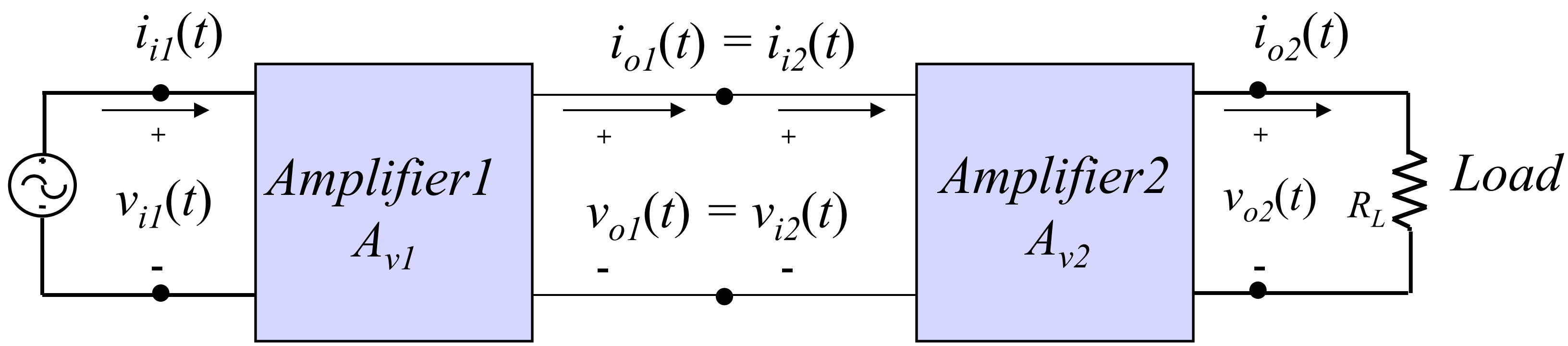
Electronic Systems

Lesson #4

Chapter 1

Cascaded Amplifiers

- Cascading is when two or more amplifiers are connected from output of the first to the input of the second, and so on.



$$A_v = \frac{v_{o2}}{v_{i1}} = \frac{v_{o2}}{v_{i2}} \times \frac{v_{i2}}{v_{i1}} = \frac{v_{o2}}{v_{i2}} \times \frac{v_{o1}}{v_{i1}} = A_{v1}A_{v2}$$

Since $v_{i2} = v_{o1}$

And

$$A_i = \frac{i_{o2}}{i_{i1}} = \frac{i_{o2}}{i_{i2}} \times \frac{i_{i2}}{i_{i1}} = \frac{i_{o2}}{i_{i2}} \times \frac{i_{o1}}{i_{i1}} = A_{i1}A_{i2}$$

Since $i_{i2} = i_{o1}$

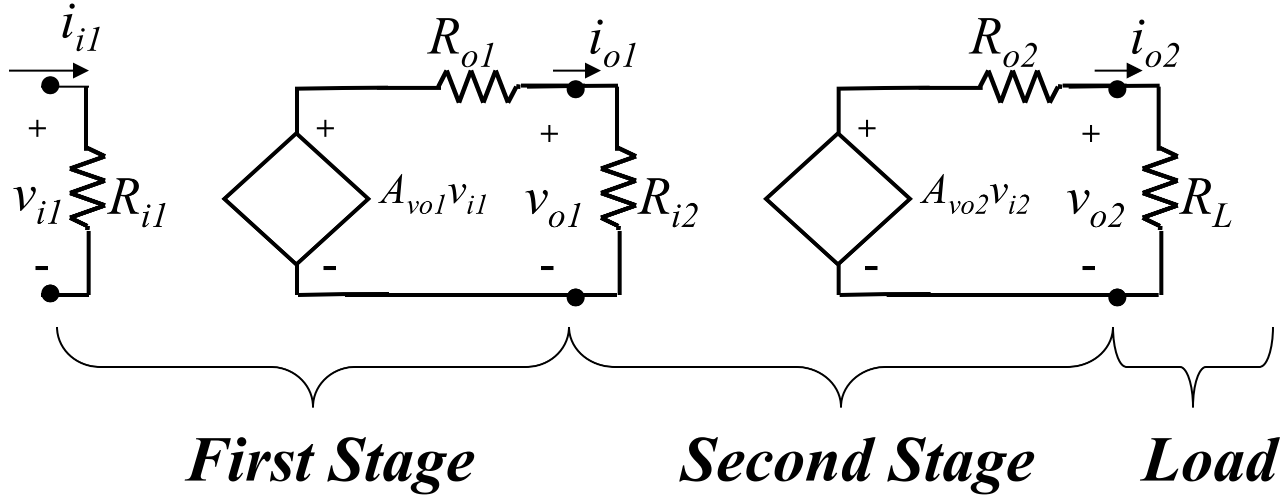
Effects of Loading on Cascaded Amplifiers

$A_{vo1} = 200; A_{vo2} = 100$

$R_{i1} = 1M\Omega; R_{o1} = 500\Omega$

$R_{i2} = 1500\Omega; R_{o1} = 100\Omega$

$R_L = 100\Omega$



$$A_{v1} = A_{vo1}\frac{R_{i2}}{R_{i2}+R_{o1}} = 200\frac{1500}{1500+500} = 150$$

$$A_{v2} = A_{vo2}\frac{R_L}{R_L+R_{o2}} = 100\frac{100}{100+100} = 50$$

$$A_v = A_{v1}\times A_{v2} = A_{vo1}\frac{R_{i2}}{R_{i2}+R_{o1}}\times A_{vo2}\frac{R_L}{R_L+R_{o2}}$$

$$= A_{vo1}A_{vo2}\frac{R_{i2}}{R_{i2}+R_{o1}}\times\frac{R_L}{R_L+R_{o2}} = 200\times100\times\frac{3}{4}\times\frac{1}{2} = 7500$$

$$A_{i1} = \frac{i_{o1}}{i_{i1}} = \frac{v_{o1}/R_{i2}}{v_{i1}/R_{i1}} = A_{v1}\frac{R_{i1}}{R_{i2}} = 150\times\frac{10^6}{1500} = 1\times10^5$$

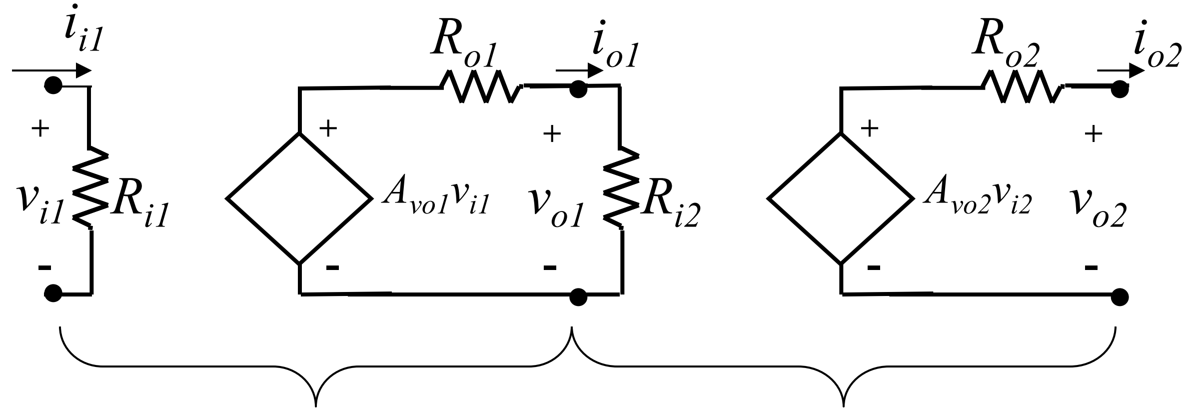
$$A_{i2} = \frac{i_{o2}}{i_{i2}} = \frac{v_{o2}/R_L}{v_{i2}/R_{i2}} = A_{v2}\frac{R_{i2}}{R_L} = 50\times\frac{1500}{100} = 750$$

$$A_i = A_{i1}\times A_{i2} = A_{v1}\frac{R_{i1}}{R_{i2}}\times A_{v2}\frac{R_{i2}}{R_L}$$

$$= A_{v1}A_{v2}\frac{R_{i1}}{R_L} = 150\times50\times\frac{10^6}{100} = 75\times10^6$$

$$G_1 = A_{v1}A_{i1} = 150\times10^5 = 1.5\times10^7; G_2 = A_{v2}A_{i2} = 50\times750 = 37.5\times10^3; G = G_1\times G_2 = 1.5\times10^7\times37.5\times10^3 = 56.25\times10^{11}$$

A Simplified Model of Two Cascaded Amplifiers



First Stage ***Second Stage***

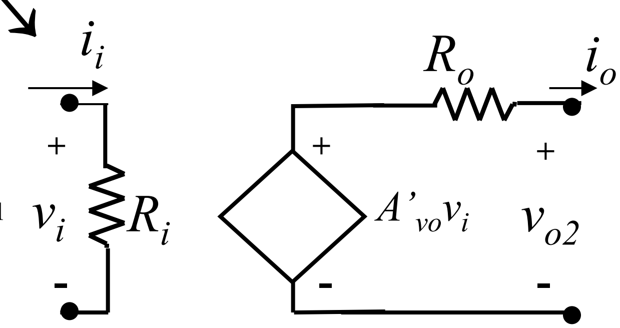
Equivalent Gain =

$$A'_{vo} = \frac{v_{o2}}{v_{i1}} = \frac{A_{vo2}v_{i2}}{v_{i1}} = \frac{A_{vo2}v_{o1}}{v_{i1}} = \frac{A_{vo2}A_{vo1}}{v_{i1}}\frac{R_{i2}}{R_{i2}+R_{o1}}v_{i1}$$

$$= A_{vo2}A_{vo1}\frac{R_{i2}}{R_{i2}+R_{o1}}$$

Equivalent Output Impedance $= R'_o = R_{o2}$

Equivalent Input Impedance $= R'_i = R_{i1}$



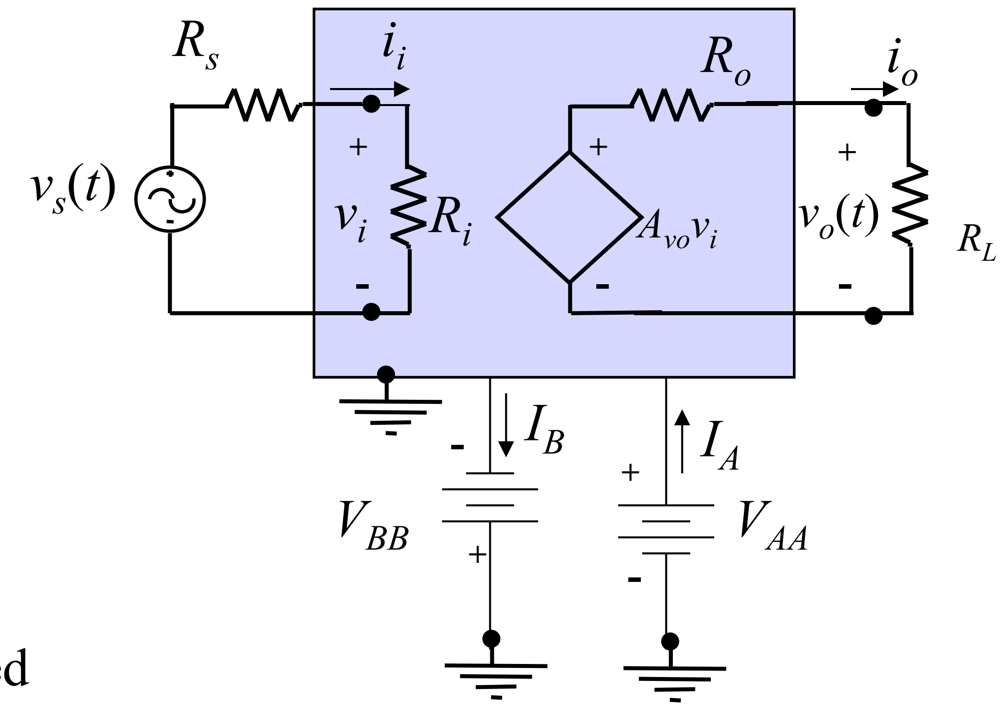
Single Stage

Power Supply Efficiency

- The power required to support the gain of an amplifier comes from DC power supplies connected to the amplifier
- The total average power supplied by the DC power supplies

$$P_{DC} = V_{AA}I_A + V_{BB}I_B$$

- The DC supplies are connected internally to the amplifier.



Power Supplies and Efficiency

- In addition to the DC power, there is power from the source which is delivered to the input of the amplifier, P_i. (note this not the total power provided by this source)
- Therefore, the total input power is $P_{DC} + P_i$
- A portion of this power is used to provide the gain and is delivered the output load, P_L
- The remainder of this power is dissipated by the components of the amplifier, P_d

$$P_{DC} + P_i = P_d + P_L$$

- The amount of from the DC source delivered to the load is called the power efficiency, η

$$\eta = P_L / P_{DC}$$

Power Supply Efficiency

$$P_i = \frac{v_i^{\,2}}{R_i}$$

$$v_i = v_s \frac{R_i}{R_i + R_s} = 2m\frac{100k}{200k} = 1mv$$

$$P_i = \frac{v_i^{\,2}}{R_i} = \frac{1\times 10^{-6}}{10^5} = 10^{-11} = 10\,pW$$

$$P_o = \frac{v_o^{\,2}}{R_L}$$

$$v_o = A_{vo} v_i \frac{R_L}{R_L + R_o} = 10^4 \times 1m \times \frac{8}{8+2} = 8V$$

$$P_o = \frac{v_o^{\,2}}{R_L} = \frac{64}{8} = 8W$$

$$P_{DC} = V_{AA}I_{AA} + V_{BB}I_{BB} = 15\times 1 + 15\times 0.5 = 22.5W$$

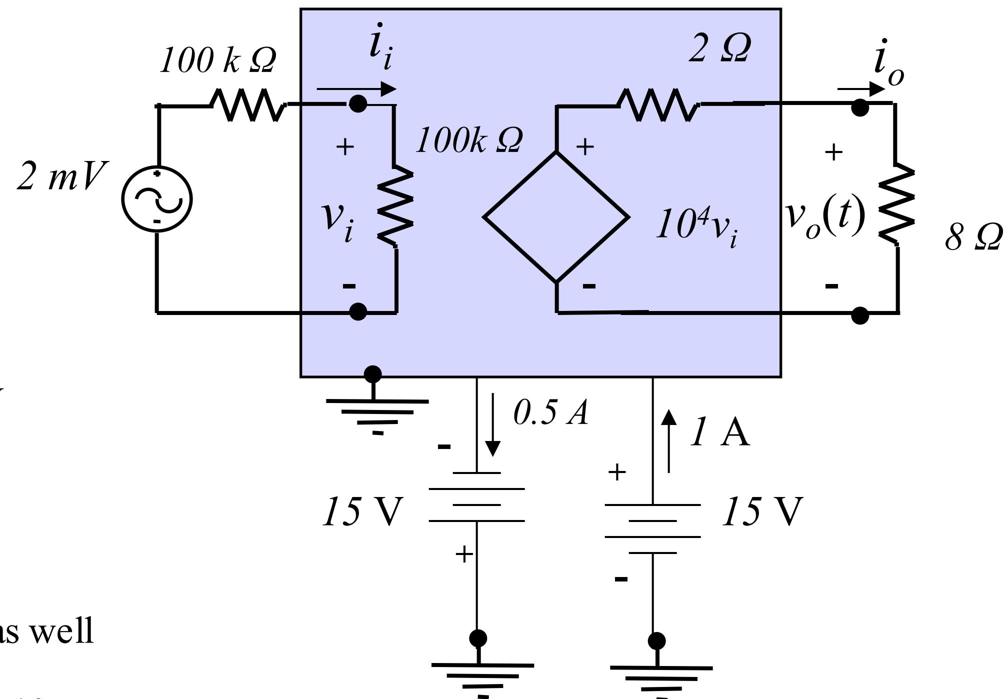
$$\eta = \frac{P_o}{P_{DC}} = \frac{8}{22.5} = 35.6\%$$

$$P_d = P_{DC} + P_i - P_o = 22.5 + 10p - 8 \approx 14.5W$$

Also the source resistor dissipates some power as well

$$i_i = \frac{v_i}{R_i} = \frac{1m}{100k} = 10^{-8}; i_i^{\,2} R_s = 10^{-16} 100k = 10^{-11} = 10\,pW$$

(But we should have known this already!!!! How?)

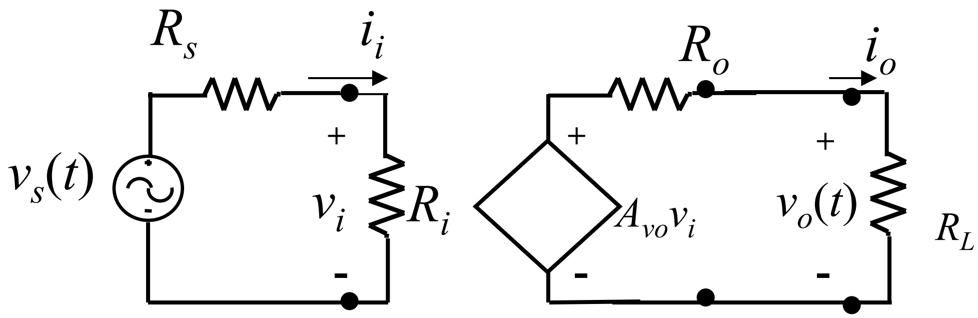


Decibel Notation

- A logarithmic scale is sometimes easier to use:
  - $G_{dB} = 10 \log G$
  - And if $G=G_1G_2$; then $G_{dB}=G_{1\ dB}+G_{2\ dB}$
- And to convert voltage and current to dBs.
  - $A_{v\ dB}= 20 \log |A_v|$
  - $A_{i\ dB}= 20 \log |A_i|$

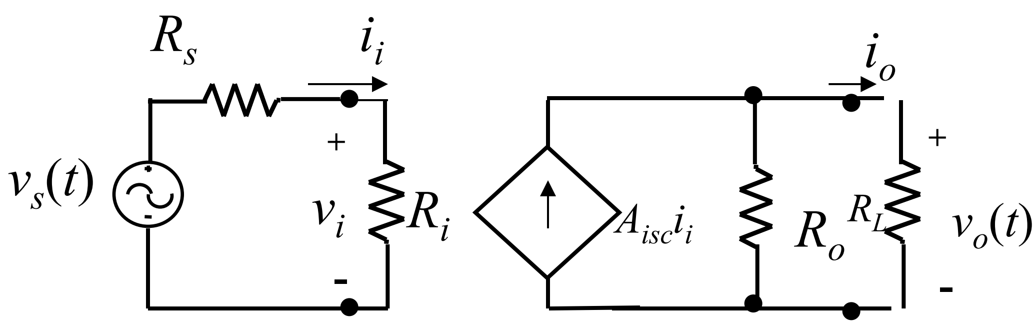
Other Amplifier Models

- Voltage-Amplifier
  - Open-circuit Voltage Gain $v_o = A_{vo}v_i$



Other Amplifier Models

- Current-Amplifier
  - Short Circuit Current Gain $i_o = A_{isc} i_i$



  - This is just the Norton Equivalent of the Voltage-Amplifier since

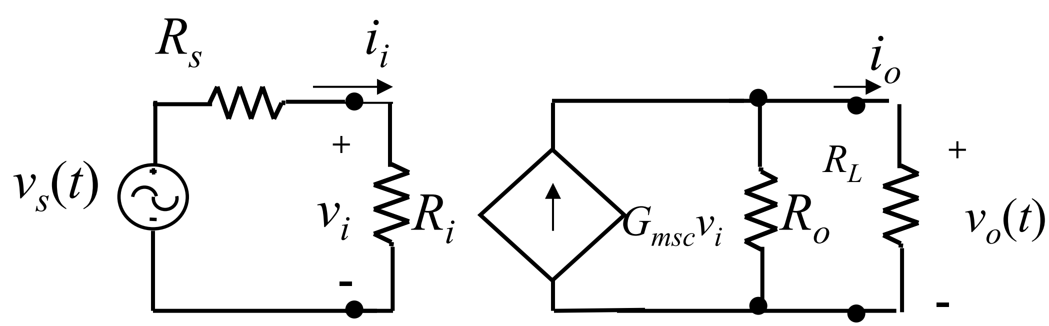
$$V_{oc} = i_o R_o = A_{isc} i_i R_o = A_{vo} v_i = A_{vo} i_i R_i$$

$$A_{isc} R_o = A_{vo} R_i$$

$$A_{isc} = A_{vo} R_i / R_o$$

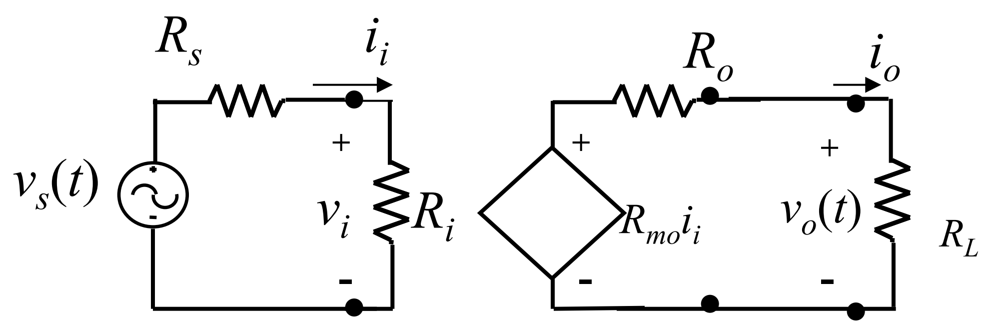
Other Amplifier Models

- Transconductance-Amplifier
  - Short Circuit Transconductance Gain $i_{osc} = G_{msc} v_i$



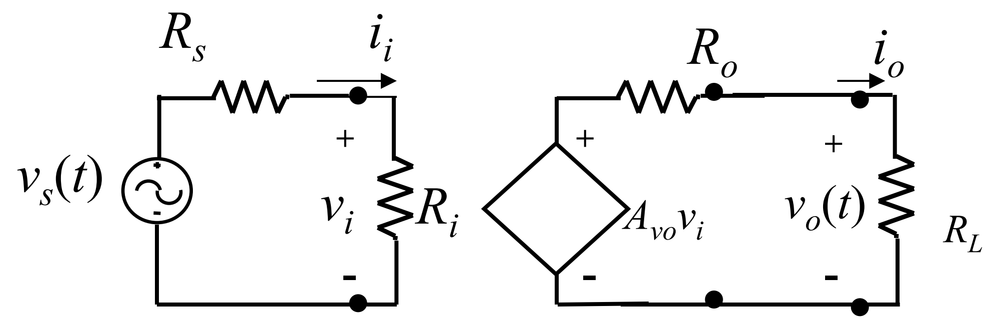
Other Amplifier Models

- Transresistance – Amplifier
  - OpenCircuit Transresistance Gain $v_{osc} = R_{moc}\, i_i$



Ideal Amplifier Models

- What is an ideal amplifier?
- Let's look at our non-ideal Voltage amplifier:
  - We saw that the output voltage is a function of the load and the output resistance.
  - We saw that the input voltage is a function of the source resistance and the input resistance.
  - This also means that the gain from the source to the output is a function of all four resistors.



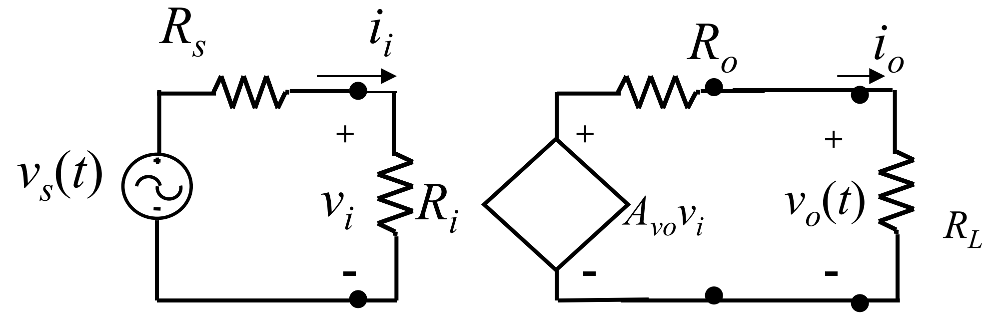
$$v_o = A_{vo} v_i \frac{R_L}{R_L + R_o}$$

$$v_i = v_s \frac{R_i}{R_i + R_s}$$

$$v_o = A_{vo} v_s \times \frac{R_i}{R_i + R_s} \times \frac{R_L}{R_L + R_o}$$

Ideal Amplifier Models

- Ideally, we would like the gain to be equal to the open-circuit gain of the amplifier
  - this will be when the maximum benefit occurs, since the gain will be independent of the input and output circuitry that is connected to the amplifier.
- We can't always choose the source resistance or the output resistance but we have control over the parameters of the amplifier.
- What happens if we make R_i infinite and R_o zero?
- These are the conditions for making a Voltage Amplifier an Ideal Amplifier



$$v_o = A_{vo}v_s \times \frac{R_i}{R_i + R_s} \times \frac{R_L}{R_L + R_o}$$

$$\approx A_{vo}v_s \times \frac{\infty}{\infty + R_s} \times \frac{R_L}{R_L + 0} = A_{vo}v_s \times 1 \times 1$$

$$= A_{vo}v_s$$

Ideal Amplifiers

Amplifier Type	Input Impedance	Output Impedance	Gain
Voltage	∞	0	A_{vo}
Current	0	∞	A_{isc}
Trans-conductance	∞	∞	G_{msc}
Trans-resistance	0	0	R_{mo}

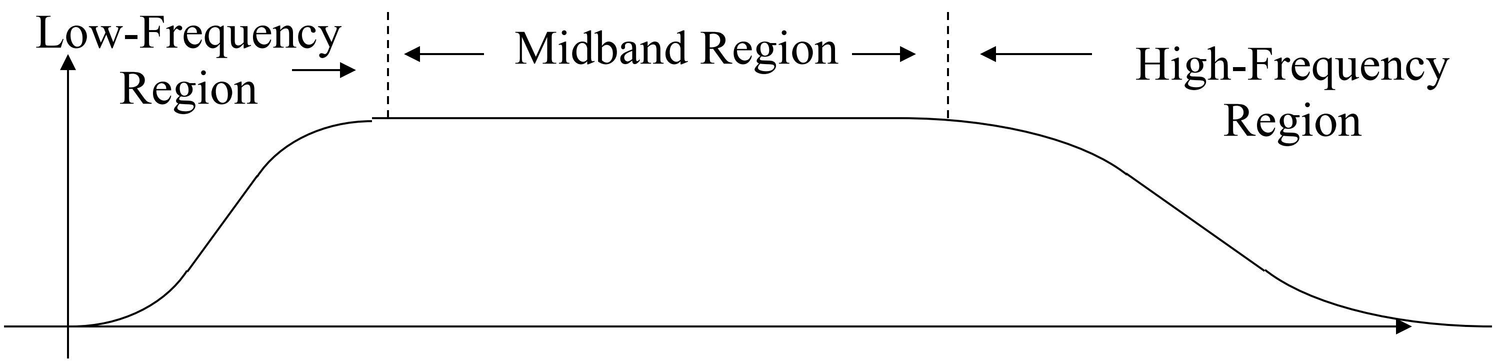
For an exercise, show that this table of ideal amplifier characteristics is correct

Frequency Response

- In general, the signals that we apply to amplifier are made up of difference frequencies.
  - ECG: 0.01 to 250Hz
  - EOG 0.1 to 50 Hz
  - EEG 0 to 150 Hz
  - EMG 0 to 10k Hz
  - Audible Sounds: 20 to 15 kHz
  - Video: DC to 4.5 MHz
  - AM radio: 540 k to 1600 kHz
  - FM radio: 88 M to 108 M Hz
- Therefore, we need to look at a spectrum (of frequencies) and how the spectrum is affected by the amplifier design
- Gain is now a complex number since both the amplitude and phase of an input signal can be affected.

Frequency Response

- We look at the spectrum in terms of regions or bands:
  - Low, mid, and high frequency bands



Frequency Response

- We define parameters associated with the spectrum of the output of an amplifier
  - Half power frequencies or (3 dB points):
    - f_L low frequency 3db point
    - f_H High frequency 3db point
  - Bandwidth (the distance in frequency between 3 dB points

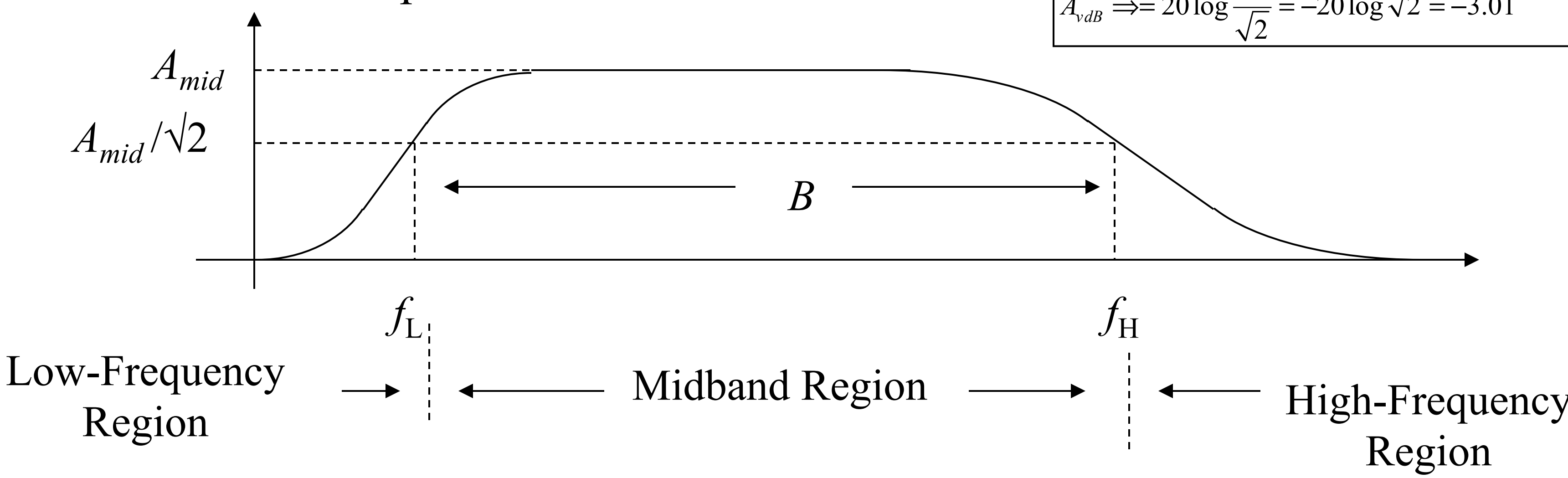
$$\text{Gain at max} = A_{mid}$$

$$\text{Power at max} = \frac{A_{mid}^{\ 2}}{R_L}; \text{Half Power} = \frac{A_{mid}^{\ 2}}{2R_L}$$

$$\frac{\text{Half Power}}{\text{Power at max}} = \frac{A_{mid}^{\ 2}}{2R_L} \Big/ \frac{A_{mid}^{\ 2}}{R_L} = \frac{1}{2}$$

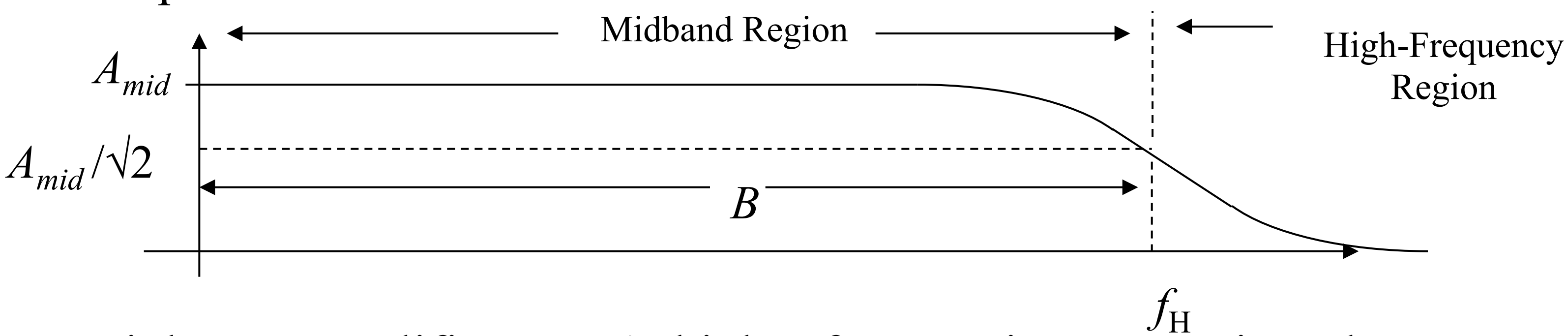
$$\text{Voltage reduction at Half Power} = \frac{1}{\sqrt{2}}$$

$$A_{vdB} \Rightarrow = 20\log\frac{1}{\sqrt{2}} = -20\log\sqrt{2} = -3.01$$

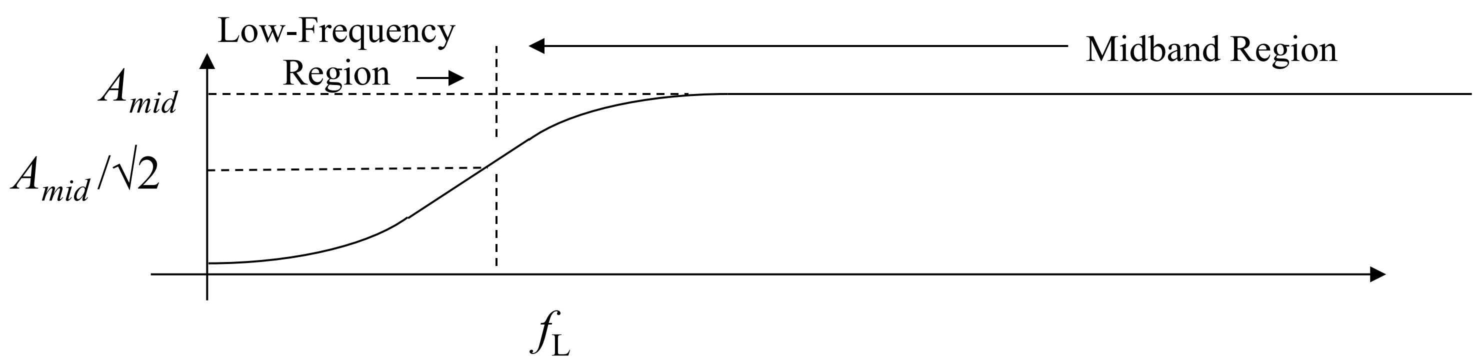


Different Types of Amplifiers

- Low pass amplifier – Let's lower frequencies pass; rejects higher frequencies

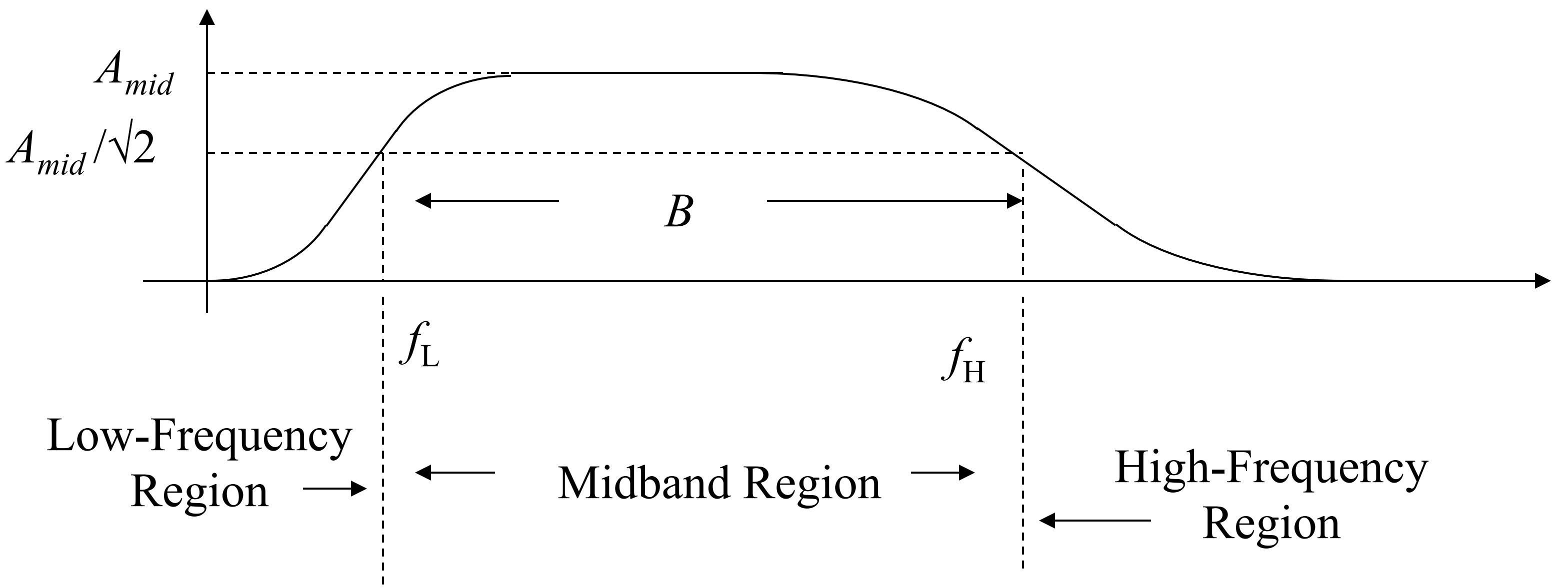


- High pass amplifier – Let's higher frequencies pass; rejects lower frequencies



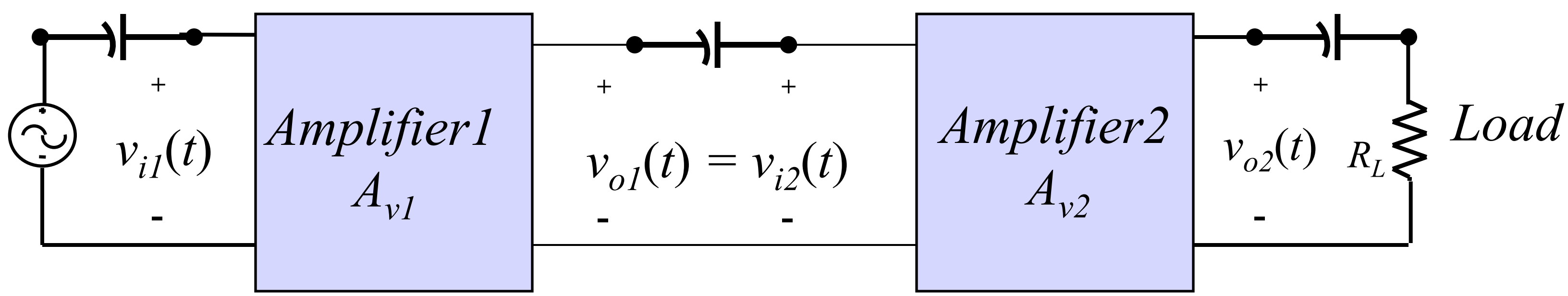
Frequency Response

- Band pass amplifier – Let's a narrow band frequencies pass; rejects higher frequencies



Frequency Response

- Coupling one stage to the next affects the frequency response since stages can be coupled either:
  - directly: DC Coupling
  - Using a capacitor when we don't want the DC to be transferred from one stage to the next: AC Coupling
  - DC coupling yields a frequency response similar to a low pass filter.
  - AC coupling produces a band pass amplifier
  - f_L is a function of the circuit elements using in the design



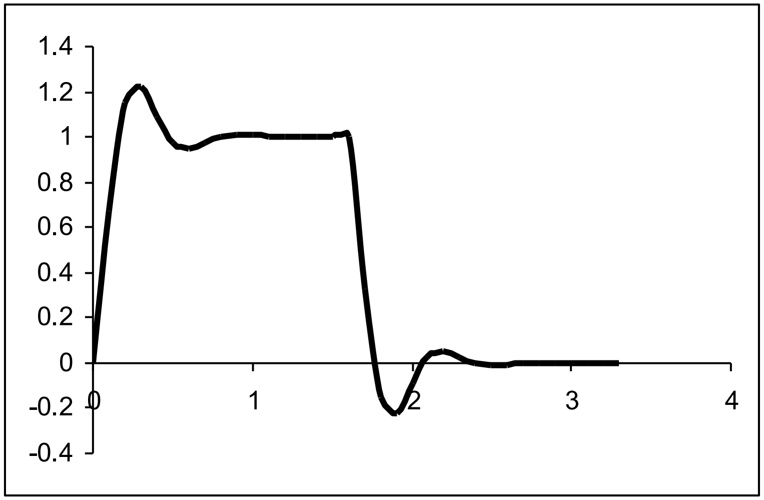
Frequency Response

- High Frequencies are affected by the small internal capacitances of the non-linear devices we use (e.g., transistors) and any stray capacitances that appear in parallel with the signal path
  - Since the impedance of a capacitor is inversely proportion to the frequency and capacitive value, the smaller the frequency or the smaller the capacitance the higher the impedance
  - Therefore, small capacitances will then look like open circuits
  - However, as the frequency increases these impedances become smaller and since they are in parallel with the signal path, they can short out the signal

Frequency Response

- For digital signals (pulses), we also look in the time domain (as opposed to the frequency domain) to see how the amplifier has affected the signal.
- Usually pulses show a slow rise and fall. Sometimes they show ringing.
- The rise time can be shown to be a function of the bandwidth, *B*, of the amplifier such that

$$t_r \approx 0.35 / B$$



Differential Amplifiers

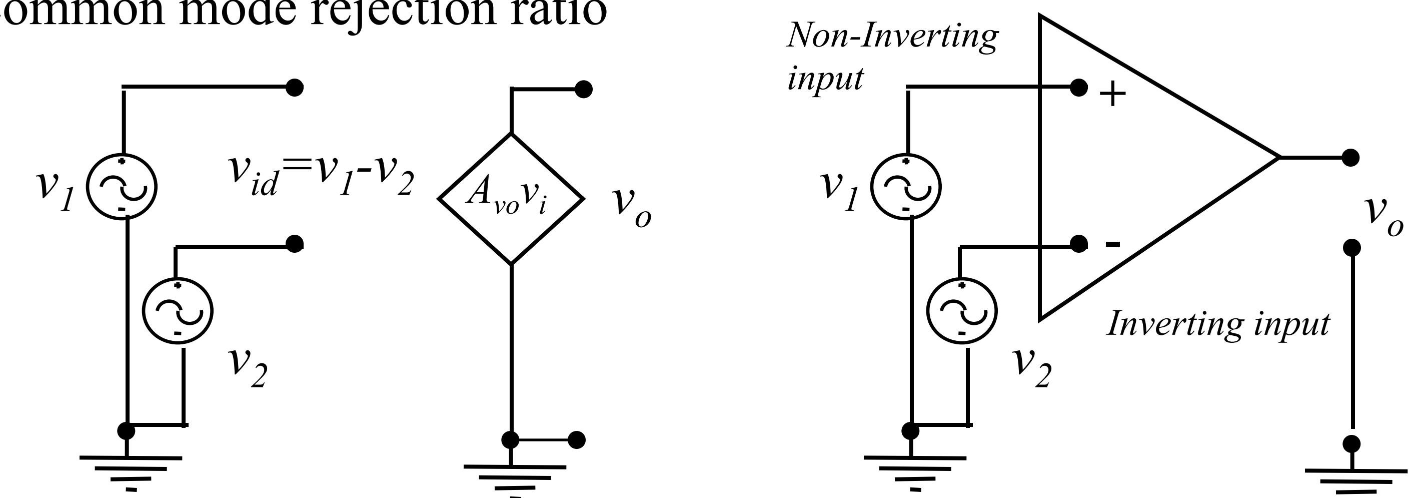
- Output of the amplifier is a function of the difference of the inputs:

$$v_o = A_d(v_{i1} - v_{i2})$$

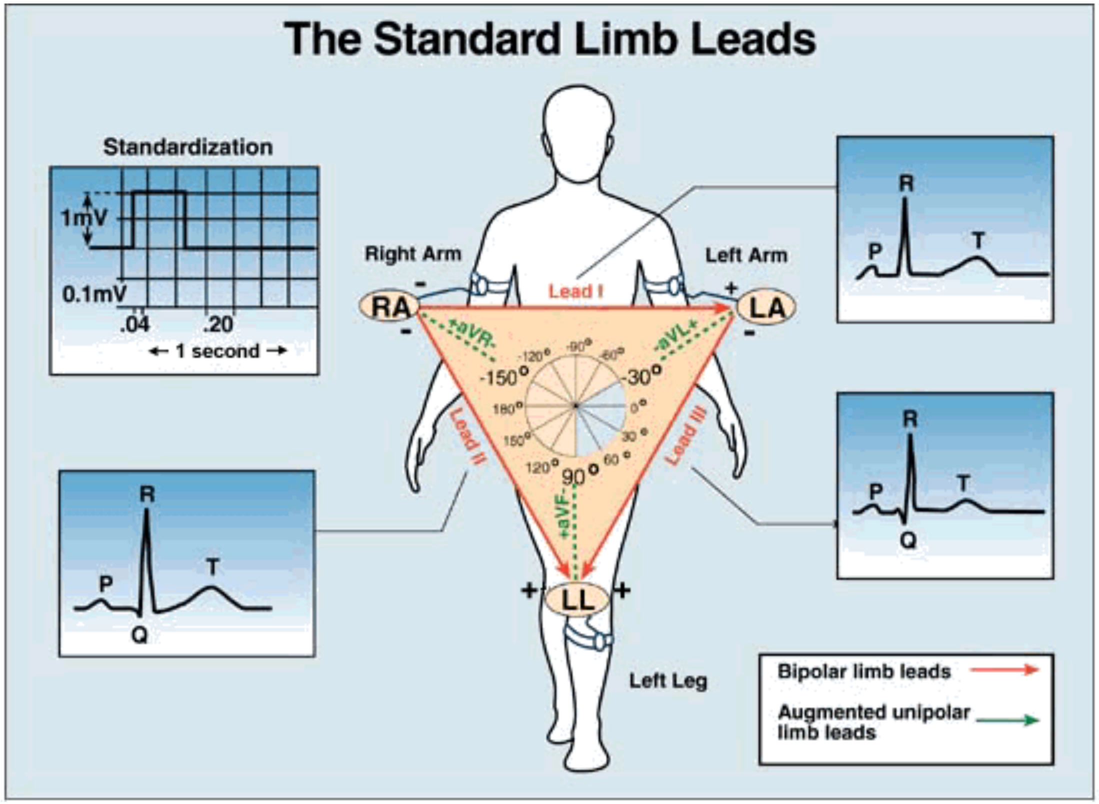
- However, most real Differential Amplifier are affected by the average of the input and we define the common mode input signal

$$v_{\text{icm}} = 1/2(v_{i1} + v_{i2})$$

- Therefore, $v_o = A_d(v_{i1} - v_{i2}) + A_m\, v_{\text{icm}}$ and the ratio of A_d to A_m is called the common mode rejection ratio



Uses of the Differential Amplifier



Common-Mode Rejection Ratio

$$v_o = A_d v_{id} + A_{cm} v_{icm}$$

$$CMRR = 20\log\frac{|A_d|}{|A_{cm}|}$$

This is usually measured at a given frequency, for example at 60Hz

For a ECG amplifier, if the $A_d = 1000,\ \ v_{id} = 1mV,\ \ \&\ \ v_{icm} = 100V\ @\ 60Hz$; calculate the CMRR when it is desired that the common mode output of the differential amp is 1% or less than the differential output.

$$v_{od} = A_d v_i = 1000 \times 1m = 1V$$

$$v_{ocm} < .01 \times 1 = 0.01V$$

$$A_{cm} = \frac{v_{ocm}}{v_{icm}} = \frac{0.01}{100} = 10^{-4} \Rightarrow 20\log 10^{-4} = -80dB$$

$$CMRR = 20\log\frac{|A_d|}{|A_{cm}|} = 20\log\frac{1000}{10^{-4}} = 20\log 10^{7} = 140dB$$

Homework

- Probs 1.30, 1.31, 1.35, 1.36, 1.37, 1.55, 1.56, 1.59, 1.60, 1.61