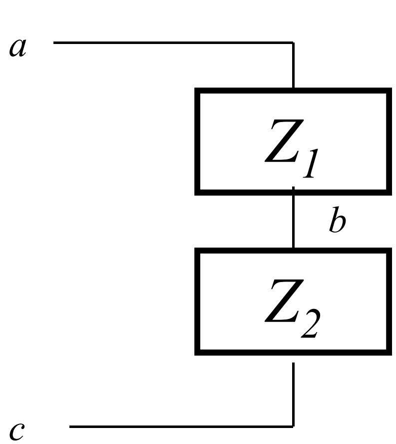
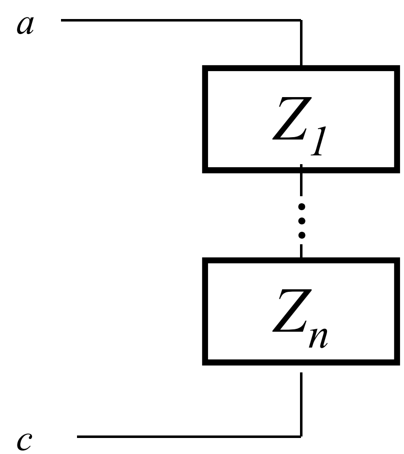
Circuit Analysis

Lesson #2

Voltage Division

- The voltage across impedances in series divides in proportion to the impedances.





$$\mathbf{V}_{ac} = \mathbf{V}_{ab} + \mathbf{V}_{bc} = \mathbf{I}(Z_1 + Z_2); \text{KVL} + \text{Ohm's Law}$$

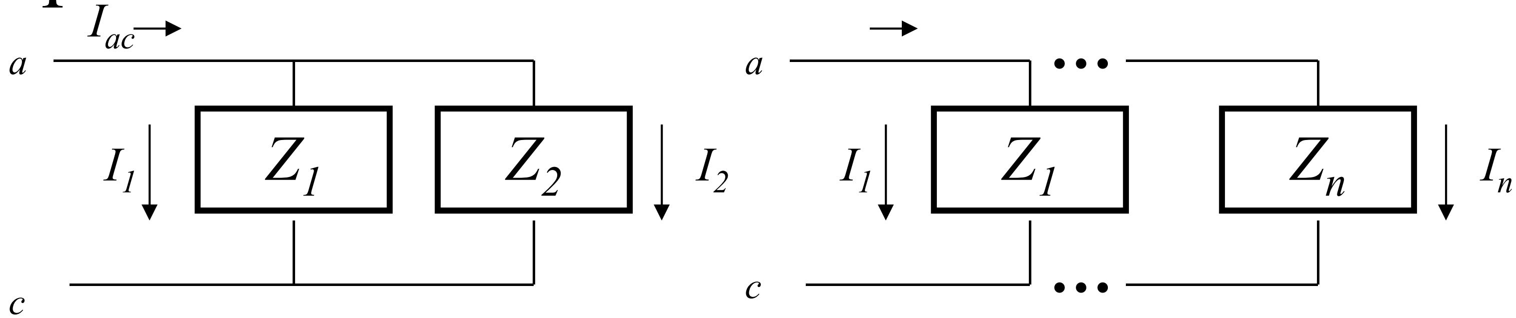
$$\mathbf{V}_{bc} = \mathbf{I}Z_2$$

$$\frac{\mathbf{V}_{bc}}{\mathbf{V}_{ac}} = \frac{Z_2}{Z_1 + Z_2}$$

$$\frac{\mathbf{V}_i}{\mathbf{V}_{ac}} = \frac{Z_i}{Z_1 + Z_2 + \cdots Z_n}$$

Current Division

- The current into impedances in parallel divides in proportion to the inverse of the impedances.



$$\mathbf{I}_{ac} = \mathbf{I}_1 + \mathbf{I}_2 = \mathbf{V}(\frac{1}{Z_1} + \frac{1}{Z_2}); \text{KCL} + \text{Ohm's Law}$$

$$\mathbf{I}_1 = \frac{\mathbf{V}}{Z_1}$$

$$\frac{\mathbf{I}_1}{\mathbf{I}_{ac}} = \frac{1/Z_1}{(1/Z_1) + (1/Z_2)} = \frac{Z_2}{Z_1 + Z_2}$$

$$\frac{\mathbf{I}_i}{\mathbf{I}_{ac}} = \frac{(1/Z_i)}{(1/Z_1) + (1/Z_2) + \cdots + (1/Z_n)}$$

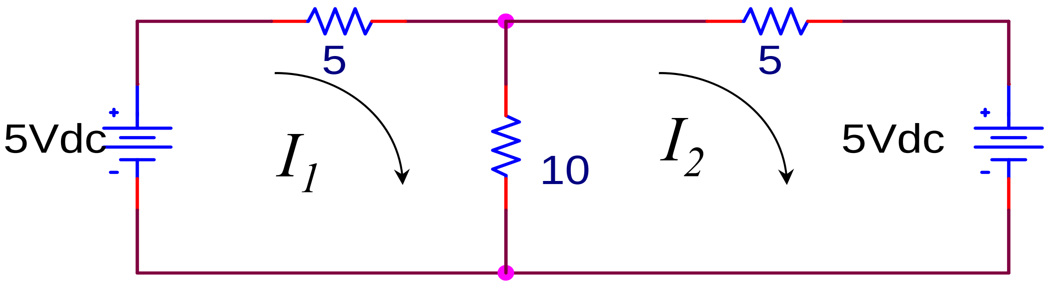
Mesh Analysis

1. Define a current in each mesh (loop) of a network. For example, in a 5 mesh network, define 5 current unknowns.
2. Using KVL, write an equation for each mesh using the unknown currents. In our 5 mesh example, you’ll have 5 equations and 5 unknown currents.
3. Solve for the unknown currents and now apply these currents to the network to find the voltages for each impedance in the network.

Nodal Analysis

1. Define a voltage at each node (junction point) of a network. For example, in a 5 node network, define 5 voltage unknowns.
2. Using KCL, write an equation for each node using the unknown voltages. In our 5 node example, you'll have 5 equations and 5 unknown voltage.
3. Solve for the unknown voltages and now apply these voltages to the network to find the currents for each impedance in the network.

Mesh Analysis Example



Mesh 1

$$0 = I_1 5 + 10(I_1 - I_2) - 5;\ 5 = 15I_1 - 10I_2$$

Mesh 2

$$0 = I_2 5 + 5 + 10(I_2 - I_1);\ -5 = 15I_2 - 10I_1$$

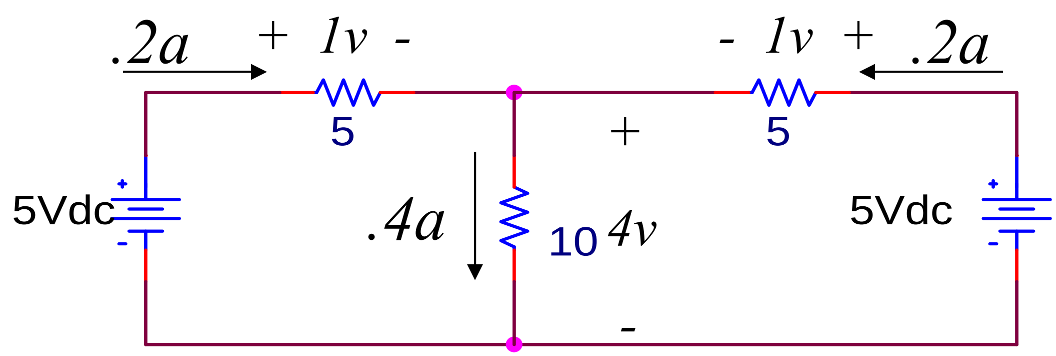
$$I_1 = \frac{1}{5};\ I_2 = -\frac{1}{5}$$

Mesh Analysis

I1(15)+I2(-10) = 5

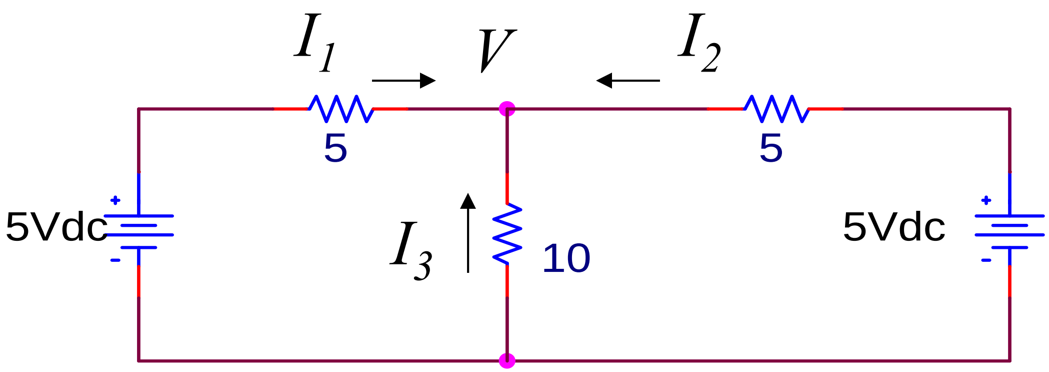
I1(-10)+I2(15) = -5

I1 = 0.2, I2 = -0.2




```
clear all;
R=[15 -10;-10 15];
V=[5;-5];
I=R\V;
grid off
axis off
text(.1,1,'Mesh Analysis')
text(.1,.9,['I1(',num2str(R(1,1)),')+I2(',num2str(R(1,2)),') = ',num2str(V(1))]);
text(.1,.8,['I1(',num2str(R(2,1)),')+I2(',num2str(R(2,2)),') = ',num2str(V(2))]);
text(.1,.7,['I1 = ',num2str(I(1)),',  I2 = ',num2str(I(2))])
```

Nodal Analysis Example



Node 1

$$I_1 + I_2 + I_3 = 0$$

$$\frac{5-V}{5} + \frac{5-V}{5} - \frac{V}{10} = 0; 2 = \frac{V}{2}$$

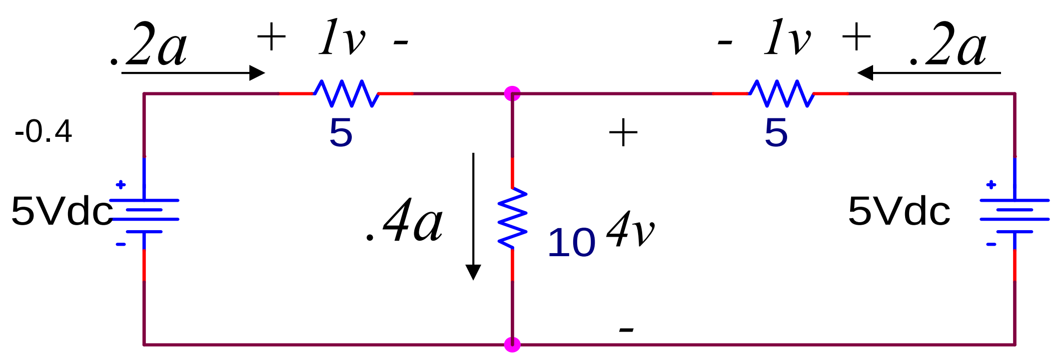
$$V = 4$$

Nodal Analysis

V = 4

I1 = 0.2, I2 = 0.2, I3 = -0.4

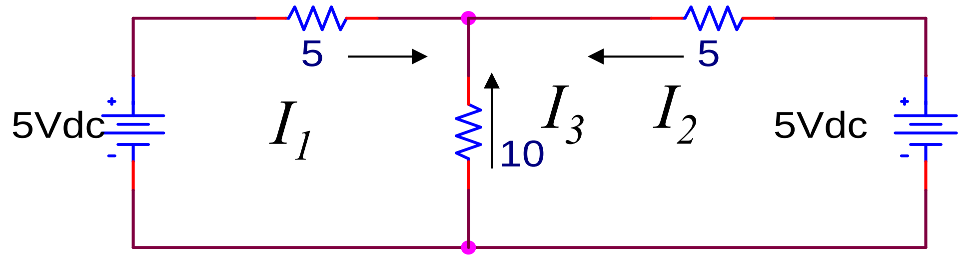
```
clear all;
R=[1/5+1/5+1/10];
VS=[10/5];
V=R\VS;
I1=(5-V(1))/5;I2=(5-V(1))/5;I3=-V(1)/10;
grid off
axis off
text(.1,1,'Nodal Analysis')
text(.1,.9,['V = ',num2str(V)]);
text(.1,.8,['I1 = ',num2str(I1),', I2 = ',num2str(I2),', I3 = ',num2str(I3)]);
```



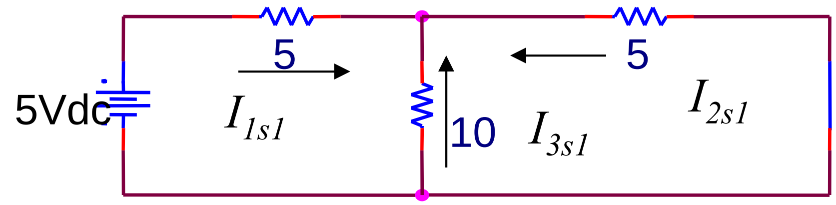
Superposition

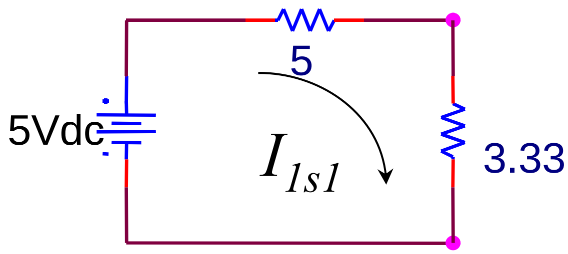
- Used to analyze a circuit with multiple sources.
- Steps:
  1. Set all sources except for one to zero (voltage sources are shorted-circuited, current sources are open-circuited)
  2. Solve for the currents and voltages for all of the circuit elements
  3. Repeat steps 1-2 for the remaining sources.
  4. Add each of the solutions to obtain the solution for the entire circuit

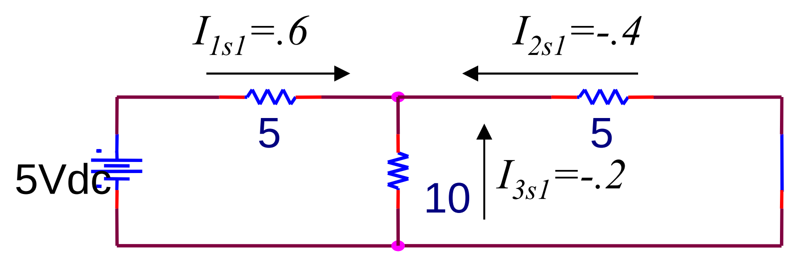
Superposition Analysis Example



Source 1







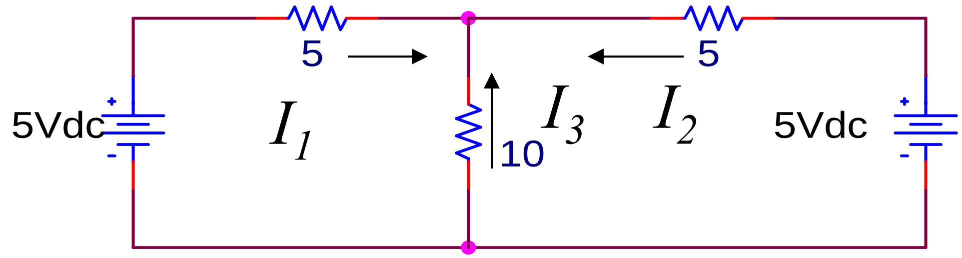
$$R_p = 10 \| 5 = \frac{10*5}{10+5} = \frac{50}{15} = \frac{10}{3} = 3.33\Omega$$

$$I_{1s1} = \frac{5}{5+\frac{10}{3}} = \frac{15}{25} = \frac{3}{5}$$

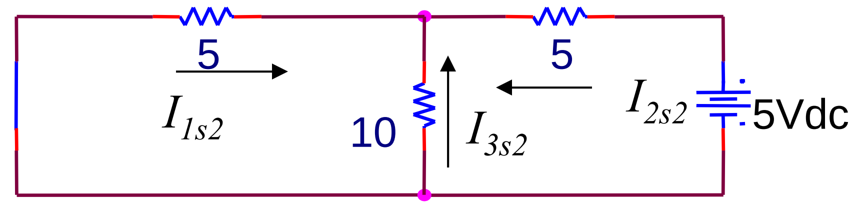
$$I_{2s1} = -\frac{10}{15}\times\frac{3}{5} = -\frac{2}{5} = -.4$$

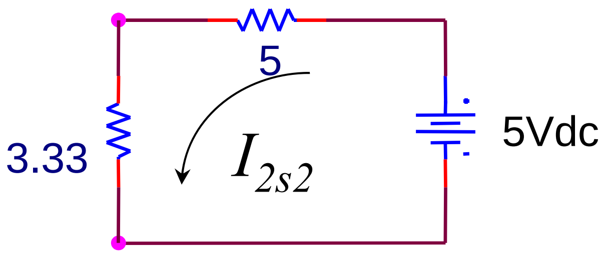
$$I_{3s1} = -\frac{5}{15}\times\frac{3}{5} = -\frac{1}{5} = -.2$$

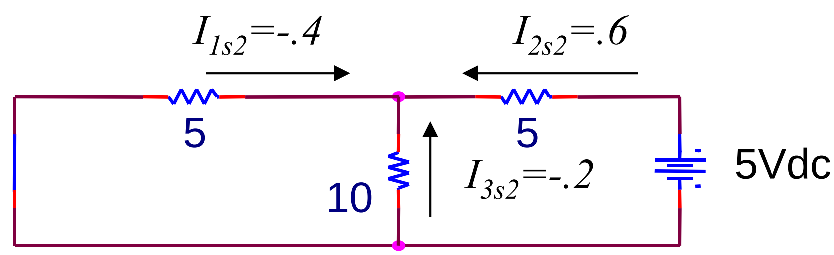
Superposition Analysis Example



Source 2







$$R_p = 10 \parallel 5 = \frac{10 * 5}{10 + 5} = \frac{50}{15} = \frac{10}{3} = 3.33\Omega$$

$$I_{2s2} = \frac{5}{5 + \frac{10}{3}} = \frac{15}{25} = \frac{3}{5}$$

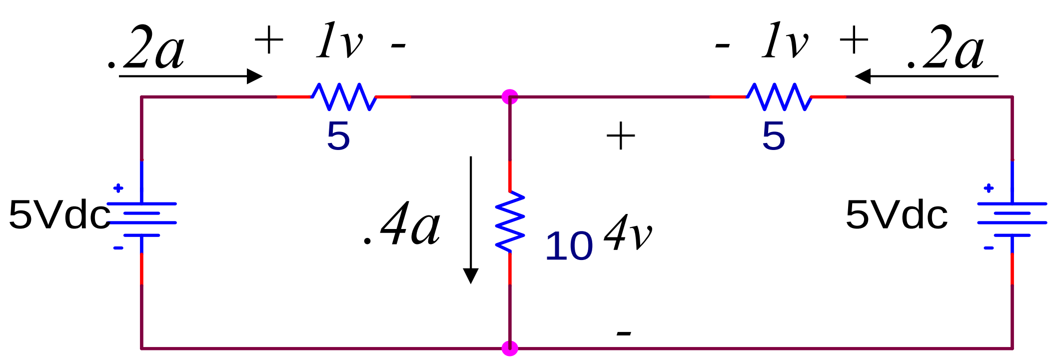
$$I_{1s2} = -\frac{10}{15} \times \frac{3}{5} = -\frac{2}{5} = -.4$$

$$I_{3s2} = -\frac{5}{15} \times \frac{3}{5} = -\frac{1}{5} = -.2$$

Superposition Analysis Example

- Summing the results of each solution:

$$I_1 = I_{1s1} + I_{1s2} = .6 - .4 = .2$$
$$I_2 = I_{2s1} + I_{2s2} = -.4 + .6 = .2$$
$$I_3 = I_{3s1} + I_{3s2} = -.2 + -.2 = -.4$$



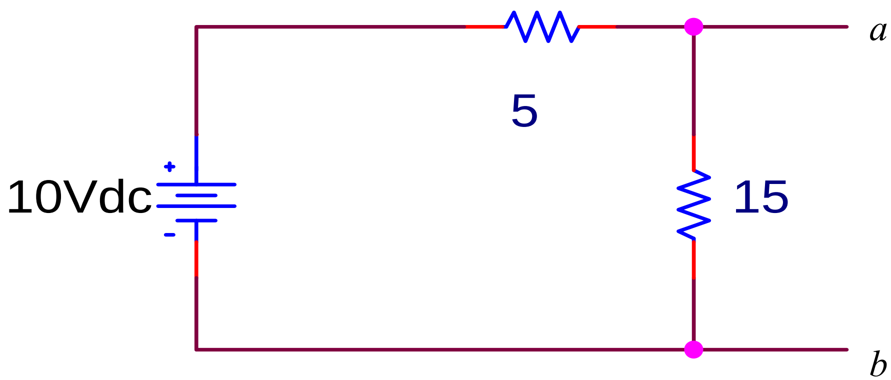
Thevenin and Norton Equivalent Circuits

- Thevenin's Theorem: Any circuit consisting of passive and active components can be represented by a voltage source in series with an equivalent set of passive components
  - The value of the voltage source equals the voltage seen at the output terminal without any load connected to it, i.e., the open-circuit voltage
  - The value of the equivalent set of passive components equals the impedance looking back into the terminals with the sources set to zero, i.e., the output impedance.

Thevenin and Norton Equivalent Circuits

- Norton's Theorem: Any circuit consisting of passive and active components can be represented by a current source in parallel with an equivalent set of passive components
  - The value of the current source equals the current seen at the output terminal shorted and without any load connected to it, i.e., the short-circuit current
  - The value of the equivalent set of passive components equals the impedance looking back into the terminals with the sources set to zero, i.e., the output impedance.
- Note that the Thevenin and Norton Equivalents Circuits are equivalent to each other when the value of the Thevenin's voltage source equals the product of the equivalent impedance times the Norton's current source

Thevenin and Norton Examples

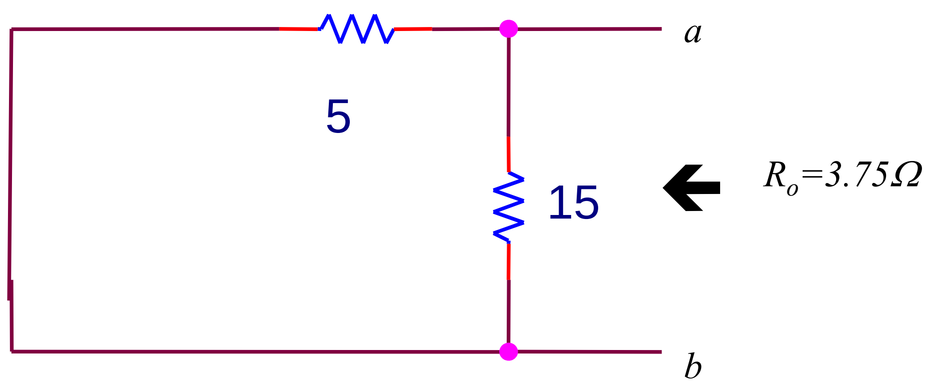


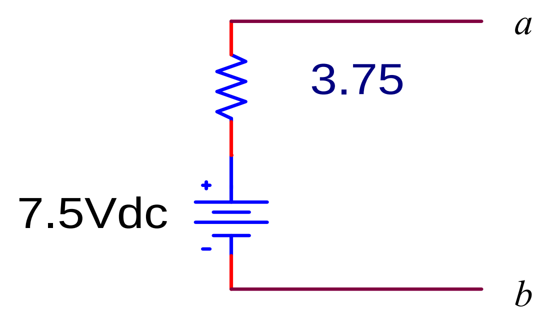
Open Circuit Voltage at terminals : ab

$$V_{abOC} = \frac{15}{5+15} \times 10 = 7.5v$$

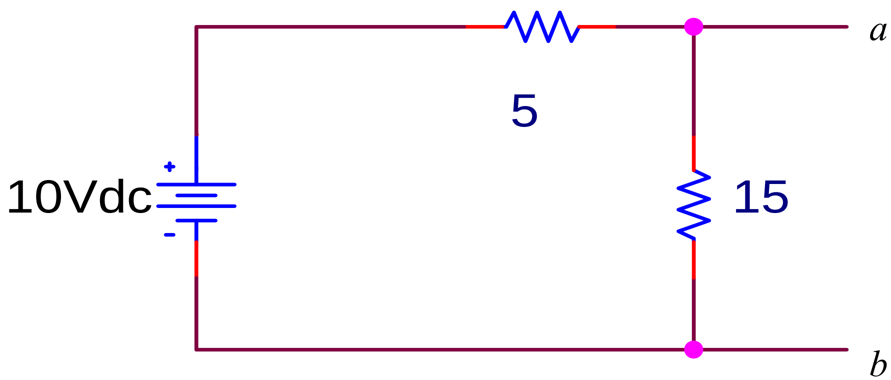
Output Impedance

$$R_o = 5 \,||\, 15 = \frac{5 \times 15}{20} = \frac{15}{4} = 3.75\Omega$$





Thevenin and Norton Examples

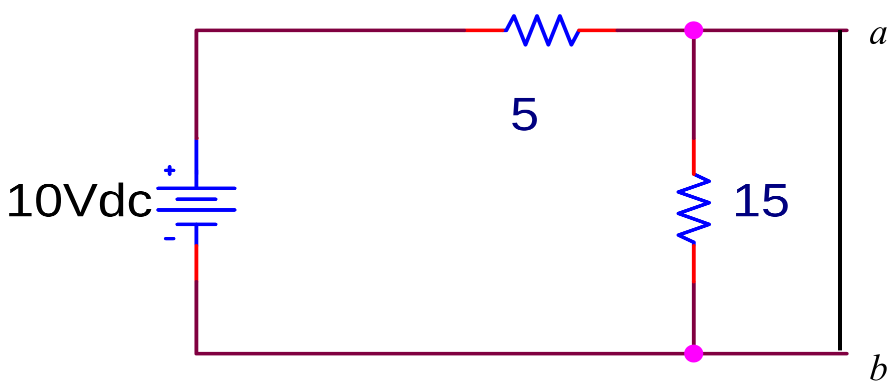


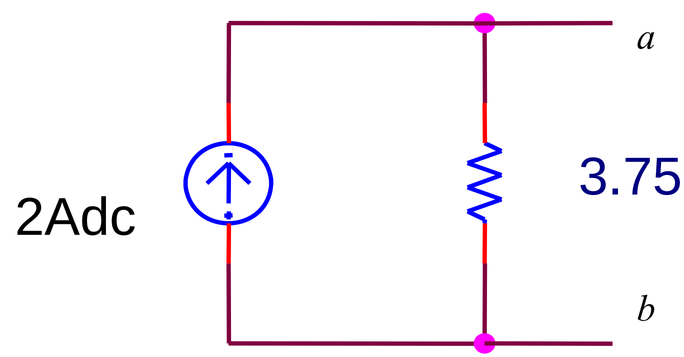
Short Circuit Current at terminals: ab

$$I_{abSC} = \frac{10}{5} = 2a$$

Output Impedance

$$R_o = 5 \,||\, 15 = \frac{5 \times 15}{20} = \frac{15}{4} = 3.75\Omega$$



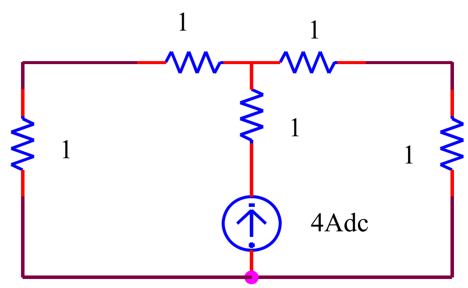


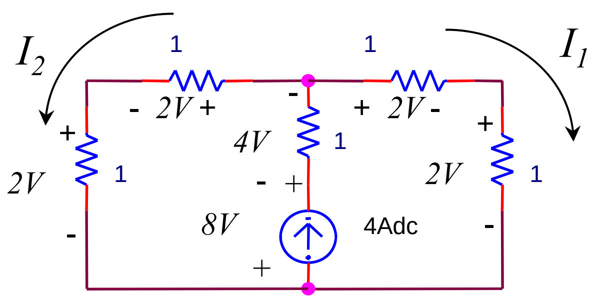
Input and Output Impedance of a Circuit

- Input impedance of a circuit is the impedance looking into the input terminals of the circuit with any load connected to the output and all internal sources are set to zero.
- Output impedance of a circuit is the impedance looking into the output terminals of the circuit without any load connected to the output and all internal and input sources are set to zero.

Examples

Find the currents and voltages in these circuits





$$I_1 = I_2 = \frac{2}{4} \times 4 = 2a$$

$$V_{1kBRANCH} = 1 \times 2a = 2V$$

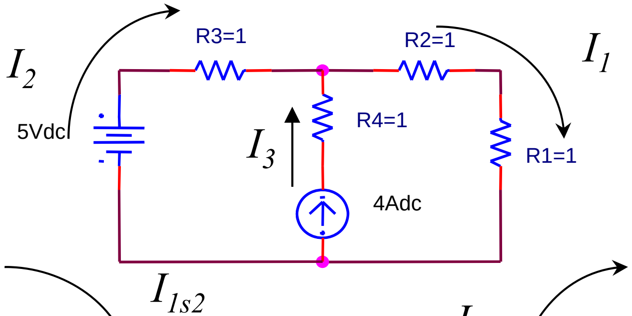
$$V_{1kSOURCEBRANCH} = 1 \times 4a = 4V$$

$$V_{CURRENTSOURCE} = 2 \times V_{1kBRANCH} + V_{1kSOURCEBRANCH} = 8V$$

Examples

$$I_{2s2} = I_{1s2} = \frac{5}{3} = 1.67A$$

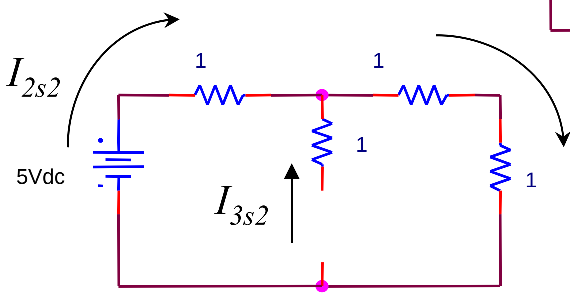
$$I_{3s2} = 0$$

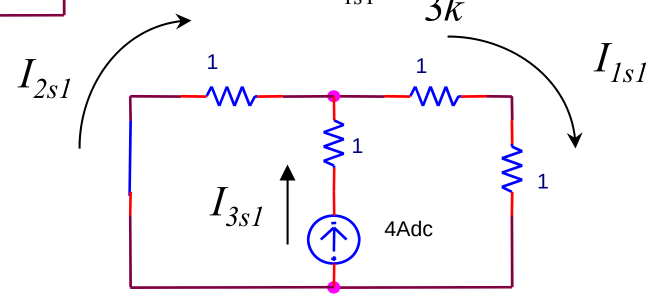


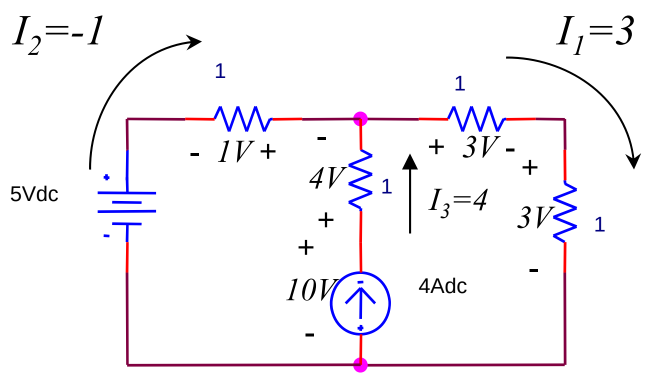
$$I_{3s1} = 4A$$

$$I_{2s1} = -\frac{2k}{3k} \times 4A = -2.67A$$

$$I_{1s1} = \frac{1k}{3k} \times 4A = 1.33A$$







$$I_1 = 1.33 + 1.67 = 3A$$

$$I_2 = -2.67 + 1.67 = -1A$$

$$I_3 = 0 + 4 = 4A$$

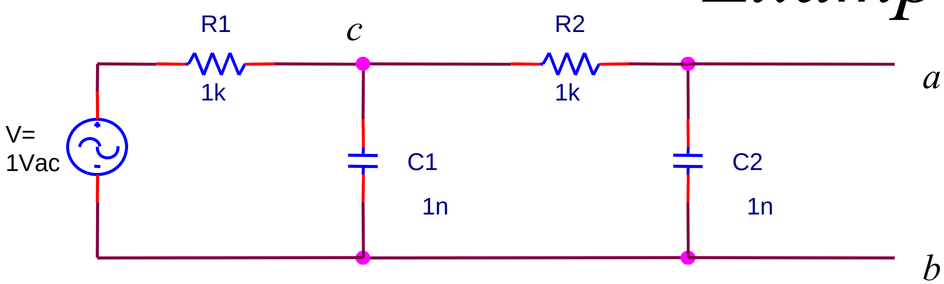
$$V_{R1} = V_{R2} = 3 \times 1 = 3V$$

$$V_{R3} = -1 \times 1 = -1V$$

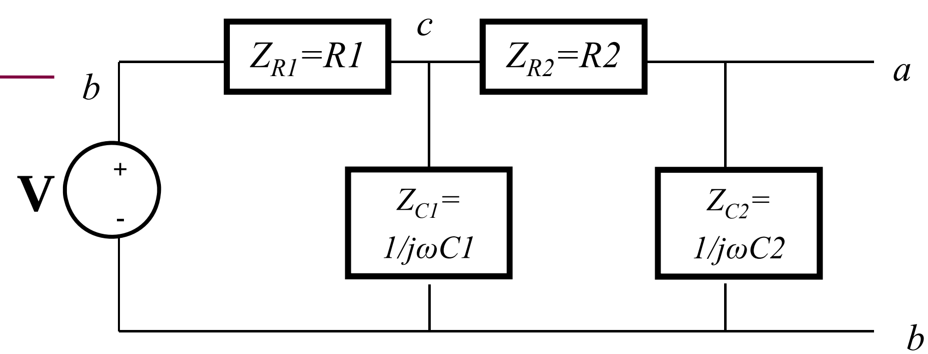
$$V_{R4} = 4 \times 1 = 4V$$

$$V_{CURRENTSOURCE} = V_{R4} + V_{R2} + V_{R1} = 4 + 3 + 3 = 10V$$

Examples



Find the voltage V_{ab} in terms of V



$$V_{ab} = \frac{Z_{c2}}{Z_{R2}+Z_{c2}} \times V_{cb}; \text{(1) using voltage division}$$

$$V_{cb} = \frac{Z_{cb}}{Z_{cb}+Z_{R1}} \times V; \text{(2) using voltage division}$$

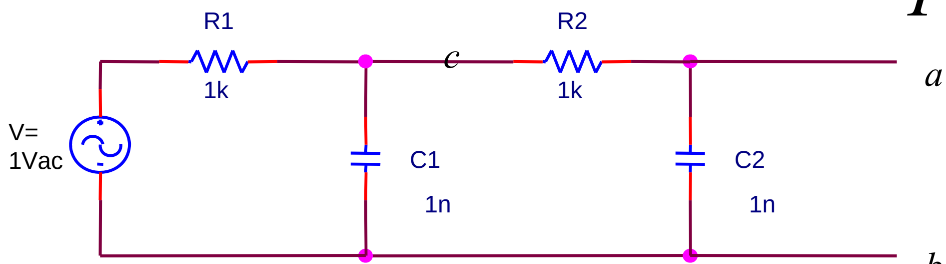
$$Z_{cb} = Z_{c1} \text{ in parallel with the series combination of } Z_{c2}+Z_{R2}$$

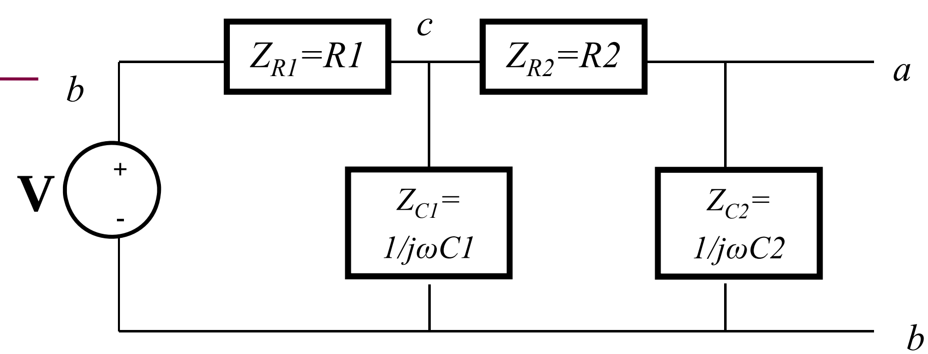
$$= Z_{c1} \,\|\, (Z_{c2}+Z_{R2}) = \frac{Z_{c1}\times(Z_{c2}+Z_{R2})}{Z_{c1}+Z_{c2}+Z_{R2}}$$

$$V_{cb} = \frac{\dfrac{Z_{c1}\times(Z_{c2}+Z_{R2})}{Z_{c1}+Z_{c2}+Z_{R2}}}{\dfrac{Z_{c1}\times(Z_{c2}+Z_{R2})}{Z_{c1}+Z_{c2}+Z_{R2}}+Z_{R1}} \times V = \frac{Z_{c1}\times(Z_{c2}+Z_{R2})}{Z_{c1}\times(Z_{c2}+Z_{R2})+Z_{R1}\times(Z_{c1}+Z_{c2}+Z_{R2})} \times V$$

Examples

Find the voltage V_{ab} in terms of V





Substituting (2) into (1)

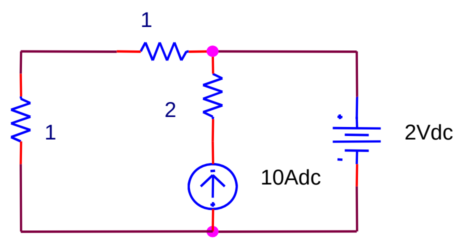
$$V_{ab} = \frac{Z_{c2}}{Z_{R2} + Z_{c2}} \times \frac{Z_{c1} \times (Z_{c2} + Z_{R2})}{Z_{c1} \times (Z_{c1} + Z_{R2}) + Z_{R1} \times (Z_{c1} + Z_{c2} + Z_{R2})} \times V$$

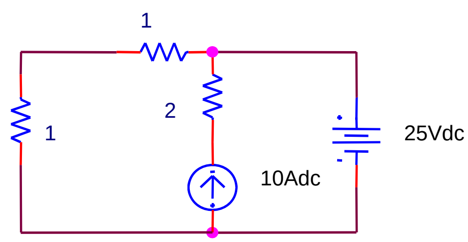
$$= \frac{Z_{c1} Z_{c2}}{Z_{c1} Z_{c1} + Z_{c1} Z_{R2} + Z_{R1} Z_{c1} + Z_{R1} Z_{c2} + Z_{R1} Z_{R2}} \times V$$

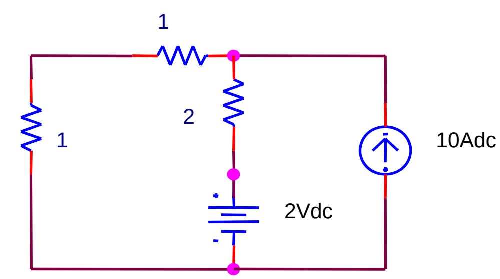
$$V_{ab} = \frac{\frac{1}{j\omega C_1}\frac{1}{j\omega C_2}}{\frac{1}{j\omega C_1}\frac{1}{j\omega C_2} + \frac{R_2}{j\omega C_2} + \frac{R_1}{j\omega C_1} + \frac{R_1}{j\omega C_2} + R_1 R_2} \times V = \frac{V}{1 - \omega^2 C_1 C_2 R_1 R_2 + j\omega (R_1 C_1 + R_1 C_2 + R_2 C_2)}$$

Homework

- Voltage and Current division
  - How does the voltage divide across two capacitors in series? Show your results.
  - How does the current divide among two capacitors in parallel? Show your results.
- Calculate the Currents and Voltages for the following circuits:



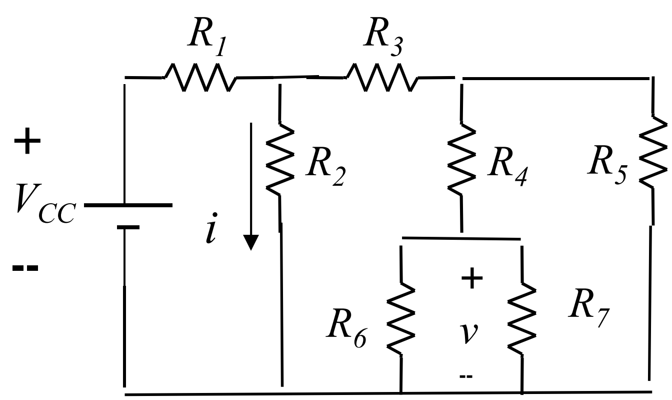




Homework

Calculate the current labeled i and the voltage labeled v in the following circuit

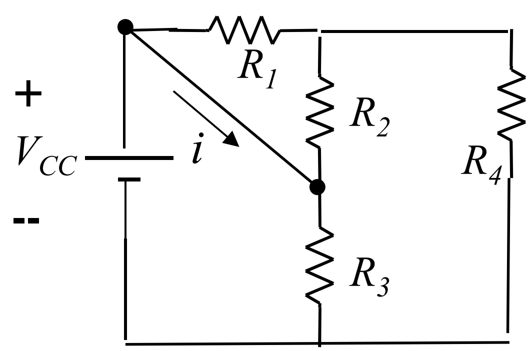
$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 1\Omega$, $R_4 = 1\Omega$, $R_5 = 2\Omega$, $R_6 = 2\Omega$, $R_7 = 2\Omega$, $V_{cc} = 4v$



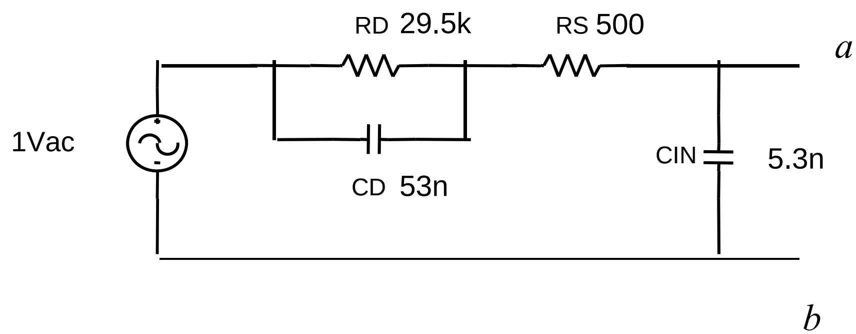
Homework

Calculate the current labe$i_4$led, i.

$R_1 = 2\Omega$, $R_2 = 2\Omega$, $R_3 = 2\Omega$, $R_4 = 3\Omega$, $V_{cc} = 2v$



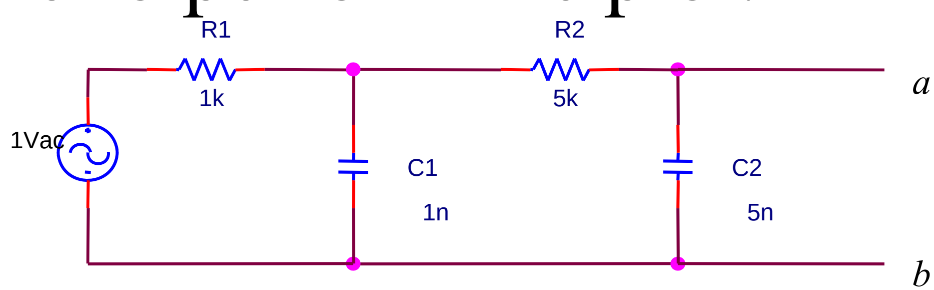
Homework



An electrode is connected to an oscilloscope which has a purely capacitance input impedance, CIN. Find and plot the output voltage $V_{ab}(j\omega)$ as function of ω. Use Matlab to perform the plot.

Homework

- Repeat the analysis of this circuit using Mesh and Nodal Analysis. That is find and plot V_{ab} as a function of frequency.   Use Matlab to perform the plot.



Homework

- Repeat the analysis of this circuit. That is find and plot V_{out}/V_{in} as a function of frequency. Assume $V_a = 0$; $C=10nF, R_1=2.2M\Omega$, $R_2 = 330k\Omega$, $R_3=2.7M\Omega$. Perform the plot using Matlab