Transistors

Lesson #10

Chapter 4

Small Signal Equivalent Circuits and Parameters for the BJT

- If the variable portion of the input signal is small (in amplitude), it is possible to approximate the (non-linear) transistor as a linear device by representing it by an equivalent circuit
- Here's how we do it:
  - Recall $i_E = i_C + i_B$

    $$\text{Define: } \alpha = \frac{i_C}{i_E}\text{; then } i_B = (1-\alpha)i_E$$

    $$\text{and } i_C = \frac{\alpha}{1-\alpha} i_B = \beta i_B \text{ where } \beta = \frac{\alpha}{1-\alpha}$$

  - Since the base-emitter junction is a forward-biased diode and the current that crosses this junction is IE, then we can use the Shockley equation

$$i_E = I_{ES}(e^{\frac{v_{BE}}{V_T}} - 1)$$

Small Signal Equivalent Circuits and Parameters for the BJT

- Then: $i_B = (1-\alpha) I_{ES}(e^{\frac{v_{BE}}{V_T}} - 1)$
- Before we continue let's define the following notation:
  - A lower case signal (voltage or current) with upper case subscripts represents the total of the DC portion of the signal and the AC portion of the signal
  - An upper case signal with upper case subscripts represents the DC portion (the Q-point)
  - A lower case signal with lower case subscripts represents the AC portion

$$i_B(t) = I_{BQ} + i_b(t)$$
$$v_{BE}(t) = V_{BEQ} + v_{be}(t)$$

Small Signal Equivalent Circuits and Parameters for the BJT

$$i_B = (1-\alpha) I_{ES} (e^{\frac{v_{BE}}{V_T}} - 1)$$

$$I_{BQ} + i_b(t) = (1-\alpha) I_{ES} (e^{\frac{V_{BE} + v_{be}(t)}{V_T}} - 1)$$

Ignoring the -1 term since it is neglible compared to the exponential term

$$I_{BQ} + i_b(t) = (1-\alpha) I_{ES} (e^{\frac{V_{BE}}{V_T}} e^{\frac{v_{be}(t)}{V_T}})$$

Noting that the Q-point values must also satisfy the Shockley equation,

$$I_{BQ} = (1-\alpha) I_{ES} e^{\frac{V_{BE}}{V_T}}$$

$$\therefore I_{BQ} + i_b(t) = I_{BQ} e^{\frac{v_{be}(t)}{V_T}}$$

Using the first two terms of the Taylor expansion of the exponential function: $e^x \approx 1 + x$

$$I_{BQ} + i_b(t) \approx I_{BQ}(1 + \frac{v_{be}(t)}{V_T}) = I_{BQ} + I_{BQ} \frac{v_{be}(t)}{V_T}$$

$$i_b(t) = I_{BQ} \frac{v_{be}(t)}{V_T}$$

Small Signal Equivalent Circuits and Parameters for the BJT

This is a relationship of the AC portion of the base current to the base - emitter voltage:

$$i_b(t) = I_{BQ}\frac{v_{be}(t)}{V_T} \Rightarrow v_{be}(t) = i_b(t)\frac{V_T}{I_{BQ}} = i_b(t)r_\pi$$

$$r_\pi = \frac{V_T}{I_{BQ}}$$

This resistance is a function of the Q - point and represents the input resistance of our equivalent circuit.

Secondly, since the collector current $i_C(t) = \beta i_B(t)$

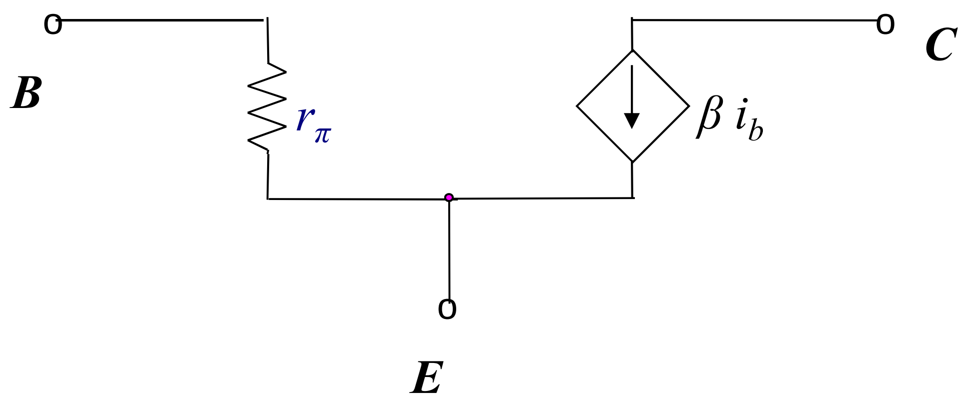
$$I_C + i_c(t) = \beta I_B + \beta i_b(t)$$

$$\therefore i_c(t) = \beta i_b(t)$$

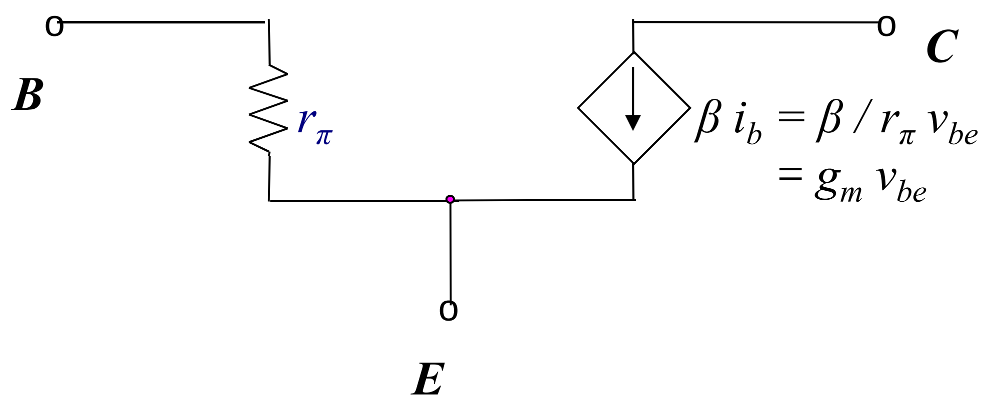
And we have a dependent current source in the collector to represent this relationship.

Small Signal Equivalent Circuits and Parameters for the BJT

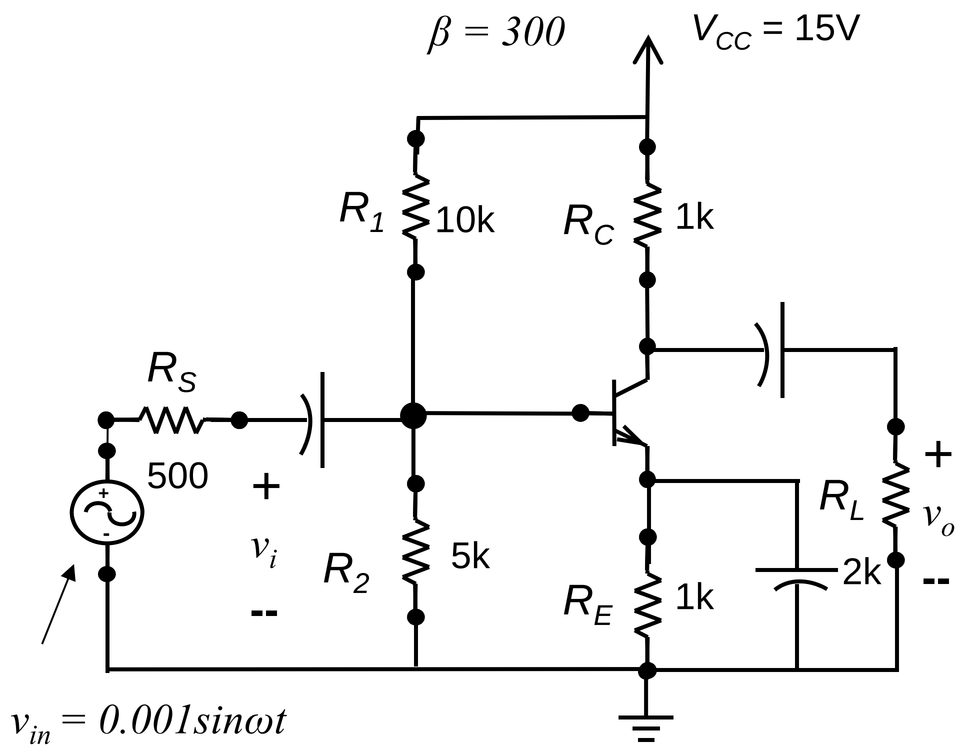
- Here is our equivalent circuit:



- Here is a second version equivalent circuit:

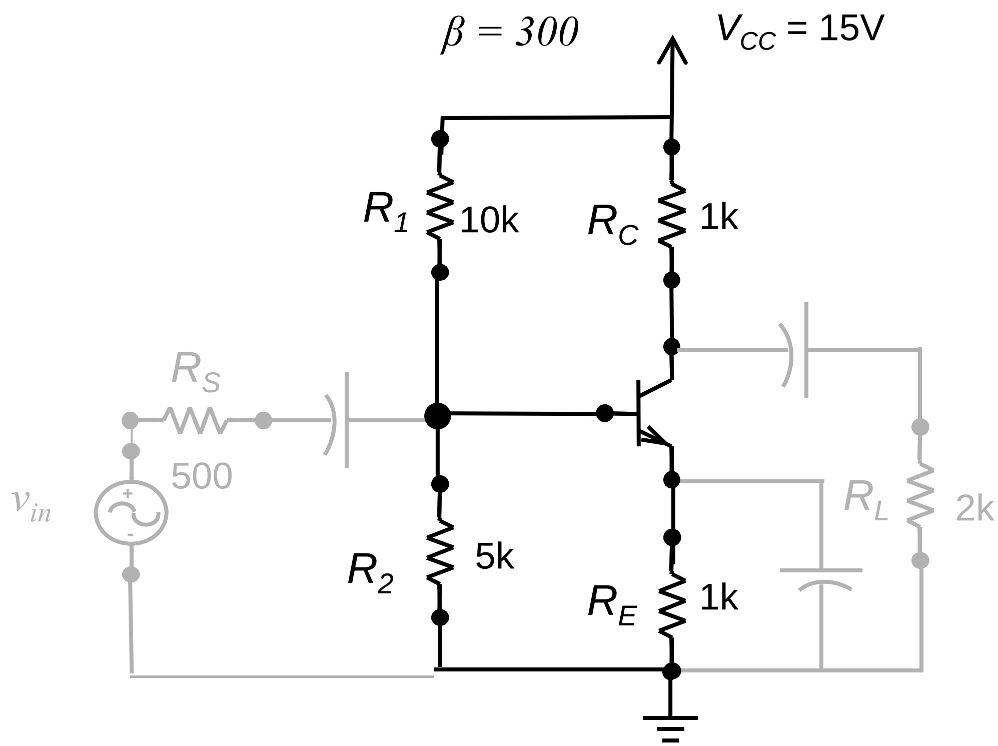


Small Signal Equivalent Circuits Example



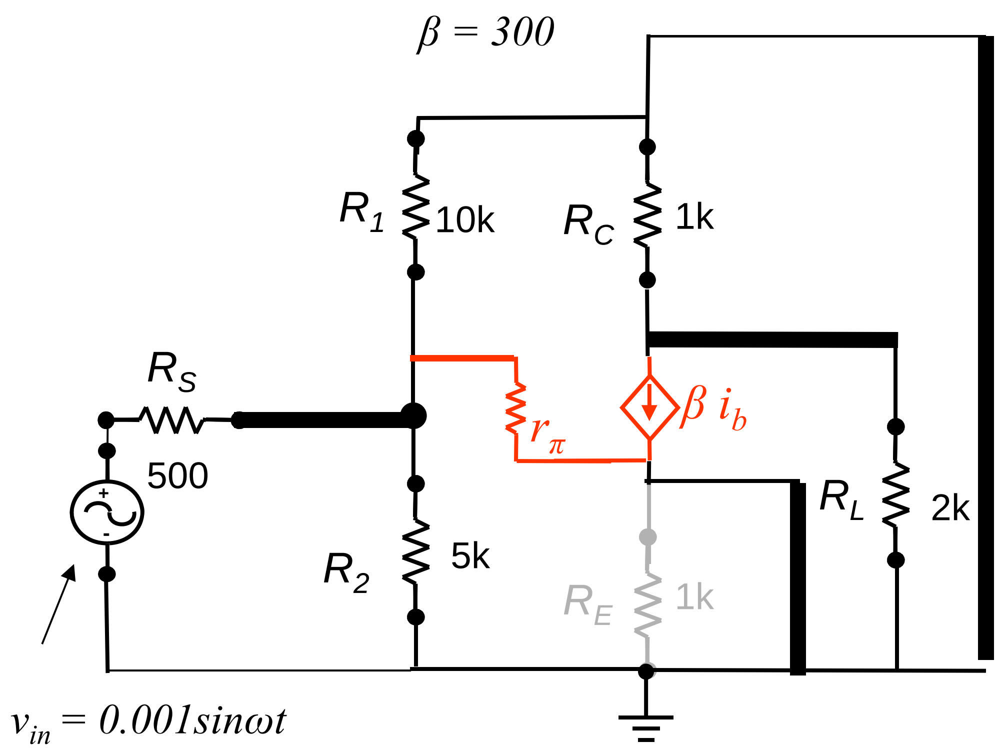
- We see that we have capacitors and extra resistors.
- The extra resistors represent
  - the load to the next stage (input impedance of the next stage)
  - the source resistance (output impedance of the previous stage)
- The extra capacitors are to:
  - protect the DC design of our amplifier
  - allow the passing the AC portion of the input signal from the source to our amplifier and on to the load (coupling capacitors)
  - eliminate the loading of the emitter resistor for the AC signal (bypass capacitors)

Small Signal Equivalent Circuits - Example DC Analysis



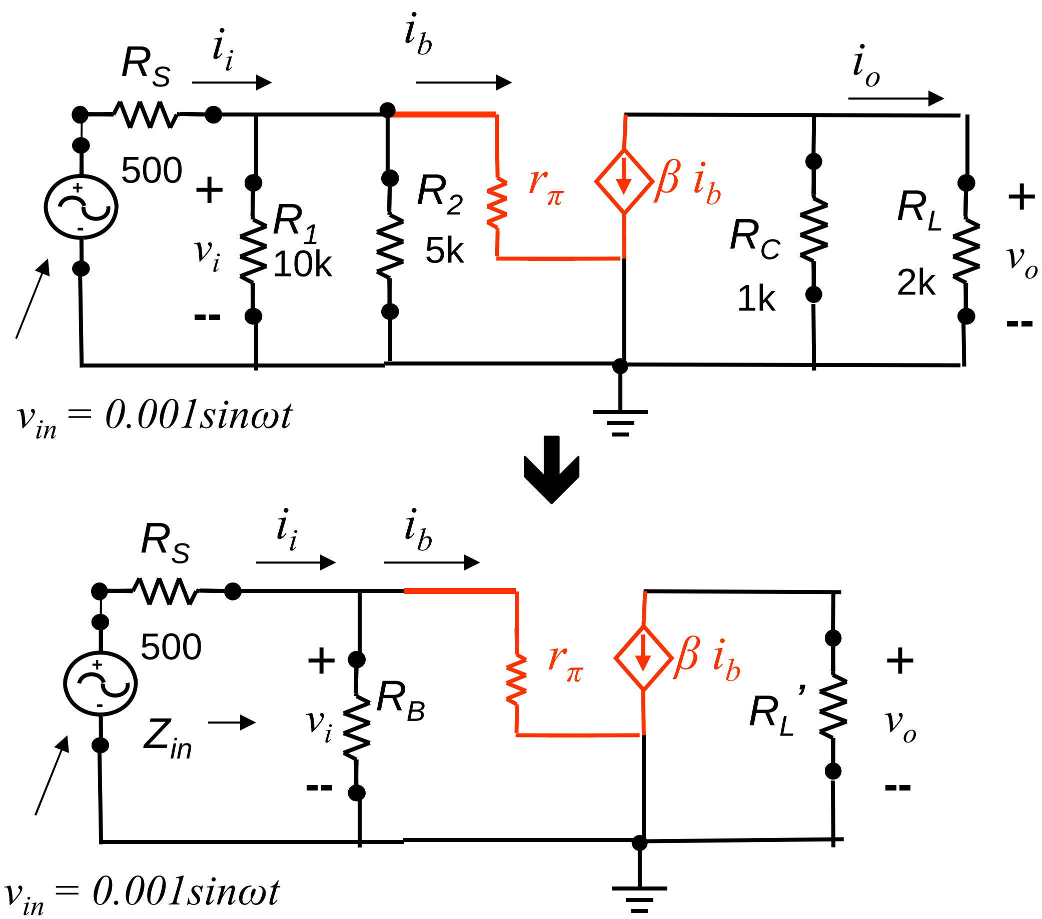
- For DC the capacitors are open-circuits so for DC analysis the circuit becomes the same circuit as we have just analyzed
- The components not affected by the DC voltage are “grayed out”.
- The next step is to perform the AC analysis.

Small Signal Equivalent Circuits-Example AC Analysis



- The next step is to perform the AC analysis by
  - Shorting any DC voltage sources
  - Opening any DC current sources
  - Shorting the capacitors
  - Eliminating any resistors which are shorted by the capacitors (e.g., the emitter resistor
  - Connecting the source, source resistance, and load resistance
  - Replace the transistor with its small signal equivalent circuit.
- Short circuits are drawn in thick lines, the removed capacitor is "greyed-out", and the equivalent circuit is in red.
- This circuit looks too messy, let's redraw

Small Signal Equivalent Circuits-Example
AC Analysis



$$R_L{}' = R_c \parallel R_L = \frac{R_c R_L}{R_c + R_L} = \frac{1 \times 2k}{1+2} = 667$$

$$R_B = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2} = \frac{5 \times 10k}{5+10} = 3.33k$$

$$r_\pi = \frac{V_T}{I_{BQ}} = \frac{26m}{14.1\mu} = 1844$$

$$v_o = -\beta i_b R_L{}'$$

$$v_i = i_b r_\pi$$

$$A_v = \frac{v_o}{v_i} = -\frac{\beta R_L{}'}{r_\pi} = -109$$

$$Z_{in} = R_B \parallel r_\pi = \frac{R_B r_\pi}{R_B + r_\pi} = 1.19k$$

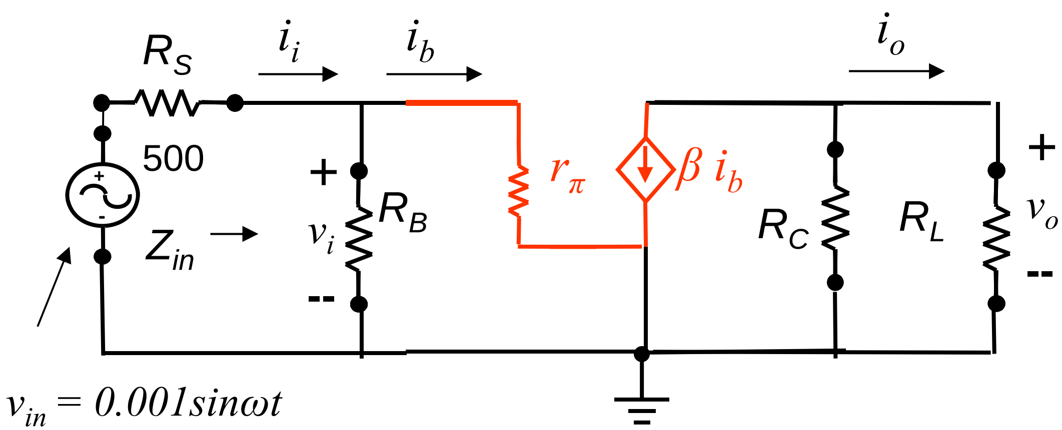
$$i_o = \frac{v_o}{R_L} = \frac{-\beta i_b R_L{}'}{R_L} = -\frac{\beta R_L{}'}{R_L}\frac{v_i}{r_\pi} = A_v \frac{v_i}{R_L}$$

$$i_i = \frac{v_i}{Z_{in}}$$

$$A_i = \frac{i_o}{i_i} = A_v \frac{\frac{v_i}{R_L}}{\frac{v_i}{Z_{in}}} = A_v \frac{Z_{in}}{R_L} = -64.4$$

$$G = A_i A_v = 7000$$

Small Signal Equivalent Circuits-Example
AC Analysis Alternative Method for A_i



$$R_L' = R_c \parallel R_L = \frac{R_c R_L}{R_c + R_L} = \frac{1 \times 2k}{1+2} = 667$$

$$R_B = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2} = \frac{5 \times 10k}{5+10} = 3.33k$$

$$r_\pi = \frac{V_T}{I_{BQ}} = \frac{26m}{14.1\mu} = 1844$$

By current division

$$i_o = \frac{R_C}{R_C + R_L}(-\beta i_b)$$

$$i_b = \frac{R_B}{R_B + r_\pi} i_i$$

$$i_o = \frac{R_C}{R_C + R_L}(-\beta \frac{R_B}{R_B + r_\pi}) i_i$$

$$A_i = \frac{i_o}{i_i} = -\beta \frac{R_C}{R_C + R_L} \times \frac{R_B}{R_B + r_\pi}$$

$$= -300 \frac{1k}{1k + 2k} \times \frac{3.33k}{3.33k + 1.844k} = -64.36$$

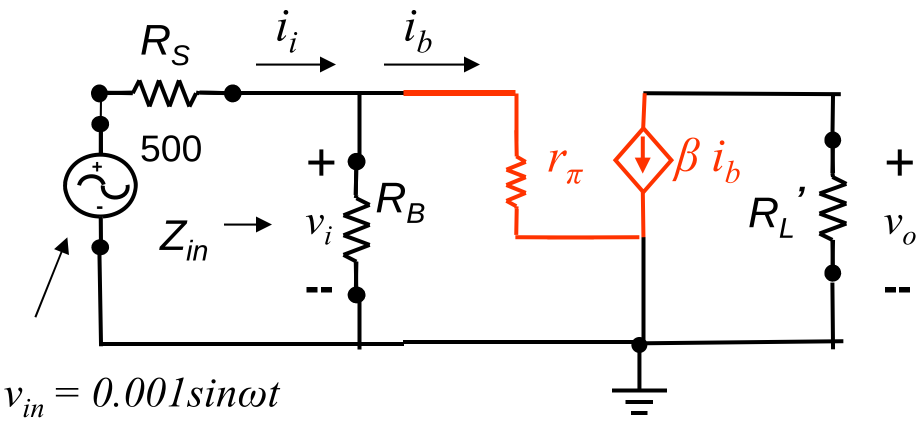
Note they are the same:

$$A_i = A_v \frac{Z_{in}}{R_L} = -\frac{\beta R_L'}{r_\pi R_L} \times \frac{R_B r_\pi}{R_B + r_\pi}$$

$$= -\frac{\beta \frac{R_C R_L}{R_C + R_L}}{r_\pi R_L} \times \frac{R_B r_\pi}{R_B + r_\pi}$$

$$= -\beta \frac{R_C}{R_C + R_L} \times \frac{R_B}{R_B + r_\pi}$$

Small Signal Equivalent Circuits-Example AC Analysis



$$A_v = \frac{v_o}{v_i} = -\frac{\beta R_L{}'}{r_\pi} = -109$$

$$Z_{in} = R_B \parallel r_\pi = \frac{R_B r_\pi}{R_B + r_\pi} = 1.19k$$

$$A_{vs} = \frac{v_o}{v_{in}} = \frac{v_o}{v_i}\frac{v_i}{v_{in}}$$

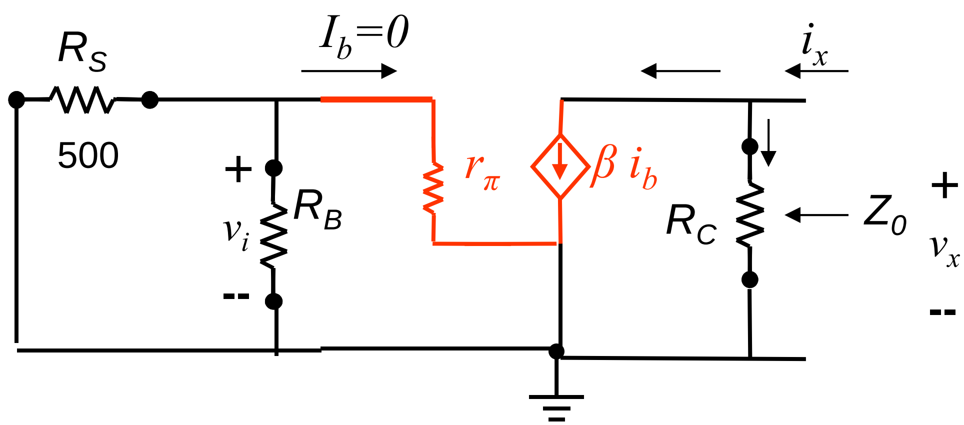
$$v_i = \frac{Z_{in}}{Z_{in} + R_s} v_{in}$$

$$\frac{v_i}{v_{in}} = \frac{Z_{in}}{Z_{in} + R_s} = .7$$

$$A_{vs} = \frac{v_o}{v_{in}} = \frac{v_o}{v_i}\frac{v_i}{v_{in}} = A_v \frac{v_i}{v_{in}} = A_v \frac{Z_{in}}{Z_{in} + R_s} = -76.4$$

$$v_o = -76.4 v_i = -76.4 \sin \omega t \; mV$$

Small Signal Equivalent Circuits-Example
AC Analysis



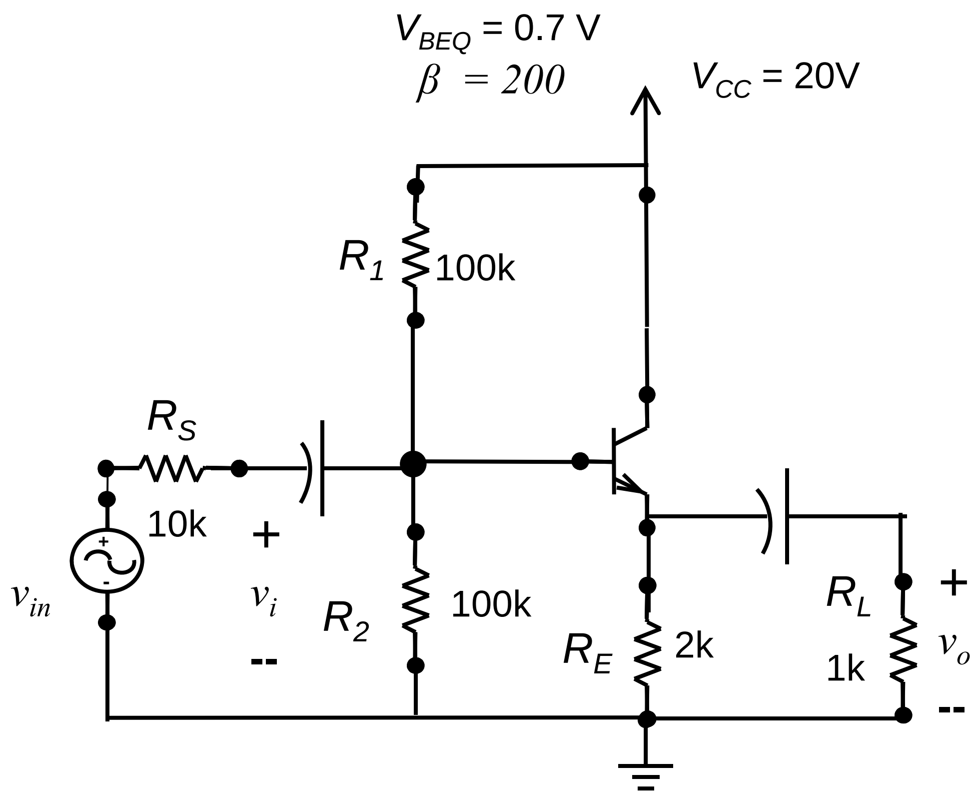
$$Z_o = \frac{v_x}{i_x}$$

$$i_x = \frac{v_x}{R_c} + \beta\, i_b$$

$$i_b = 0;\ i_x = \frac{v_x}{R_c}$$

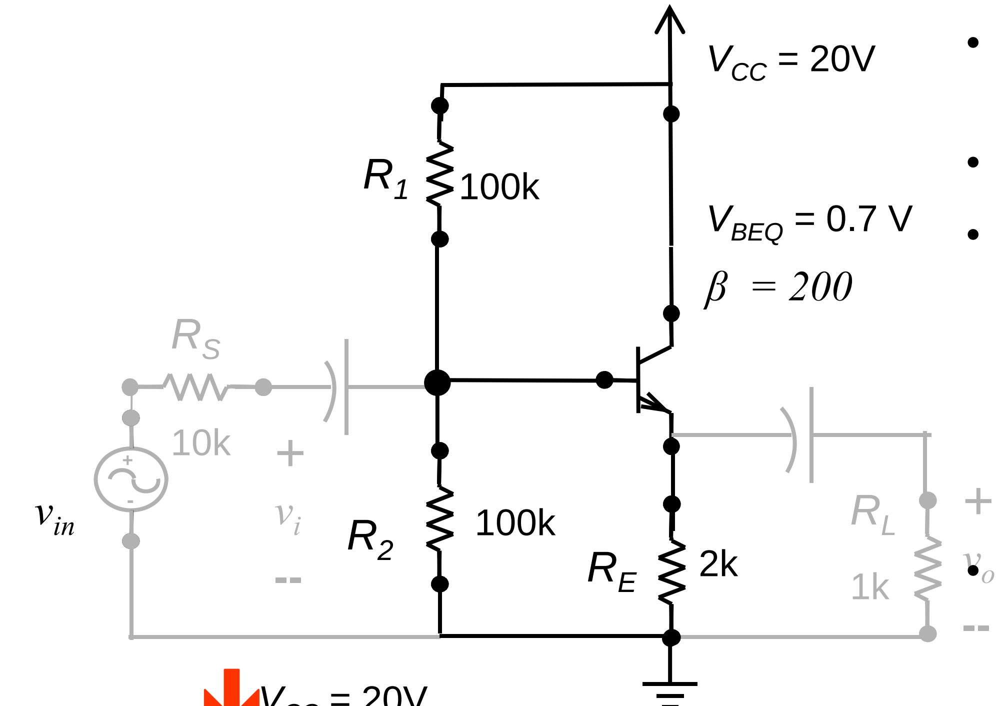
$$Z_o = \frac{v_x}{i_x} = R_c = 1k$$

Small Signal Equivalent Circuits
Emitter Follower Example



- We see that the load is across the emitter resistor
- The extra resistors and capacitors are represent
  - the load to the next stage (input impedance of the next stage)
  - the source resistance (output impedance of the previous stage)
  - Coupling capacitors
- Our analysis plan
  - First, DC analysis to determine Q-point and equivalent circuit parameters
  - AC analysis to calculate the gains

Small Signal Equivalent Circuits
Emitter Follower Example DC Analysis

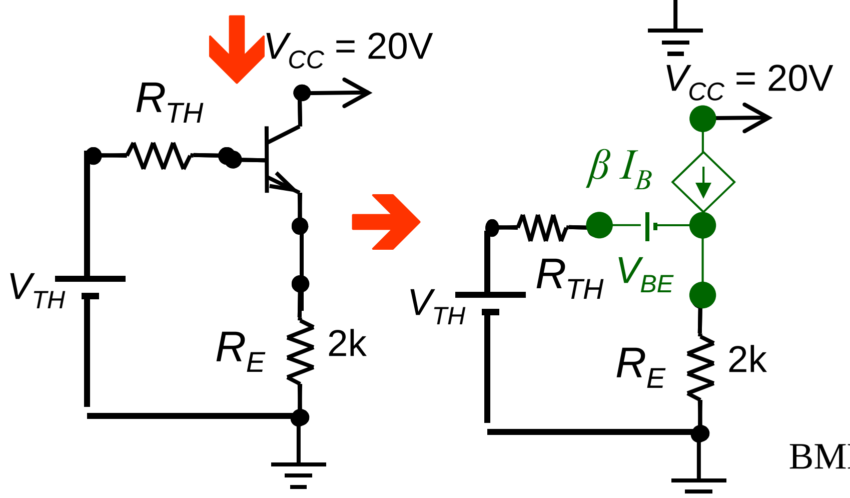


- Remove the circuit elements which are not affects by the DC voltages
- Opening the coupling capacitors
- Redraw the circuit and replace the base circuit with its Thevinen's equivalent

$$R_{TH} = R_1 \parallel R_2 = 50k$$

$$V_{TH} = \frac{R_2}{R_2 + R_1} V_{CC} = 10V$$

- Next, use the DC equivalent circuit for the active region and then write KVL for the base circuit



$$V_{TH} = I_{BQ}R_{TH} + V_{BEQ} + I_{EQ}R_E = I_{BQ}R_{TH} + V_{BEQ} + (1+\beta)I_{BQ}R_E$$

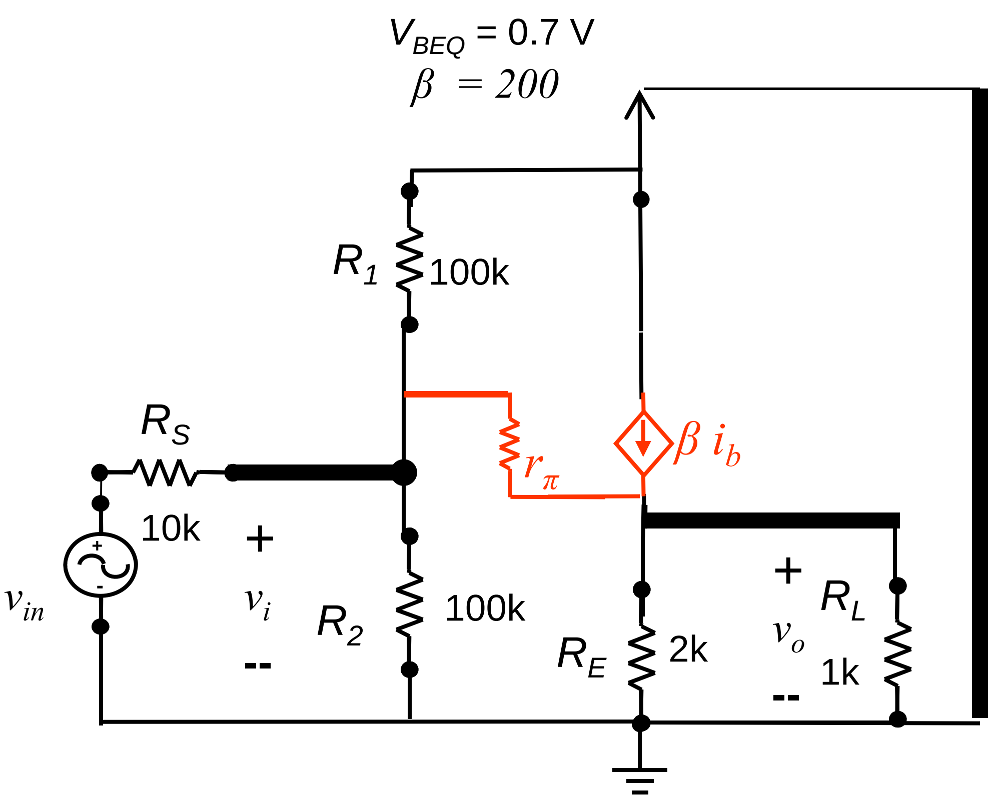
$$I_{BQ} = \frac{V_{TH} - V_{BEQ}}{R_T + (1+\beta)R_E} = \frac{10 - 0.7}{50k + (1+200)2k} = 20.6\mu A$$

$$I_{CQ} = \beta\, I_{BQ} = 4.12mA;\ I_{EQ} = (1+\beta)I_{BQ} = 4.14mA$$

$$V_{CEQ} = V_{CC} - I_{EQ}R_E = 11.7V;\ r_\pi = \frac{V_T}{I_{BEQ}} = \frac{26m}{20.6\mu} = 1260$$

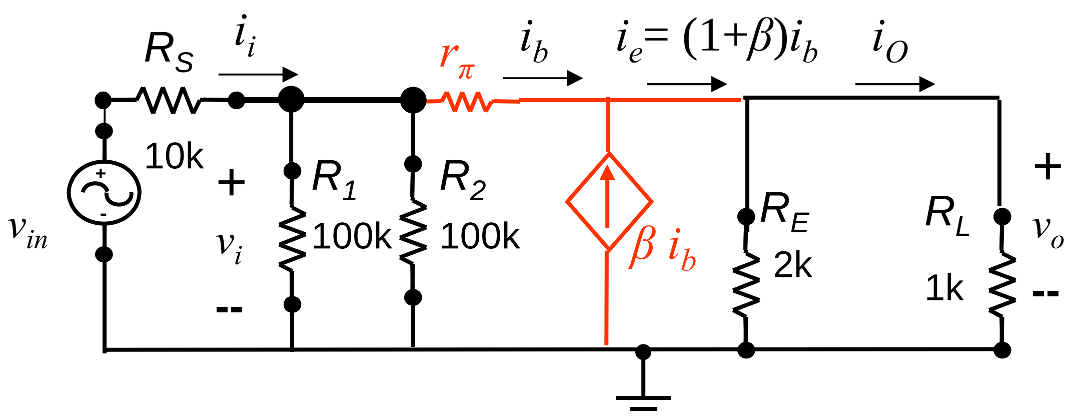
100k

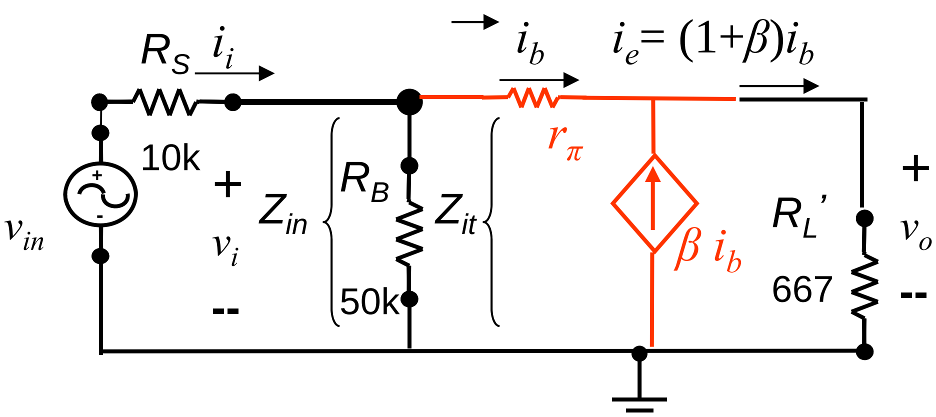
Small Signal Equivalent Circuits
Emitter Follower Example AC Analysis



- The next step is to perform the AC analysis by
  - Shorting any DC voltage sources
  - Opening any DC current sources
  - Shorting the capacitors
  - Connecting the source, source resistance, and load resistance
  - Replace the transistor with its small signal equivalent circuit.
- And Redraw to simplify

Small Signal Equivalent Circuits
Emitter Follower Example AC Analysis





$$R_B = R_1 \,\|\, R_2 = 50k$$

$$R_L' = R_E \,\|\, R_L = 667$$

$$v_o = (1+\beta) i_b R_L'$$

$$v_i = i_b r_\pi + v_o = \frac{v_o}{(1+\beta)R_L'} r_\pi + v_o$$

$$A_v = \frac{v_o}{v_i} = \frac{(1+\beta)R_L'}{(1+\beta)R_L' + r_\pi}$$

$$A_v = \frac{(1+200)\times 667}{(1+200)\times 667 + 1260} = .991$$

$$Z_{it} = \frac{v_i}{i_b} = r_\pi + (1+\beta)R_L' = 134k$$

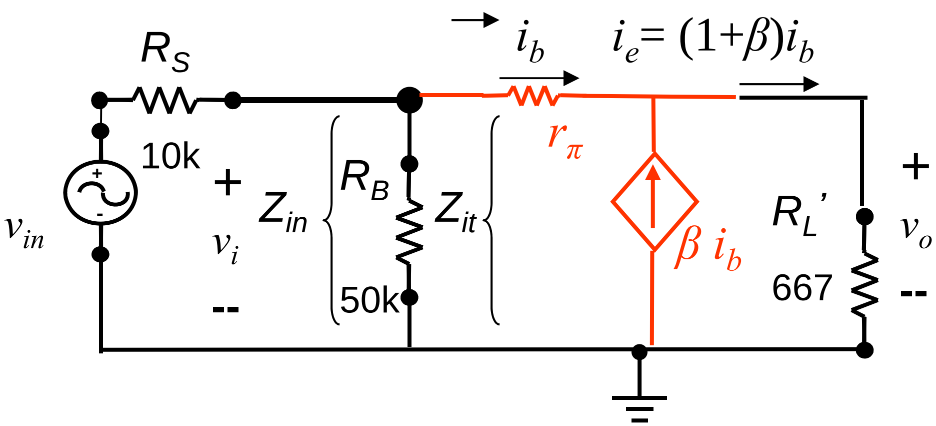
$$Z_{in} = R_B \,\|\, Z_{it} = 36.5k$$

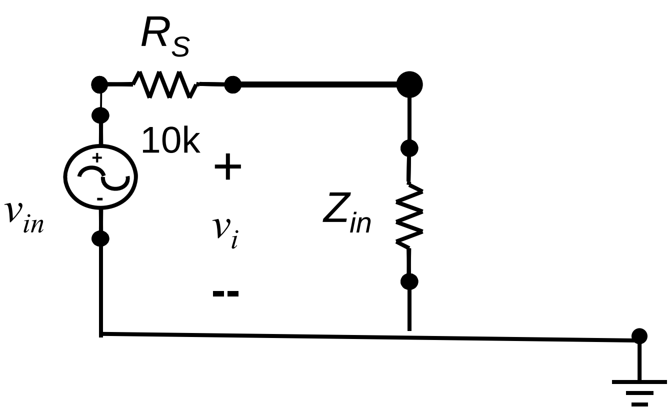
$$i_o = \frac{v_o}{R_L} = \frac{A_v v_i}{R_L}; \; i_i = \frac{v_i}{Z_{in}}$$

$$A_i = \frac{i_o}{i_i} = A_v \frac{Z_{in}}{R_L} = 36.2$$

$$G = A_v A_i = 35.8$$

Small Signal Equivalent Circuits
Emitter Follower Example AC Analysis





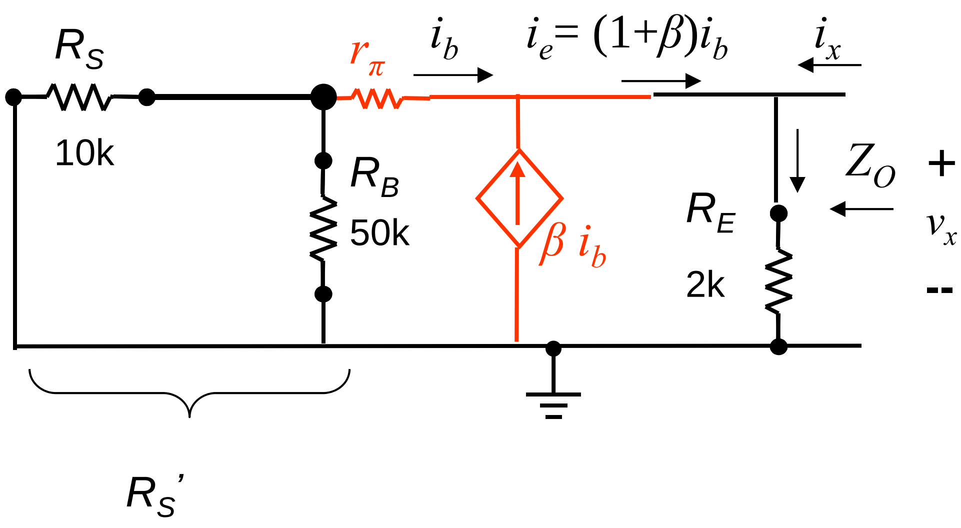
$$A_v = \frac{v_o}{v_i} = \frac{(1+200)\times 667}{(1+200)\times 667 + 1260} = .991$$

$$A_{vs} = \frac{v_o}{v_s} = \frac{v_o}{v_i}\frac{v_i}{v_s} = 0.991\frac{v_i}{v_s}$$

$$\frac{v_i}{v_s} = \frac{Z_{in}}{Z_{in}+R_s} = \frac{36.5}{36.5+10} = 0.78$$

$$A_{vs} = \frac{v_o}{v_s} = \frac{v_o}{v_i}\frac{v_i}{v_s} = 0.991\times 0.78 = 0.78$$

Small Signal Equivalent Circuits Emitter Follower Example AC Analysis



$$Z_o = \frac{v_x}{i_x}$$

$$i_x = \frac{v_x}{R_E} - i_e = \frac{v_x}{R_E} - (1+\beta)i_b$$

$$R_S' = R_S \parallel R_B = 8.33k$$

$$v_x = -i_b(R_S' + r_\pi)$$

$$i_x = \frac{v_x}{R_E} + (1+\beta)\frac{v_x}{R_S' + r_\pi}$$

$$v_x = i_x\left(\frac{1}{1/R_E + (1+\beta)/(R_S' + r_\pi)}\right)$$

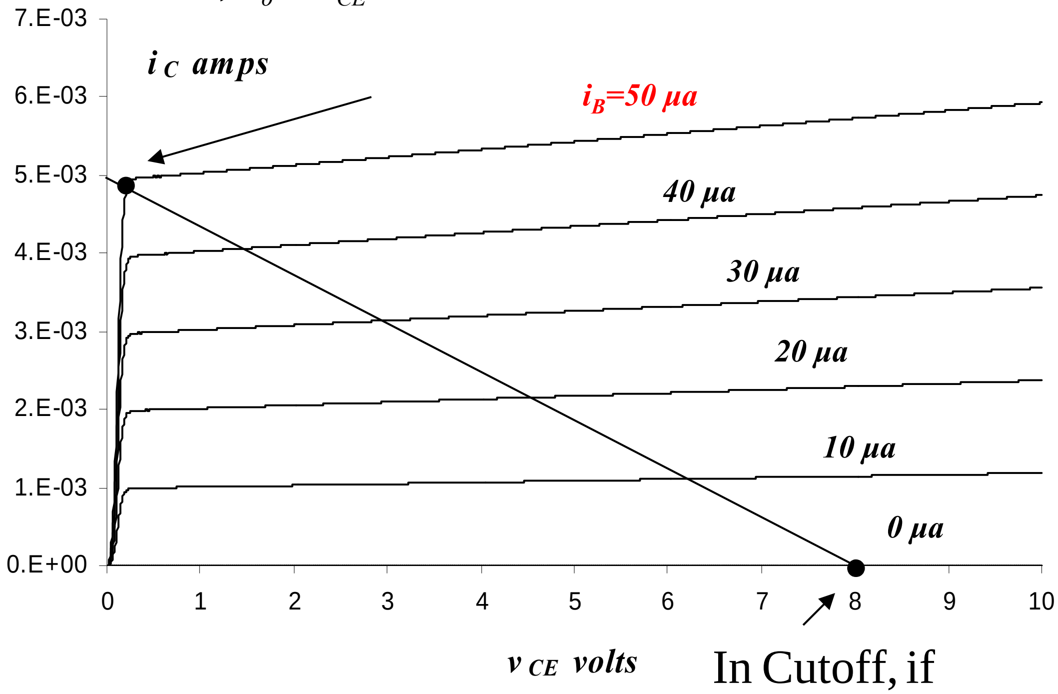
$$Z_o = \frac{1}{1/R_E + (1+\beta)/(R_S' + r_\pi)} \Rightarrow R_E \parallel \frac{R_S' + r_\pi}{1+\beta} = 46.6$$

The BJT as a Digital Switch

In Saturation, if

$$V_o = V_{CE} = V_{CC} - R_C i_C = V_{CC} - R_C \beta\ i_B = V_{CC} - R_C \beta\ \frac{V_{in} - V_{BEQ}}{R_B} < .2V$$

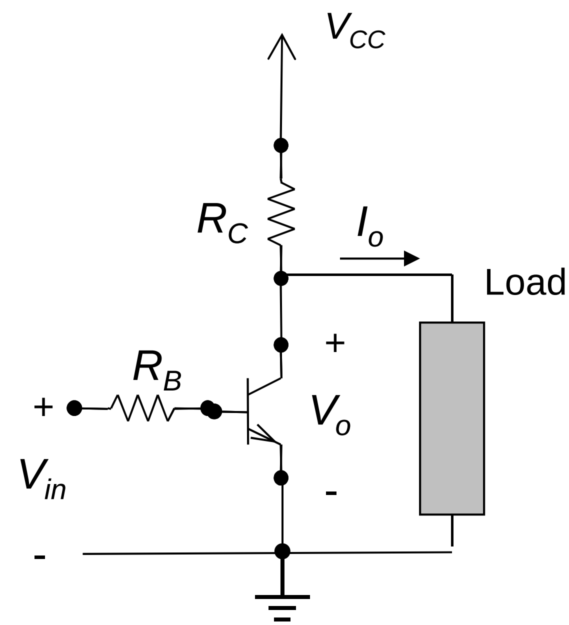
Then, $V_o = V_{CE} = .2$



In Cutoff, if

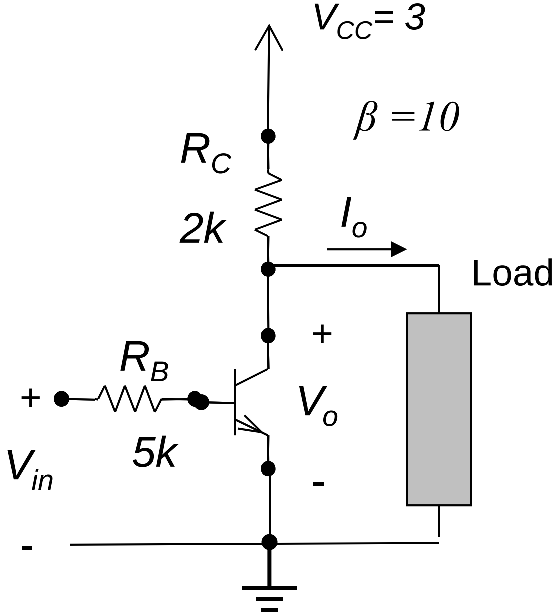
$V_{in} < .7V$

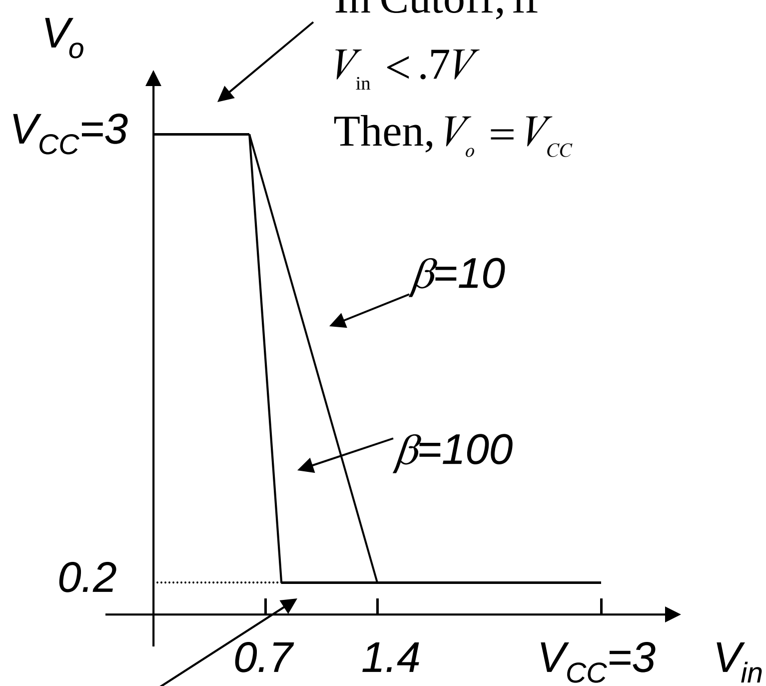
Then, $V_o = V_{CC}$



- Operating between cutoff and saturation (i.e., bypassing the active region), the BJT acts like an inverter.
- From this behavior, logic circuits such as NOR gates can be developed

The BJT as a Digital Switch





In Cutoff, if

$$V_{in} < .7V$$

Then, $V_o = V_{CC}$

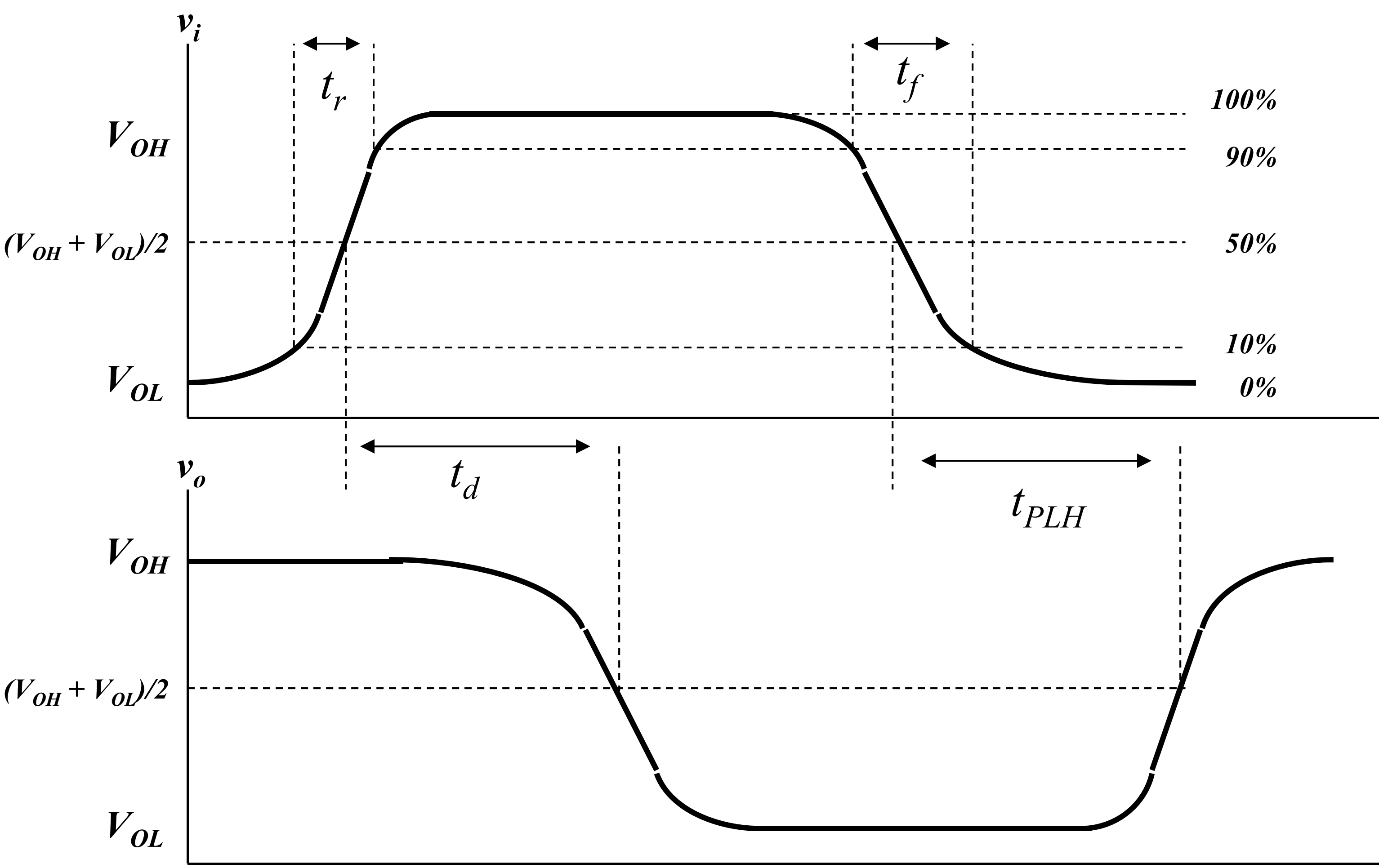
In Saturation, if

$$V_o = V_{CC} - R_C\beta\frac{V_{in} - V_{BEQ}}{R_B} < .2V$$

$$V_o = 3 - 2\times\beta\frac{V_{in} - .7}{5} = .2$$

$$V_{in} = 5\times\frac{3-.2}{2\beta} + .7 = \frac{7}{\beta} + .7$$

Switching and Timing



t_r = rise time

t_f = fall time

t_d = delay time (propagation delay from High to Low)

t_s = storage time (propagation delay from Low to High)

Homework

- Probs. 4.40, 4.42, 4.43, 4.45, 4.46, 4.51, 4.53, 4.54, 4.56,