$



$$I_{ab} = I_1 + I_2 = C_1\frac{dV}{dt} + C_2\frac{dV}{dt}$$
$$= (C_1 + C_2)\frac{dV}{dt} = C_T\frac{dV}{dt}$$

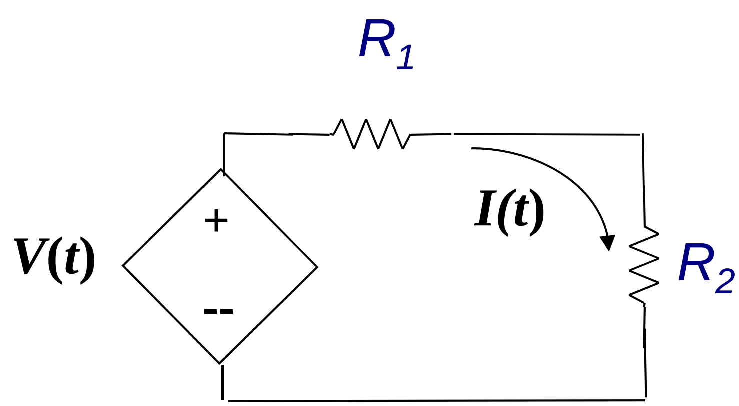
Combining Circuit Elements Kirchhoff's Laws

- Kirchhoff Voltage Law: The sum of the voltages around a loop must equal zero
- Kirchhoff Current Law: The sum of the currents leaving (entering) a node must equal zero

Combining Rs, Ls and Cs

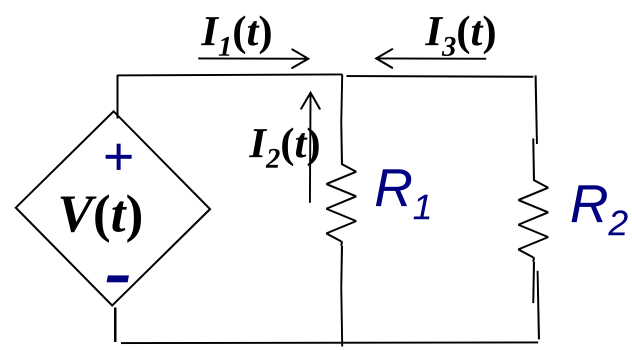
- We can use KVL or KCL to write and solve an equation associated with the circuit.
  - Example: a series Resistive Circuit

$$V(t) = I(t)R_1 + I(t)R_2$$
$$V(t) = I(t)(R_1 + R_2)$$



Combining Rs, Ls, and Cs

- We can use KVL or KCL to write and solve an equation associated with the circuit.
  - Example: a series Resistive Circuit



$$I_1(t)+I_2(t)+I_3(t)=0$$

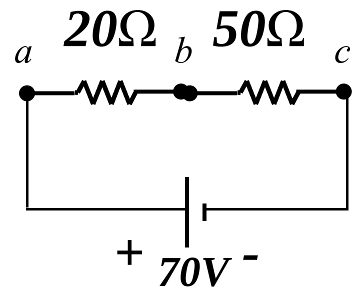
$$I_2(t)=-V(t)/R_1;\ I_3(t)=-V(t)/R_2;$$

$$I_1(t) - V(t)/R_1 - V(t)/R_2=0$$

$$I_1(t)=V(t)/R_1+V(t)/R_2=V(t)[1/R_1+1/R_2]$$

Series Circuits

- Resistors



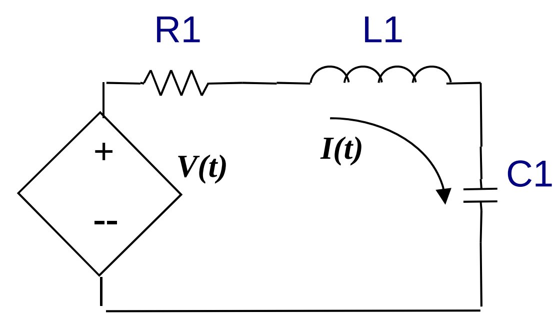
$$R_T = 20 + 50 = 70\Omega$$

$$V_{ac} = V_{ab} + V_{bc} = I20 + I50$$
$$= I(20 + 50) = I70 = 70V$$
$$I = 1A$$

Combining Rs, Ls and Cs

– Example: a series RLC circuit

$$V(t) = I(t)R_1 + L_1 \frac{dI(t)}{dt} + \frac{1}{C_1}\int I(t)dt$$



– Or to simplify this analysis, we can concentrate on special cases

Impedances

- Our special case, signals of the form: $V(t)$ or $I(t) = Ae^{st}$ where s can be a real or complex number

$$V(t) = I(t)R_1 + L_1\frac{dI(t)}{dt} + \frac{1}{C_1}\int I(t)dt$$

Let's assume:

$$V(t) = 10e^{5t}; R_1 = 10\Omega; L_1 = 5h; C_1 = .2f$$

Let's try:

$$I(t) = Ae^{5t}$$

$$10e^{5t} = I(t)10 + 5\frac{dI(t)}{dt} + \frac{1}{.2}\int I(t)dt$$

$$10e^{5t} = Ae^{5t}10 + 5\frac{dAe^{5t}}{dt} + 5\int Ae^{5t}dt$$

$$10e^{5t} = A(e^{5t}10 + 5\times 5e^{5t} + \frac{5e^{5t}}{5})$$

$$= A(e^{5t}36)$$

$$A = \frac{10}{36}; I(t) = \frac{10}{36}e^{5t}$$

- This is only one portion of the solution and does not include the transient response.

Impedances

- Since the derivative [and integral] of $Ae^{st} = sAe^{st}$ [$=(1/s)Ae^{st}$], we can define the impedance of a circuit element as $Z(s)=V/I$ where Z is only a function of s since the time dependency drops out.

Impedances

For an inductor, let's assume $I(t) = Ae^{st}$; then $V(t) = L\dfrac{dI(t)}{dt} = LsAe^{st}$;

$$Z(s) = \frac{V}{I} = \frac{sLAe^{st}}{Ae^{st}} = sL$$

For a capacitor, let's assume $V(t) = Ae^{st}$; then $I(t) = C\dfrac{dV(t)}{dt} = CsAe^{st}$;

$$Z(s) = \frac{V}{I} = \frac{Ae^{st}}{sCAe^{st}} = \frac{1}{sC}$$

For a resistor, let's assume $I(t) = Ae^{st}$; then $V(t) = RI(t) = RAe^{st}$;

$$Z(s) = \frac{V}{I} = \frac{RAe^{st}}{Ae^{st}} = R$$

Impedances

- What about signals of the type: $\cos(\omega t+\theta)$;

- Recall Euler's formula $e^{j\theta} = \cos\theta + j\sin\theta$ where j is the imaginary number = $\sqrt{-1}$

- A special case of our special case is for sinusoidal inputs, where $s=j\omega$

Complex Numbers

- Complex numbers: What are they?
- What is the solution to this equation?

$$ax^2+bx+c=0$$

- This is a second order equation whose solution is:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

What is the solution to?

1. $x^2+4x+3=0$

$$x_{1,2} = \frac{-4 \pm \sqrt{4^2 - 4\times 3}}{2} = \frac{-4 \pm \sqrt{16-12}}{2}$$

$$= \frac{-4 \pm \sqrt{4}}{2} = \frac{-4 \pm 2}{2} = -1, -3$$

What is the solution to?

2. $x^2+4x+5=0$

$$x_{1,2} = \frac{-4 \pm \sqrt{4^2 - 4 \times 5}}{2} = \frac{-4 \pm \sqrt{16-20}}{2}$$

$$= \frac{-4 \pm \sqrt{-4}}{2} \quad ?????$$

What is the Square Root of a Negative Number?

- We define the square root of a negative number as an imaginary number
- We define

$$\sqrt{-1} \Rightarrow j \text{ for engineers } (i \text{ for mathematicans})$$

- Then our solution becomes:

$$x_{1,2} = \frac{-4 \pm \sqrt{4^2 - 4 \times 5}}{2} = \frac{-4 \pm \sqrt{16 - 20}}{2}$$

$$= \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm j\sqrt{4}}{2} = \frac{-4 \pm j2}{2} = -2 + j1, -2 - j1$$

The Complex Plane

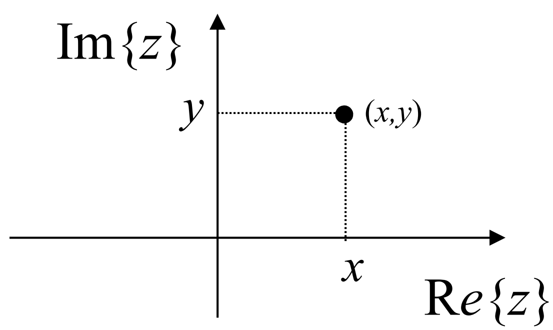
- $z = x+jy$ is a complex number where:

  $x = \mathrm{Re}\{z\}$ is the real part of z

  $y = \mathrm{Im}\{z\}$ is the imaginary part of z

- We can define the complex plane and we can define 2 representations for a complex number:

$z = x+jy$



Rectangular Form

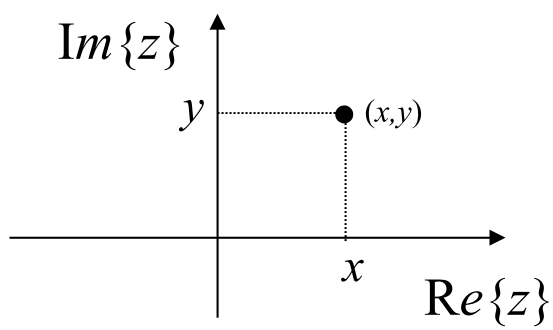
- Rectangular (or cartesian) form of a complex number is given as

$$z = x + jy$$

$x = \mathrm{Re}\{z\}$ is the real part of z

$y = \mathrm{Im}\{z\}$ is the imaginary part of z

$z = x + jy$

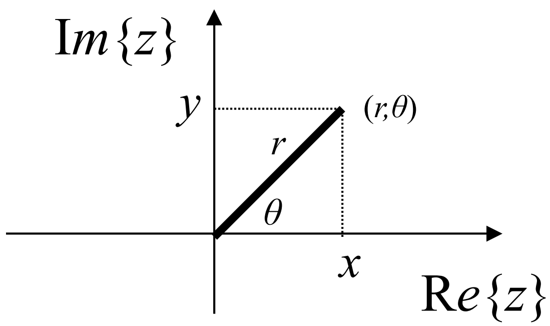


Rectangular or Cartesian

Polar Form

- $z = re^{j\theta} = r\angle\theta$ is a complex number where:
- r is the magnitude of z
- θ is the angle or argument of z (arg z)

$$z = r\, e^{j\theta}$$



Polar

Relationships between the Polar and Rectangular Forms

$$z = x + jy = r\,e^{j\theta}$$

- Relationship of Polar to the Rectangular Form:

$$x = \mathrm{R}e\{z\} = r\cos\theta$$

$$y = \mathrm{I}m\{z\} = r\sin\theta$$

- Relationship of Rectangular to Polar Form:

$$r = \sqrt{x^2 + y^2} \quad \text{and} \quad \theta = \arctan(\frac{y}{x})$$

Addition of 2 complex numbers

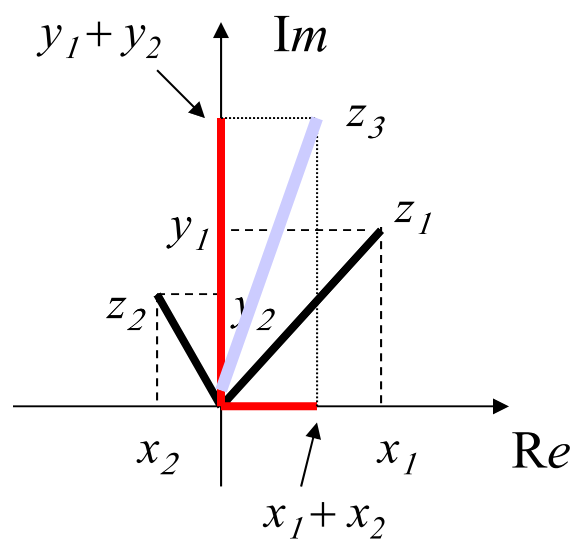
- When two complex numbers are added, it is best to use the rectangular form.
- The real part of the sum is the sum of the real parts and imaginary part of the sum is the sum of the imaginary parts.
- Example: $z_3 = z_1 + z_2$

$$z_1 = x_1 + jy_1; z_2 = x_2 + jy_2$$

$$z_3 = z_1 + z_2 = x_1 + jy_1 + x_2 + jy_2$$

$$= x_1 + x_2 + jy_1 + jy_2$$

$$= (x_1 + x_2) + j(y_1 + y_2)$$



Multiplication of 2 complex numbers

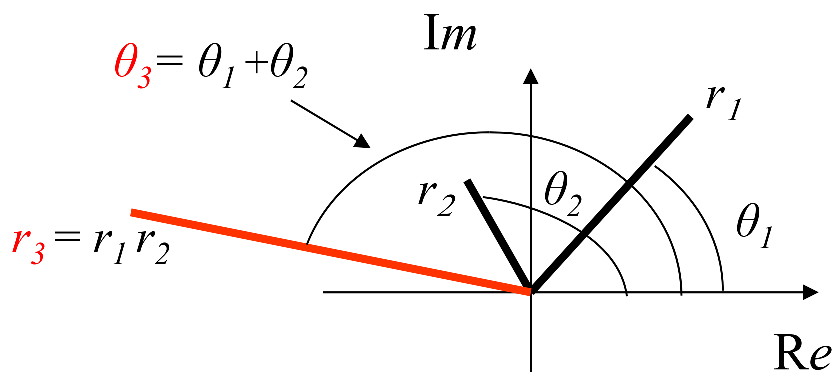
- When two complex numbers are multiplied, it is best to use the polar form:
- Example: $z_3 = z_1 \times z_2$

$$z_1 = r_1 e^{j(\theta_1)}; z_2 = r_2 e^{j(\theta_2)}$$

$$z_3 = z_1 \times z_2 = r_1 e^{j(\theta_1)} \times r_2 e^{j(\theta_2)}$$

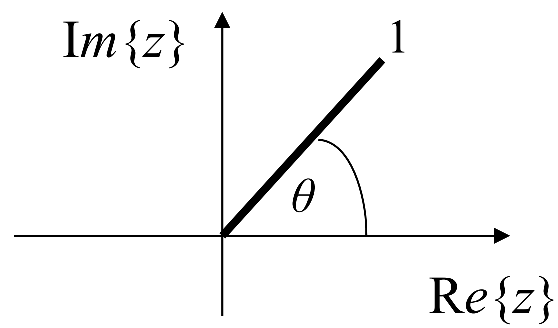
$$= r_1 r_2 e^{j(\theta_1)} e^{j(\theta_2)} = r_1 r_2 e^{j(\theta_1 + \theta_2)}$$

- We multiply the magnitudes and add the phase angles



Euler's Formula

$$e^{j\theta} = \cos\theta + j\sin\theta$$



- We can use Euler's Formula to define complex numbers

$$z = r\,e^{j\theta} = r\cos\theta + j\,r\sin\theta$$
$$= x + j\,y$$

Complex Exponential Signals

- A complex exponential ***signal*** is define as:

$$z(t) = Ae^{j(\omega_o t + \phi)}$$

- Note that it is defined in polar form where
  - the magnitude of $z(t)$ is $|z(t)| = A$
  - the angle (or argument, arg $z(t)$) of $z(t) = (\omega_o t + \phi)$
    - Where ω_o is called the radian frequency and ϕ is the phase angle (phase shift)

Complex Exponential Signals

- Note that by using Euler's formula, we can rewrite the complex exponential signal in rectangular form as:

$$z(t) = Ae^{j(\omega_o t+\phi)}$$
$$= A\cos(\omega_o t + \phi) + jA\sin(\omega_o t + \phi)$$

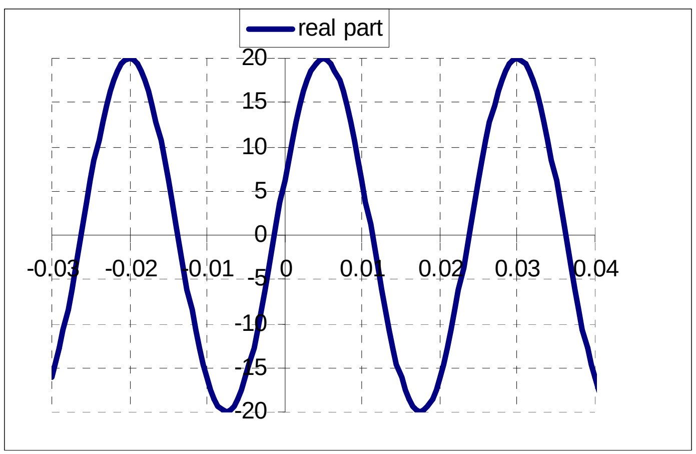
- Therefore real part is the cosine signal and imaginary part is a sine signal both of radial frequency ω_o and phase angle of ϕ

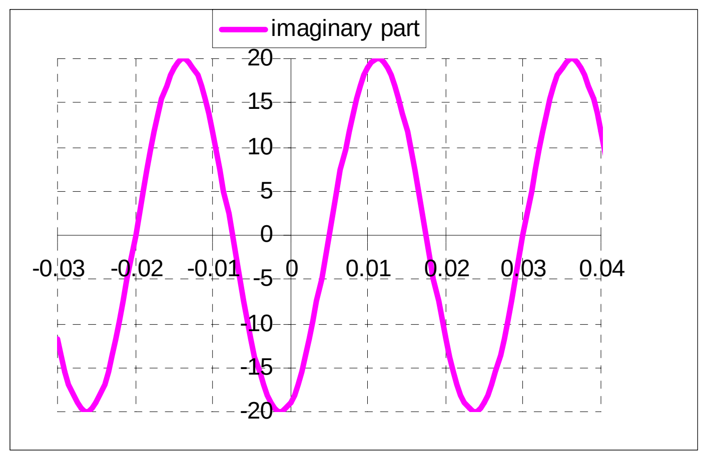
Plotting the waveform of a complex exponential signal

- For an complex signal, we plot the real part and the imaginary part separately.
- Example:

$$z(t) = 20e^{j(2\pi(40)t-0.4\pi)} = 20e^{j(80\pi t-0.4\pi)}$$

$$= 20\cos(80\pi t-0.4\pi) + j20\sin(80\pi t-0.4\pi)$$





NOTE!!!!

- The reason why we prefer the complex exponential representation of the real cosine signal:

$$x(t) = \Re e\{z(t)\} = \Re e\{Ae^{j(\omega_o t+\phi)}\}$$
$$= A\cos(\omega_o t + \phi)$$

- In solving equations and making other calculations, it easier to use the complex exponential form and then take the Real Part.

Complex Exponential Function as a function of time

- Let's look at this $z(t) = 1e^{j2\pi(1)t} = e^{j2\pi t} = \cos 2\pi t + j\sin 2\pi t$

t=8/8 seconds

arg(z(t))=2π x8/8 = 2π ; z(t)= 1+ j0

t=2/8 seconds

arg(z(t))=2π x2/8= π/2; z(t)= 0 + j1

t=3/8 seconds

arg(z(t))=2π x3/8 = 3π/4;

z(t)= -0.707+ j0.707

t=1/8 seconds

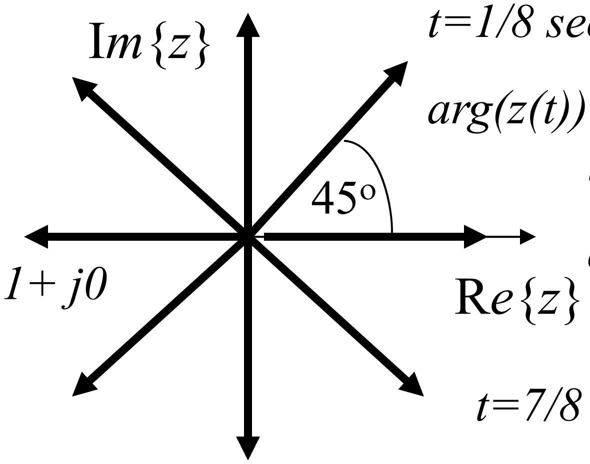
arg(z(t))=2π x1/8=π/4; z(t)=0.707+j 0.707

t=0 seconds

arg(z(t))=2π x0=0; z(t)=1+ j0

t=4/8 seconds

arg(z(t))=2π x4/8 = π; z(t)= -1+ j0



t=5/8 seconds

arg(z(t))=2π x5/8 = 5π/4;

z(t)= -0.707 - j0.707

t=7/8 seconds

arg(z(t))=2π x7/8= 7π/4;

z(t) = 0 .707- j0.707

t=6/8 seconds

arg(z(t))=2π x6/8 = 3π/2; z(t) = 0 - j

Phasor Representation of a Complex Exponential Signal

- Using the multiplication rule, we can rewrite the complex exponential signal as

$$z(t) = Ae^{j(\omega_o t+\phi)} = Ae^{j\omega_o t}e^{j\phi} = Ae^{j\phi}e^{j\omega_o t} = \mathbf{X}e^{j\omega_o t}$$

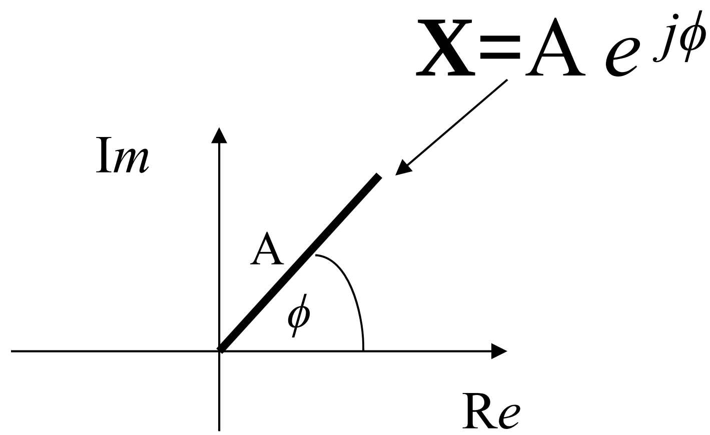
where **X** is a complex number equal to

$$\mathbf{X} = Ae^{j\phi}$$

- **X** is complex amplitude of the complex exponential signal and is also called a phasor

Graphing a phasor

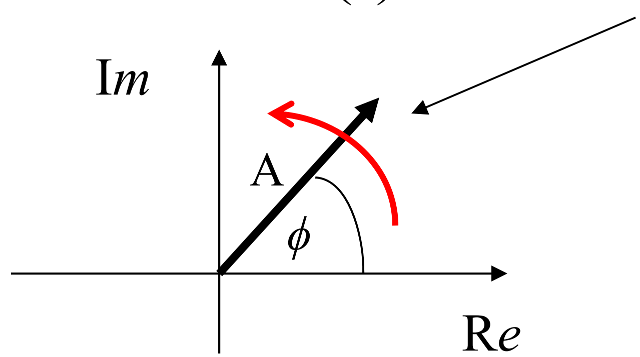
- $\mathbf{X}$=A $e^{j\phi}$ can be graphed in the complex plane with magnitude A and angle ϕ:



Graphing a Complex Signal in terms of its phasors

- Since a complex signal, $z(t)$, is a phasor multiplying a complex exponential signal $e^{j\omega_o t}$, then a complex signal can be viewed as a phasor rotating in time:

$$z(t) = Ae^{j(\omega_o t+\phi)} = \mathbf{X}e^{j\omega_o t}$$



Rotating Phasor

- Let's look at this $z(t) = Ae^{j(2\pi t + \frac{\pi}{4})} = Ae^{j\frac{\pi}{4}}e^{j2\pi t} = \mathbf{X}e^{j2\pi t}$

$$\mathbf{X} = Ae^{j\frac{\pi}{4}}$$

t=1 seconds

arg(z(t))=2π x1+π/4 = 9π/4 = π/4

t=1/8 seconds

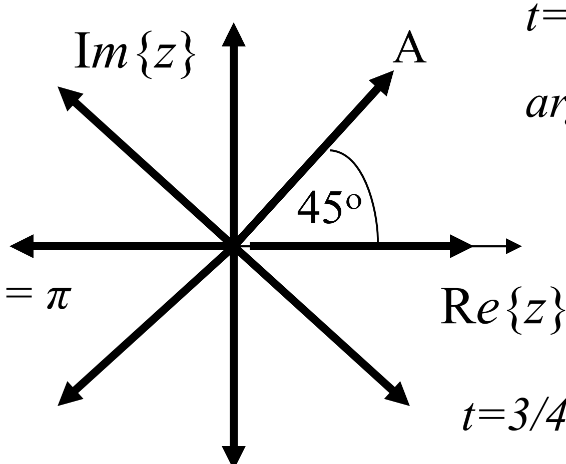
arg(z(t))=2π x1/8+π/4 = π/2

t=1/4 seconds

arg(z(t))=2π x1/4+π/4 = 3π/4

t=0 seconds

arg(z(t))=2π x0+π/4 = π/4



t=7/8 seconds

arg(z(t))=2π x7/8+π/4 = 8π/4 = 2π

t=3/8 seconds

arg(z(t))=2π x3/8+π/4 = 4π/4 = π

t=1/2 seconds

arg(z(t))=2π x1/2+π/4 = 5π/4

t=3/4 seconds

arg(z(t))=2π x3/4+π/4 = 7π/4

t=5/8 seconds

arg(z(t))=2π x5/8+π/4 = 6π/4 = 3π/2

Sinusoidal Steady State

- If $V(t) = A\cos(\omega t+\theta)$, then we can represent $V(t)$ as

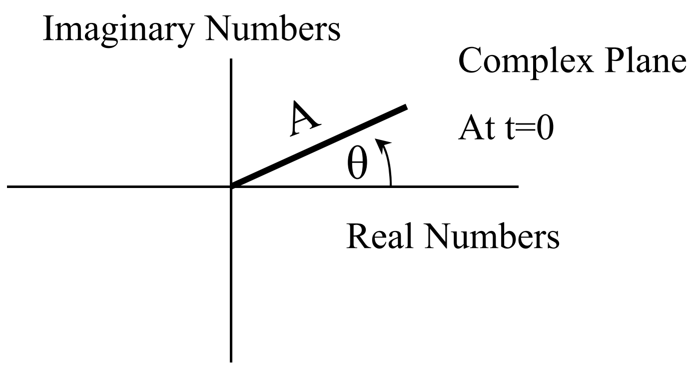
$$\mathrm{Re}\{Ae^{j(\omega t+\theta)}\} \quad = \quad \mathrm{Re}\{Ae^{j\theta}\, e^{j\omega t}\}$$

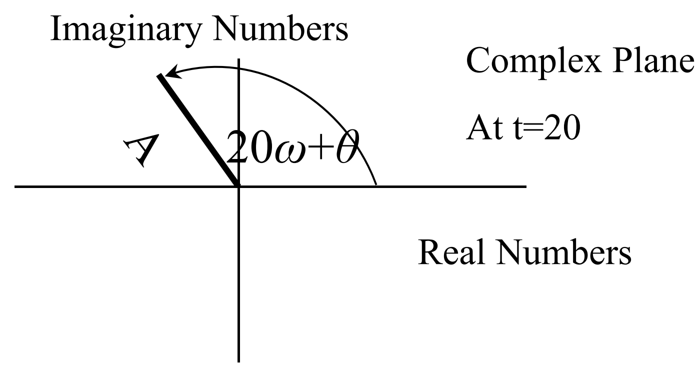
Since from Euler's formula:

$$A\, e^{\,j(\omega t+\theta)} = A\cos(\omega t+\theta) + j\, A\sin(\omega t+\theta)$$

Sinusoidal Steady State

- What is $Ae^{j(\omega t+\theta)}$?
  - First, it is a complex function since it is a function of a complex number. If we plot on the complex plane, it has a magnitude of *A* and angle of $\omega t+\theta$. It can be viewed as a vector which rotates in time around the origin of the complex plane at angular velocity ω and at $t=0$ is at θ degrees from the real axis.





  - We can **represent** this function by a **PHASOR** in terms of rectangular coordinates or polar coordinates

$$MAGNITUDE\angle ANGLE \text{ (phasor notation)} \Rightarrow \text{ or in this case } V = A\angle\theta$$

Sinusoidal Steady State Continued

- We define the Voltage phasor as **V** and current phasor as **I**
- Define SSS impedance as Z = **V** / **I** using Ohm's Law
- Then the impedances become:

– For an inductor V= $j\omega L$ I

$Z_L = j\omega L \Rightarrow \omega L \angle \frac{\pi}{2}$; Here we say the voltage across an inductor leads the current through it by 90°.

– For a capacitor V= $\frac{1}{j\omega C}$ I

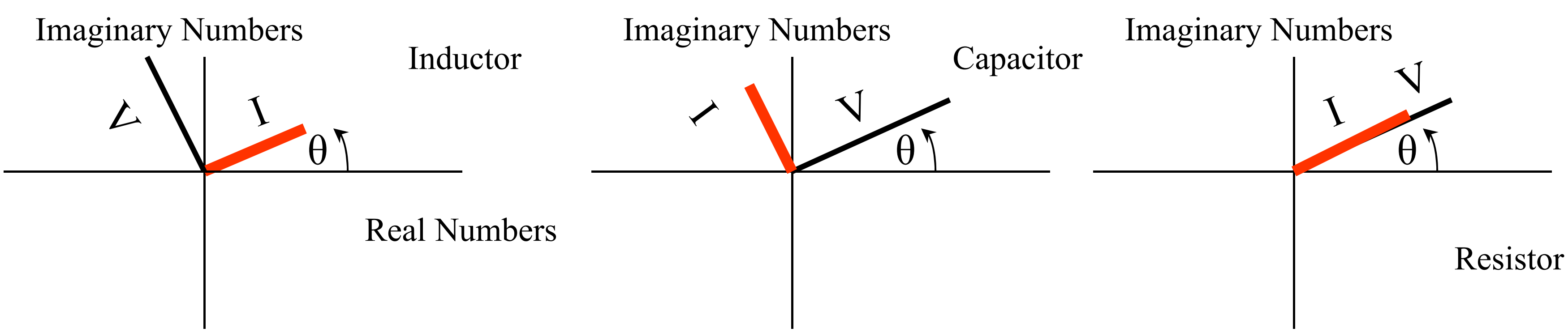
$Z_C = \frac{1}{j\omega C} \Rightarrow \frac{1}{\omega C} \angle -\frac{\pi}{2}$; Here we say the voltage across a capacitor lags the current through it by 90°.

– For a resistor V= R I

$Z_R = R \Rightarrow R\angle 0$; Here we say the voltage across a resistor is in phase with the current through it.

Sinusoidal Steady State Continued

- For an inductor, $Z_L = j\omega L \Rightarrow \omega L\angle\frac{\pi}{2}$
- For a capacitor, $Z_C = \frac{1}{j\omega C} \Rightarrow \frac{1}{\omega C}\angle-\frac{\pi}{2}$.
- For a resistor, $Z_R = R \Rightarrow R\angle 0$



Sinusoidal Steady State Continued

For $V(t)=A\cos\omega t$, using phasor notation for $V(t) \Rightarrow \mathbf{V} = A\angle 0$ and $I(t) \Rightarrow \mathbf{I}$, our equation can be re-written:

$$V(t) = I(t)R_1 + L_1\frac{dI(t)}{dt} + \frac{1}{C_1}\int I(t)dt$$

Converting to Phasor representation

$$\mathbf{V} = A\angle 0 = \mathbf{I}R_1 + j\omega L_1\mathbf{I} + \frac{1}{j\omega C_1}\mathbf{I}$$

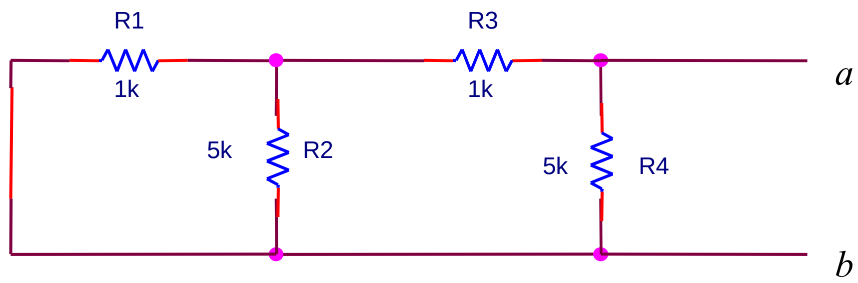
Sinusoidal Steady State Continued

$$\mathbf{I} = \frac{A\angle 0}{R_1 + j\omega L_1 + \frac{1}{j\omega C_1}} = \frac{A\angle 0}{R_1 + j(\omega L_1 - \frac{1}{\omega C_1})} = \frac{A}{\sqrt{R_1^{\,2} + (\omega L_1 - \frac{1}{\omega C_1})^2}}\angle - \tan^{-1}[\frac{(\omega L_1 - \frac{1}{\omega C_1})}{R_1}]$$

Converting back to the time representation,

$$I(t) = \frac{A}{\sqrt{R_1^{\,2} + (\omega L_1 - \frac{1}{\omega C_1})^2}}\cos(\omega t - \tan^{-1}[\frac{(\omega L_1 - \frac{1}{\omega C_1})}{R_1}])$$

Homework

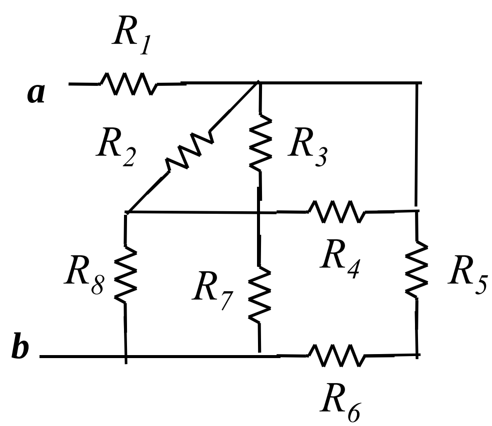


Find the total resistance

Homework

Find the total resistance R_{ab} where

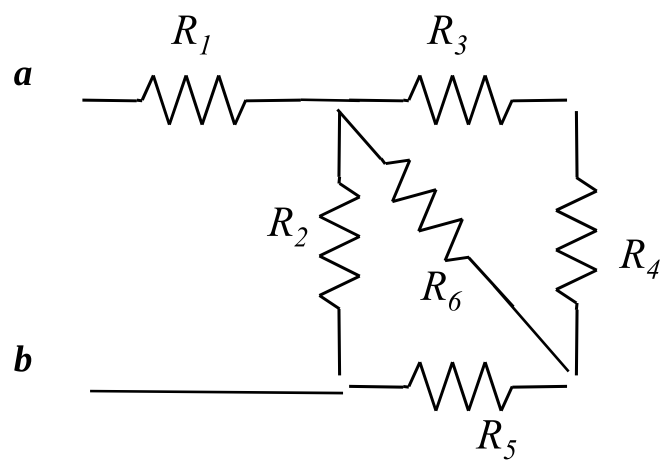
$R_1 = 3\Omega$, $R_2 = 6\Omega$, $R_3 = 12\Omega$, $R_4 = 4\Omega$, $R_5 = 2\Omega$, $R_6 = 2\Omega$, $R_7 = 4\Omega$, $R_8 = 4\Omega$



Homework

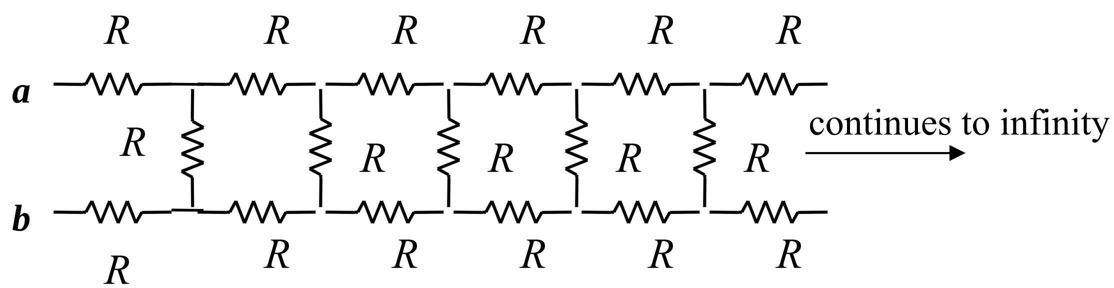
Find the total resistance R_{ab} where

$R_1 = 2\Omega$, $R_2 = 4\Omega$, $R_3 = 2\Omega$, $R_4 = 2\Omega$, $R_5 = 2\Omega$, $R_6 = 4\Omega$,

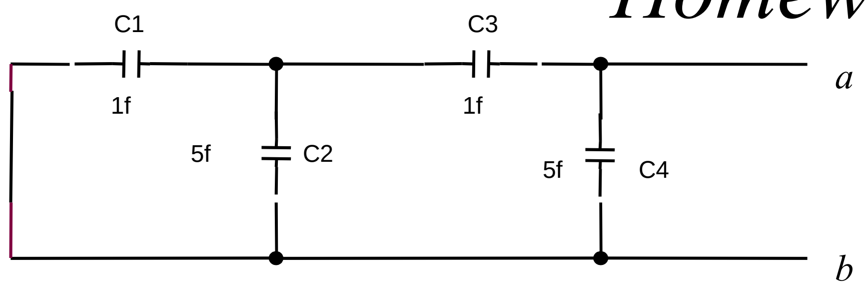


Homework

Find the total resistance R_{ab} for this infinite resistive network



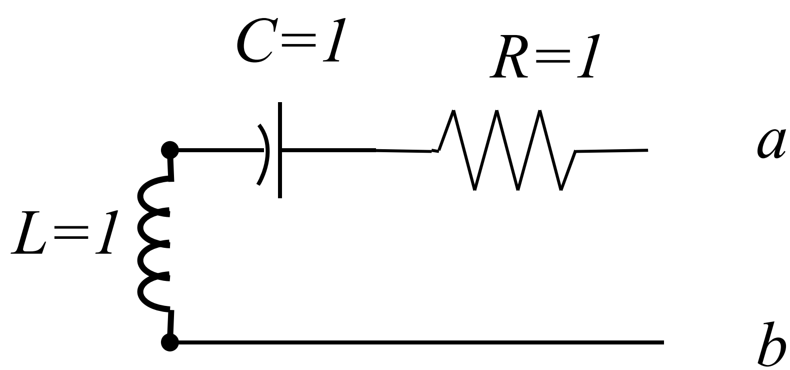
Homework



Find the total capacitance

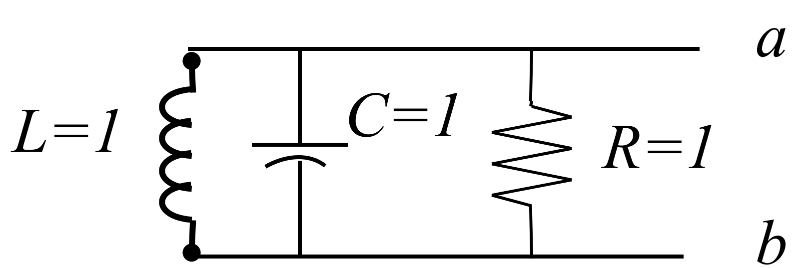
Homework

Find and plot the impedance $Z_{ab}(j\omega)$ as a function of frequency. Use Matlab to perform the plot.

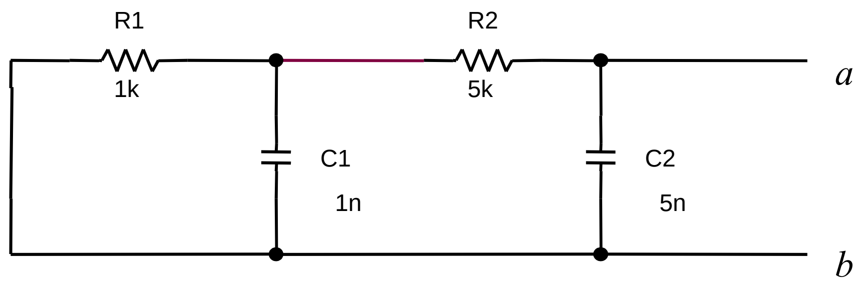


Homework

Find and plot the impedance $Z_{ab}(j\omega)$ as a function of frequency. Use Matlab to perform the plot.

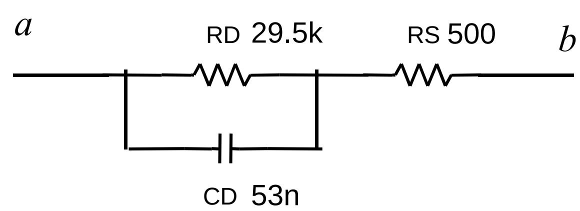


Homework



Find and plot the impedance $Z_{ab}(j\omega)$ as function of ω. Use Matlab to perform the plot.

Homework



The circuit shown is an equivalent circuit of an electrode where RD and CD are the resistance and capacitance associated with the interface of the electrode and the body and RS is the resistance of the device itself. Find and plot the impedance $Z_{ab}(j\omega)$ as function of ω. Use Matlab to perform the plot.