# Operational Amplifiers 

## Lesson \#5 <br> Chapter 2

## Operational Amplifiers

- An operational Amplifier is an ideal differential with the following characteristics:
- Infinite input impedance
- Infinite gain for the differential signal
- Zero gain for the common-mode signal
- Zero output impedance
- Infinite Bandwidth



## Operational Amplifier Feedback

- Operational Amplifiers are used with negative feedback
- Feedback is a way to return a portion of the output of an amplifier to the input
- Negative Feedback: returned output opposes the source signal
- Positive Feedback: returned output aids the source signal
- For Negative Feedback
- In an Op-amp, the negative feedback returns a fraction of the output to the inverting input terminal forcing the differential input to zero.
- Since the Op-amp is ideal and has infinite gain, the differential input will exactly be zero. This is called a virtual short circuit
- Since the input impedance is infinite the current flowing into the input is also zero.
- These latter two points are called the summing-point constraint.

Operational Amplifier Analysis Using the Summing Point Constraint

- In order to analyze Op-amps, the following steps should be followed:

1. Verify that negative feedback is present
2. Assume that the voltage and current at the input of the Op-amp are both zero (Summingpoint Constraint
3. Apply standard circuit analyses techniques such as Kirchhoff's Laws, Nodal or Mesh Analysis to solve for the quantities of interest.

## Example: Inverting Amplifier

1. Verify Negative Feedback: Note that a portion of $v_{o}$ is fed back via $R_{2}$ to the inverting input. So if $v_{i}$ increases and, therefore, increases $v_{o}$, the portion of $v_{o}$ fed back will then have the affect of reducing $v_{i}$ (i.e., negative feedback).
2. Use the summing point constraint.
3. Use KVL at the inverting input node for both the branch connected to the source and the branch connected to the output
$v_{\text {in }}=i_{1} R_{1}+0$ since $v_{i}$ is zero due to the summing - point constraint
$i_{1}=i_{2}$ due to the summing - point constraint

$$
Z_{i n}=\frac{v_{i n}}{i_{1}}=\frac{i_{1} R_{1}}{i_{1}}=R_{1}
$$

$v_{0}=-i_{2} R_{2}+0$ since $v_{i}$ is zero
$=-\frac{R_{2}}{R_{1}} v_{\text {in }}$ which is independent of $R_{L}$ (note that the output is opposite to the input :inverted)

## Op-amp

- Because we assumed that the Op-amp was ideal, we found that with negative feedback we can achieve a gain which is:

1. Independent of the load
2. Dependent only on values of the circuit parameter
3. We can choose the gain of our amplifier by proper selection of resistors.

## Another Example: Inverting Amplifier

1. Verify Negative Feedback:

2. Use the summing point constraint.
3. Use KVL at the inverting input node for the branch connected to the source and KCL \& KVL at the node where the 3 resistors are connected
$v_{i n}=i_{1} R_{1}+0$ since $v_{i}$ is zero due to the summing - point constraint
$i_{1}=i_{2}$ due to the summing - point constraint
$v_{i}=i_{2} R_{2}-i_{3} R_{3}=0 \Rightarrow i_{2} R_{2}=i_{3} R_{3}$ since $v_{i}$ is zero
$i_{4}=i_{3}+i_{2}$
$v_{o}=-R_{4} i_{4}-R_{3} i_{3}$

## Another Example: Continued



$$
\begin{aligned}
i_{2} R_{2} & =i_{3} R_{3} \Rightarrow i_{3}=\frac{R_{2}}{R_{3}} i_{2} \\
v_{i n} & =i_{1} R_{1}+0 ; i_{1}=i_{2} \Rightarrow i_{2}=\frac{v_{i n}}{R_{1}} \\
i_{4} & =i_{3}+i_{2}=\left(\frac{R_{2}}{R_{3}}+1\right) i_{2}=\left(\frac{R_{2}}{R_{3} R_{1}}+\frac{1}{R_{1}}\right) v_{i n} \\
v_{o} & =-R_{4} i_{4}-R_{3} i_{3}=-R_{4}\left(\frac{R_{2}}{R_{3} R_{1}}+\frac{1}{R_{1}}\right) v_{i n}-R_{3} \frac{R_{2}}{R_{3}} \frac{v_{3}}{R_{1}} \\
& =-v_{\text {in }}\left(\frac{R_{2} R_{4}}{R_{3} R_{1}}+\frac{R_{4}}{R_{1}}+\frac{R_{2}}{R_{1}}\right) \\
A_{v} & =\frac{v_{o}}{v_{i n}}=-\left(\frac{R_{2} R_{4}}{R_{3} R_{1}}+\frac{R_{4}}{R_{1}}+\frac{R_{2}}{R_{1}}\right)
\end{aligned}
$$

$$
v_{i n}=i_{1} R_{1}+0
$$

$$
i_{1}=i_{2}
$$

$$
i_{2} R_{2}=i_{3} R_{3}
$$

Why would we use this design over the simpler one?

$$
i_{4}=i_{3}+i_{2}
$$

1. Same gain but with smaller values of resistance

$$
v_{o}=-R_{4} i_{4}-R_{3} i_{3}
$$

2. Higher gain

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## Non-inverting Amp

1. First check: negative feedback?
2. Next apply, summing point constraint
3. Use circuit analysis

$$
\begin{aligned}
& v_{i n}=v_{i}+v_{f}=0+v_{f}=v_{f} \\
& v_{f}=\frac{R_{1}}{R_{1}+R_{2}} v_{o}=v_{i n} \\
& A_{v}=\frac{v_{o}}{v_{i n}}=\frac{R_{2}+R_{1}}{R_{1}}=1+\frac{R_{2}}{R_{1}}
\end{aligned}
$$

Note:
Since $i_{\text {in }}=0 ; Z_{i n}=\frac{v_{i n}}{i_{i n}}=\infty \quad 1$.


## Non-inverting Amp Special Case

What happens if $R_{2}=0$ ?

$$
\begin{aligned}
& v_{i n}=v_{i}+v_{f}=0+v_{f}=v_{f} \\
& v_{f}=\frac{R_{1}}{R_{1}+0} v_{o}=v_{o}=v_{i n} \\
& A v=\frac{v_{o}}{v_{i n}}=\frac{0+R_{1}}{R_{1}}=1
\end{aligned}
$$



Since $i_{i n}=0 ; Z_{i n}=\frac{v_{i n}}{i_{i n}}=\infty$
This is a unity gain amplifier and is also called a voltage follower.

## Some Practical Issues when Designing Op-amps

- Since ratios of resistor values determines the gain, choosing the proper resistor values is crucial
- Too small means large currents drawn
- Too large yields another set of problems
- Open Loop Gain is not constant but a function of frequency
- Non-linearities of the amplifier
- Voltage clipping
- Slew rate
- DC imperfections
- Offsets
- Bias Currents


## Selecting Resistor Values

- Let say we want a gain of 10 . This means that $R_{2}=9 R_{1}$.
- If we chose $R_{l}=1 \Omega$, then for a 10 volt output, there will be 1 A flowing threw $R_{1}$ and $R_{2}$.
- THIS IS DANGEROUS!!!!
- On the other hand if $R_{l}=10 \mathrm{M} \Omega$, then there may be unwanted effects due to pickup of induced signals
- Therefore choosing values between
 $100 \Omega$ and $1 \mathrm{M} \Omega$ is optimum


## Frequency Issues

- Let's assume that the open loop gain of our Op-amp is a function of frequency.
$A_{O L}(f)=\frac{A_{o O L}}{1+j\left(f / f_{B O L}\right)}$
where $A_{\text {ooL }}$ is the open-loop gain for $f=0$, $f_{B O L}$ is called the open-loop break
frequency since when $f=f_{B O L}$
then $\left|A_{O L}(f)\right|=A_{o O L} / \sqrt{2}$ or at the half power point.

Note that when $f=A_{\text {ooL }} f_{B O L},\left|A_{O L}(f)\right| \rightarrow 1$


This is will define an important relationship for the amplifier when feedback is used

## Frequency Issues $\quad I_{i n}=0$

$A_{O L}(f)=\frac{A_{o O L}}{1+j\left(f / f_{B O L}\right)}$
Using phasors:
$\mathbf{V}_{f}=\frac{R_{1}}{R_{1}+R_{2}} \times \mathbf{V}_{O}=\beta \mathbf{V}_{O}$; where $\beta=\frac{R_{1}}{R_{1}+R_{2}}$
$\mathbf{V}_{i n}=\mathbf{V}_{i}+\beta \mathbf{V}_{O}=\frac{\mathbf{V}_{O}}{A_{O L}}+\beta \mathbf{V}_{O} ;$ since $\mathbf{V}_{o}=\mathbf{V}_{i} A_{O L}$

$\therefore A_{C L}=\frac{\mathbf{V}_{O}}{\mathbf{V}_{\text {in }}}=\frac{A_{O L}}{1+\beta A_{O L}}$

$$
A_{C L}(f)=\frac{A_{O L}(f)}{1+\beta A_{O L}(f)}=\frac{A_{O L}(f)}{1+\beta A_{O L}(f)}=\frac{1+j\left(f / f_{B O L}\right)}{1+\frac{\beta A_{O L}}{1+j\left(f / f_{B O L}\right)}}
$$

(Note for the ideal Op-amp $A_{O L} \rightarrow \infty$ and $A_{C L} \rightarrow \frac{1}{\beta}=\frac{R_{1}+R_{2}}{R_{1}}=1+\frac{R_{2}}{R_{1}}$ which what we expected.)

$$
=\frac{\frac{A_{o O L}}{1+\beta A_{o O L}}}{1+j\left(\frac{f}{f_{B O L}\left(1+\beta A_{o o L}\right)}\right)}=\frac{A_{o C L}}{1+j\left(\frac{f}{f_{B C L}}\right)}
$$

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A interesting factor now comes to light:
$A_{o C L} f_{B C L}=\frac{A_{o O L}}{1+\beta A_{o O L}} f_{B O L}\left(1+\beta A_{o O L}\right)=A_{o O L} f_{B O L}$

Where:
$A_{o C L}=\frac{A_{o O L}}{1+\beta A_{o O L}}$ and $f_{B C L}=f_{B O L}\left(1+\beta A_{o O L}\right)$

The gain bandwidth product is constant!!!

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## Example

- For an amplifier with Open-loop dc gain of 100 dB with Open-loop breakpoint of 40 Hz the Bandwidth product for $\beta=1,0.1,0.01$

| $\beta$ | $A_{O C L}$ | $A_{O C L}(d B)$ | $f_{\mathrm{BCL}}$ |
| :---: | :---: | :---: | :---: |
| 1 | 0.9999 | 0 | 4 MHz |
| 0.1 | 9.9990 | 20 | 400 kHz |
| 0.01 | 99.90 | 40 | 40 kHz |



- The Op-amp has a limit as to how large an output voltage and current can be produced. (See figure 2.28)
- For example for the circuit shown it has the following limitations: $\pm 12 \mathrm{~V}$ and $\pm 25 \mathrm{~mA}$


## Voltage clipping $\xrightarrow{I_{i n}=0}$

a) If the load resistor is $10 \mathrm{k} \Omega$, what is the maximum input voltage which can be handled without clipping
b) Repeat for $100 \Omega$
$A_{v C L}=1+3 / 1=4$
a) $I_{o \text { max }}=\frac{V_{o \text { max }}}{R_{L}}+\frac{V_{o \text { max }}}{R_{1}+R_{2}}=\frac{12}{10^{4}}+\frac{12}{3000+1000}$
$=4.2 \mathrm{~mA}$
For this case, maximum output voltage will occur before maximum output current is reached.
$V_{\text {in } \max }=12 / 4=3 \mathrm{~V}$
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## Slew Rate

- Slew Rate is a phenomenon which occurs when the Op-Amp can not keep up the change in the input.
- Therefore, we identify the maximum rate of change of the Op-amp as the Slew Rate SR


## DC imperfections

- We saw that we have to provide DC voltages to an amplifier in order to provide it with the power to support amplification.
- For a differential amplifier which must handle both positive and negative voltages
- The process of designing this DC circuitry is called biasing
- As a result, biasing currents flow through the amplifier which affects its performance.
- In particular, a voltage during to the biasing will appear at the output without any input signal.
- These extra voltage can be due to:
- Bias currents flowing in the feedback circuitry
- Bias current differentials
- Voltage offsets due to the fact that the Op-amp circuitry is not ideal


## Special Amplifiers

- Summer (Homework Problem)
- Instrumentation Amplifier
- Uses 3 Op-amps
- One as a differential amplifier
- Two Non-inverting Amps using for providing gain


## Medical Instrumentation Amplifier

Non-inverting Amplifier


Differential Amplifier
$\geqslant$
Non-inverting Amplifier

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## Medical Instrumentation Amplifier

## Non-inverting Amplifier



Differential Amplifier

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## Medical Instrumentation Amplifier

## Differential Amplifier



Chose $\frac{R_{5}+R_{6}}{R_{5}} \frac{R_{4}}{R_{3}+R_{4}}=1$
$R_{4} R_{5}+R_{4} R_{6}=R_{5} R_{3}+R_{5} R_{4}$
$R_{4} R_{6}=R_{5} R_{3}$
$\frac{R_{6}}{R_{5}}=\frac{R_{3}}{R_{4}}$

$$
\begin{aligned}
& \frac{v_{2 D}-v_{x}}{R_{6}}=\frac{v_{x}-v_{o}}{R_{5}} \\
& \frac{v_{2 D}}{R_{6}}-v_{x}\left(\frac{1}{R_{6}}+\frac{1}{R_{5}}\right)=\frac{-v_{o}}{R_{5}} \\
& \frac{v_{2 D}}{R_{6}}-v_{x}\left(\frac{R_{5}+R_{6}}{R_{6} R_{5}}\right)=\frac{-v_{o}}{R_{5}} \\
& v_{y}=\frac{R_{4}}{R_{3}+R_{4}} v_{1 D}=v_{x}-v_{i}=v_{x} \\
& \frac{v_{2 D}}{R_{6}}-\frac{R_{4}}{R_{3}+R_{4}} v_{1 D}\left(\frac{R_{5}+R_{6}}{R_{6} R_{5}}\right)=\frac{-v_{o}}{R_{5}} \\
& v_{o}=\frac{R_{5}}{R_{6}}\left(v_{1 D} \frac{R_{5}+R_{6}}{R_{5}} \frac{R_{4}}{R_{3}+R_{4}}-v_{2 D}\right) \\
& \text { Chose } \frac{R_{5}+R_{6}}{R_{5}} \frac{R_{4}}{R_{3}+R_{4}}=1 \\
& v_{o}=\frac{R_{5}}{R_{6}}\left(v_{1 D}-v_{2 D}\right)
\end{aligned}
$$

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## Medical Instrumentation Amplifier

Non-inverting Amplifier


## Medical Instrumentation Amplifier

 Non-inverting Amplifier

## Medical Instrumentation Amplifier

Non-inverting Amplifier


## Wheatstone Bridge as a sensor



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## Wheatstone Bridge



Using Thevinin's Theorem on the Wheatstone Bridge Left side
Right side

$$
V_{2}=\frac{r_{B}}{r_{B}+r_{C}} V ; r_{2}=\frac{r_{B} r_{C}}{r_{B}+r_{C}} \quad V_{1}=\frac{r_{A}}{r_{A}+r_{D}} V ; r_{1}=\frac{r_{A} r_{D}}{r_{A}+r_{D}}
$$



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## Wheatstone Bridge


$V_{\text {Bridge }}=V_{2}-V_{1}=\left(\frac{r_{B}}{r_{B}+r_{C}}-\frac{r_{A}}{r_{A}+r_{D}}\right) V$
When bridge is balanced $\frac{r_{B}}{r_{B}+r_{C}}=\frac{r_{A}}{r_{A}+r_{D}} \Rightarrow r_{B} r_{D}=r_{A} r_{C} \Rightarrow \frac{r_{A}}{r_{D}}=\frac{r_{B}}{r_{C}}$
(nominally, $r_{A}=r_{B}=r_{C}=r_{D}$ )
and $V_{\text {Bridge }}=0$

## Wheatstone Bridge



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## Wheatstone Bridge



$$
\begin{aligned}
& \text { Chose } \frac{R_{1}+R_{2}+r_{2}}{R_{2}} \frac{R_{4}}{R_{3}+r_{1}+R_{4}}=1 \\
& \frac{R_{1}+R_{2}+r_{2}}{R_{2}}=\frac{R_{3}+r_{1}+R_{4}}{R_{4}} \\
& \frac{R_{1}+R_{2}+\frac{r_{B} r_{C}}{r_{B}+r_{C}}}{R_{2}}=\frac{R_{3}+\frac{r_{A} r_{D}}{r_{A}+r_{D}}+R_{4}}{R_{4}}
\end{aligned}
$$

Hard to deal with; then let's make sure that $r_{2} \ll R_{1}+R_{2}$ and $r_{1} \ll R_{3}+R_{4}$
$\frac{R_{1}+R_{2}}{R_{2}}=\frac{R_{3}+R_{4}}{R_{4}} \Rightarrow R_{2} R_{3}+R_{2} R_{4}=R_{4} R_{1}+R_{4} R_{2} \Rightarrow R_{2} R_{3}=R_{4} R_{1}$
$\frac{R_{4}}{R_{3}}=\frac{R_{2}}{R_{1}}$
$v_{o}=\frac{R_{2}}{R_{1}}\left(\frac{r_{A}}{r_{A}+r_{D}}-\frac{r_{B}}{r_{B}+r_{C}}\right) V$
Note that the output depends on the unbalance of the bridge and when the bridge is balanced $v_{o}=0$ Gain is still $\frac{R_{2}}{R_{1}}$

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## Integrators and Differentiators



$$
\begin{aligned}
& i_{1}(t)=\frac{v_{i n}(t)}{R}=i_{2}(t) \\
& v_{o}=-\frac{1}{C} \int_{0}^{t} i_{2}(x) d x=-\frac{1}{R C} \int_{0}^{t} v_{i n}(x) d x
\end{aligned}
$$



$$
\begin{aligned}
& i_{1}(t)=\frac{C d v_{i n}(t)}{d t}=i_{2}(t) \\
& v_{o}=-i_{2}(t) R=-R C \frac{d v_{i n}(t)}{d t}
\end{aligned}
$$

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## Frequency Analysis



## Frequency Response



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## Frequency Response



$$
\frac{V_{\text {out }}}{V_{\text {in }}}=-\frac{Z_{2}}{Z_{1}}=-\frac{R_{2}}{R_{1}} \frac{1}{\left(1+j \omega C_{2} R_{2}\right)}=\frac{R_{2}}{R_{1}} \frac{1}{\sqrt{1+\left(\omega C_{2} R_{2}\right)^{2}}} \angle \pi-\tan ^{-1}\left(\omega C_{2} R_{2}\right)
$$



$\frac{V_{\text {out }}}{V_{\text {in }}}=-\frac{Z_{2}}{Z_{1}}=-\frac{R_{2}}{R_{1}} \frac{j \omega C_{1} R_{1}}{\left(1+j \omega C_{1} R_{1}\right)}=\frac{R_{2}}{R_{1}} \frac{\omega C_{1} R_{1}}{\sqrt{1+\left(\omega C_{1} R_{1}\right)^{2}}} \angle-\frac{\pi}{2}-\tan ^{-1}\left(\omega C_{1} R_{1}\right)$
R1 51 C1 10mf R2 100k


## Integrators and Differentiators

- Integrators and Differentiators are used in analog computers
- An analog computer solves a differential equations
- By using Integrators and Differentiators one can "program" a particular differential equation to be solved
- Usually only integrators are used since the gain of a differentiator occurs at high frequencies while the opposite is true for the integrator.
- Since the frequency response of an real Op-amp attenuates high frequencies, using a differentiator conflicts with the characteristics with a real Op-amp
- Also noise (high frequencies) are amplified by differentiators



FIGURE 6.12 Circvit diagram of coniplese A-wave detectot. (From Thakor, N. V.
197s. Retlabla R-wave detection frem ambelasory subjects. (iomed. Sel. Insifum, 197. Reliabio R .wave detection frem ame
$14: 67-72$. Used with permiation of 15 A )

$$
\begin{aligned}
f_{4} & =\frac{1}{2 \pi R_{1} C_{1}} \\
& =\frac{1}{2 \pi \times 6501 \mathrm{KA} \times 4 \% \mathrm{nF}} \\
& =0.5 \mathrm{~Hz}
\end{aligned}
$$

## R-Wave Detector



A-wat file

## Positive Feedback

- What happens to our Op-amp circuit if positive feedback is used?


Using KCL at the input :
$i_{1}+i_{2}+i_{\text {in }}=0$
$i_{i n}=0$; since Amplifier has infinite input impedance
$\frac{v_{i}-v_{i n}}{R}+\frac{v_{i}-v_{o}}{R}=0$
$v_{o}=A_{O L} v_{i}$ where $A_{O L}$ is the open loop gain
$v_{i}=\frac{1}{2}\left(v_{i n}+v_{o}\right)=\frac{1}{2}\left(v_{i n}+A_{O L} v_{i}\right)$

- Even if $v_{i n}=0$ and there is a slight voltage at $v_{i}$ then
- $v_{o}=A_{O L} v_{i}$ will increase $v_{i}$
- $v_{o}$ will grow even larger
- Eventually this will reach an extreme since there is not infinite energy in the circuit


## Positive Feedback

- Then our positive feedback design will operate between its positive and negative extremes, say $\pm 5 \mathrm{~V}$


$$
v_{i}=\frac{1}{2}\left(v_{i n}+v_{o}\right)
$$

$$
\text { For } v_{o} \text { to be at }+5 V, v_{i}>0
$$

$$
v_{i}=\frac{1}{2}\left(v_{i n}+v_{o}\right)=\frac{1}{2}\left(v_{i n}+5\right)>0
$$

$$
\text { This is true as long as } v_{i n}>-5 \mathrm{~V}
$$

$$
\text { For } v_{o} \text { to be at }-5 V, v_{i}<0
$$

$$
v_{i}=\frac{1}{2}\left(v_{i n}+v_{o}\right)=\frac{1}{2}\left(v_{i n}-5\right)<0
$$

This is true as long as $v_{i n}<5 \mathrm{~V}$

## Homework

- Probs 2.2, 2.5, 2.6, 2.10, 2.22, 2.24, 2.25, 2.28
- Calculate and plot the output vs frequency for these circuits. $R_{1}=1 \mathrm{k}, R_{2}=3 \mathrm{k}, C=1 \mu \mathrm{f}$ Use Matlab to perform the plot



## Homework



## Wheatstone Bridge - Strain Gauge

- A strain gauge shown here is used with a difference amplifier. Calculate the amplifier output signal as a function of $\Delta R$ and difference amplifier resistors. Assume that $\Delta R \ll R$.
- For the strain gauge calculate the value of each of the difference amplifier resistors for a value of $\mathrm{R}=10 \Omega$ and a percent change in R of $\pm 10 \%$ if the output of the amplifier is less than $\pm 5$ volts. Assume that the bridge is powered with 5 volts.


## Homework

Calculate and plot the output vs frequency for this circuits. $R_{1}=1 \mathrm{k}, R_{2}=3 \mathrm{k}, \mathrm{C}=1 \mu \mathrm{f}$. Use Matlab to perform the plot.


