Asymptote

2 Problems

2 Methods

Problems

Assume we have the following transfer function which has a zero at $f = 0$, a pole at $f = f_1$ and a pole at $f = f_2$. We are going to look at two problems: problem 1 is where $f_2 >> f_1$ and problem 2 is where $f_1 >> f_2$.

We will look at two methods to solve these two problems: one where the coefficient of the zero is $1/f_1$ and the other where it is $1/f_2$.

The goal is to determine the best way to calculate the Bode magnitude plot of the transfer function using asymptotes.

$$A = \frac{j\dfrac{f}{f_2}}{(1+j\dfrac{f}{f_1})(1+j\dfrac{f}{f_2})}$$

The process we will use is first to determine the asymptotes associated with this transfer function. Then divide the frequency spectrum into segments where these asymptotes defined. Then determine the composite asymptote by adding the asymptotes defined within each segment.

Problem 1: $f_2 >> f_1$
Method 1: Coefficient of the Zero = $1/f_2$

$$A_{db} = 20\log\left[\frac{\frac{f}{f_2}}{\sqrt{1+(\frac{f}{f_1})^2}\sqrt{1+(\frac{f}{f_2})^2}}\right] = 20\log(\frac{f}{f_2}) - 20\log\sqrt{1+(\frac{f}{f_1})^2} - 20\log\sqrt{1+(\frac{f}{f_2})^2}$$

$$= 20\log(\frac{f}{f_2}) - 10\log(1+(\frac{f}{f_1})^2) - 10\log(1+(\frac{f}{f_2})^2)$$

where $f_2 >> f_1$

3 Asymptotes can be determined from the logarithmic form

Asymptote1=$20\log(\frac{f}{f_2}) \Rightarrow$ Zero at $f=0$

Asymptote2=$-20\log(\frac{f}{f_1}) \Rightarrow$ Pole at $f=f_1$

Asymptote3=$-20\log(\frac{f}{f_2}) \Rightarrow$ Pole at $f=f_2$

Problem 1: $f_2 >> f_1$
Method 1: Coefficient of the Zero = $1/f_2$

To determine the composite, let's look are three regions (see the following drawing):

For $f > f_1$

Composite = Asymptote1 = $20\log(\frac{f}{f_2}) \Rightarrow$ Zero $\Rightarrow$ slope=+20db/decade

For $f_2 > f > f_1$

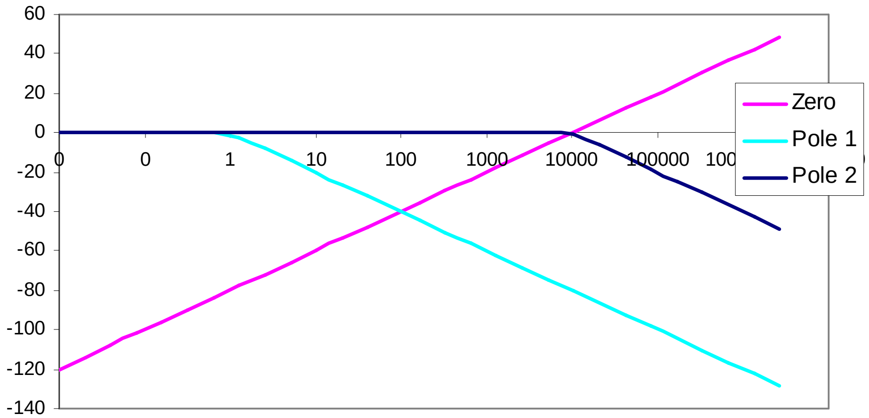
Composite = Asymptote1 + Asymptote2 = $20\log(\frac{f}{f_2}) - 20\log(\frac{f}{f_1}) \Rightarrow$ Zero + Pole at $f_1 \Rightarrow$ slope=0

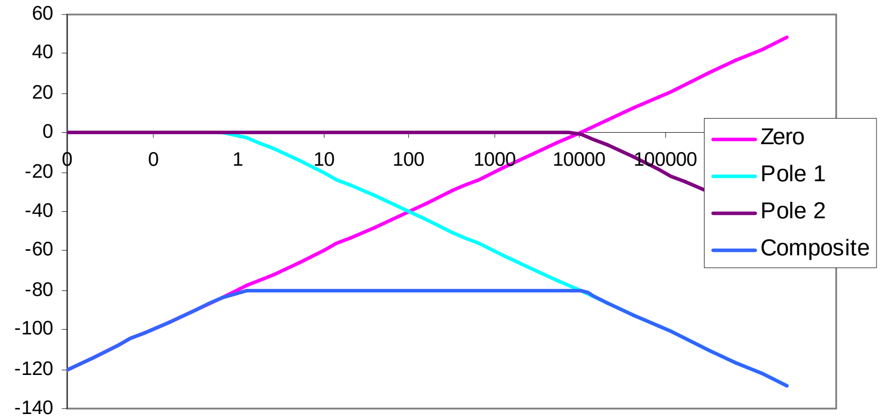
For $f > f_2$

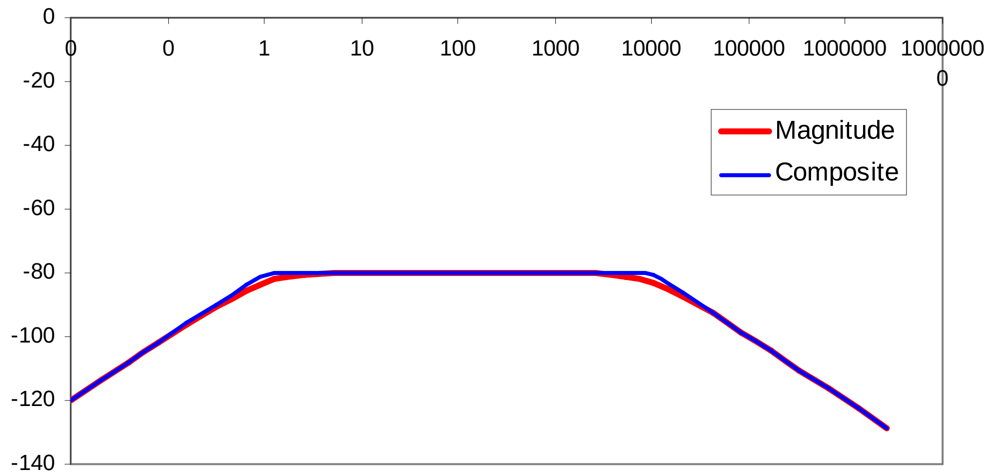
Composite = Asymptote1 + Asymptote2 + Asymptote3

$= 20\log(\frac{f}{f_2}) - 20\log(\frac{f}{f_1}) - 20\log(\frac{f}{f_2})$Zero + Pole at f_1 + Pole at $f_1 \Rightarrow$ slope=-20db/decade

Problem 1: $f_2 >> f_1$
Method 1: Coefficient of the Zero = $1/f_2$







Problem 1: $f_2 >> f_1$
Method 2: Coefficient of the Zero = $1/f_1$

To make the coefficient of the zero = $1/f_1$, we multiply the transfer function by f_1/f_1 and we get.

$$A = \frac{j\frac{f}{f_2}}{(1+j\frac{f}{f_1})(1+j\frac{f}{f_2})} = \frac{j\frac{f}{f_1}\frac{f_1}{f_2}}{(1+j\frac{f}{f_1})(1+j\frac{f}{f_2})} = \frac{f_1}{f_2}\frac{\frac{f}{f_1}}{\sqrt{1+(\frac{f}{f_1})^2}\sqrt{1+(\frac{f}{f_2})^2}}\angle\frac{\pi}{2} - \tan^{-1}(\frac{f}{f_1}) - \tan^{-1}(\frac{f}{f_2})$$

$$A_{db} = 20\log[\frac{f_1}{f_2}\frac{\frac{f}{f_2}}{\sqrt{1+(\frac{f}{f_1})^2}\sqrt{1+(\frac{f}{f_2})^2}}] = 20\log(\frac{f_1}{f_2}) + 20\log(\frac{f}{f_2}) - 20\log\sqrt{1+(\frac{f}{f_1})^2} - 20\log\sqrt{1+(\frac{f}{f_2})^2}$$

$$= 20\log(\frac{f_1}{f_2}) + 20\log(\frac{f}{f_2}) - 10\log(1+(\frac{f}{f_1})^2) - 10\log(1+(\frac{f}{f_2})^2)$$

where $f_2 >> f_1$

Problem 1: $f_2 >> f_1$
Method 2: Coefficient of the Zero = $1/f_1$

$$A_{db} = 20\log(\frac{f_1}{f_2}) + 20\log(\frac{f}{f_2}) - 10\log(1 + (\frac{f}{f_1})^2) - 10\log(1 + (\frac{f}{f_2})^2)$$

We now find there are 4 Asymptotes

$$\text{Asymptote1} = 20\log(\frac{f_1}{f_2}) \Rightarrow \text{Constant}$$

$$\text{Asymptote2} = 20\log(\frac{f}{f_2}) \Rightarrow \text{Zero at } f = 0$$

$$\text{Asymptote3} = -20\log(\frac{f}{f_1}) \Rightarrow \text{Pole at } f = f_1$$

$$\text{Asymptote4} = -20\log(\frac{f}{f_2}) \Rightarrow \text{Pole at } f = f_2$$

Problem 1: $f_2 >> f_1$
Method 2: Coefficient of the Zero = $1/f_1$

For $f > f_1$

$$\text{Composite} = \text{Asymptote1} + \text{Asymptote2} = 20\log(\frac{f_1}{f_2}) + 20\log(\frac{f}{f_2})$$

$\Rightarrow$ Constant + Zero $\Rightarrow$ slope=-20db/decade

For $f_2 > f > f_1$

$$\text{Composite} = \text{Asymptote1} + \text{Asymptote2} + \text{Asymptote3} = 20\log(\frac{f_1}{f_2}) + 20\log(\frac{f}{f_2}) - 20\log(\frac{f}{f_1})$$

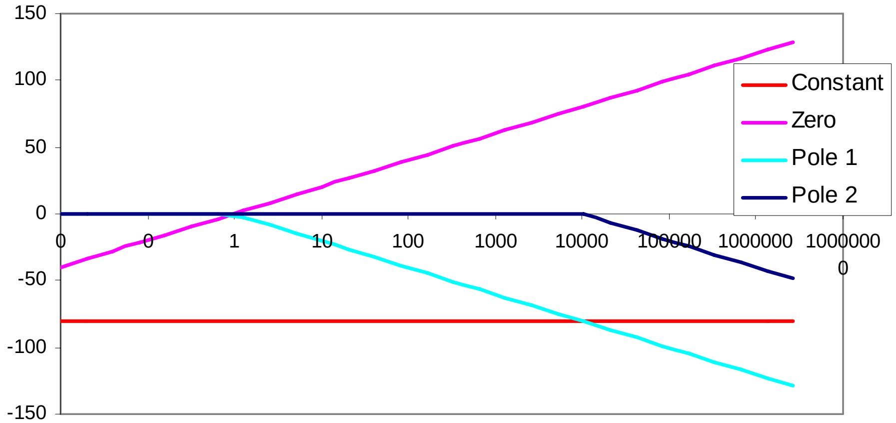
$\Rightarrow$ Constant + Zero + Pole at f_2 $\Rightarrow$ slope=0

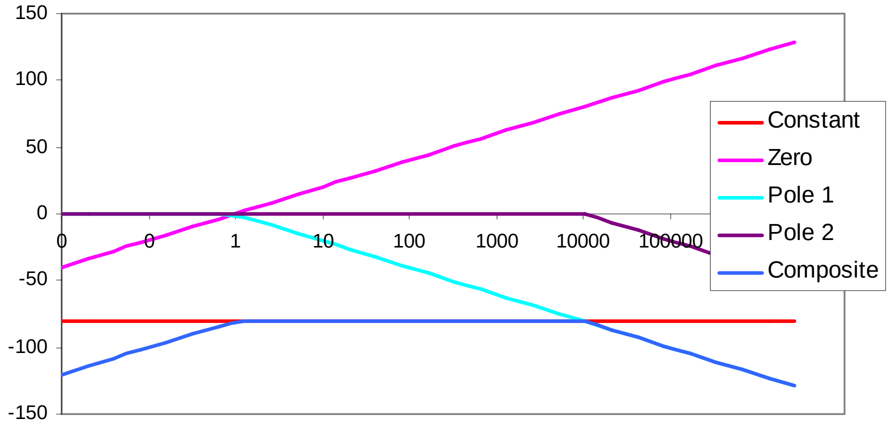
For $f > f_2$

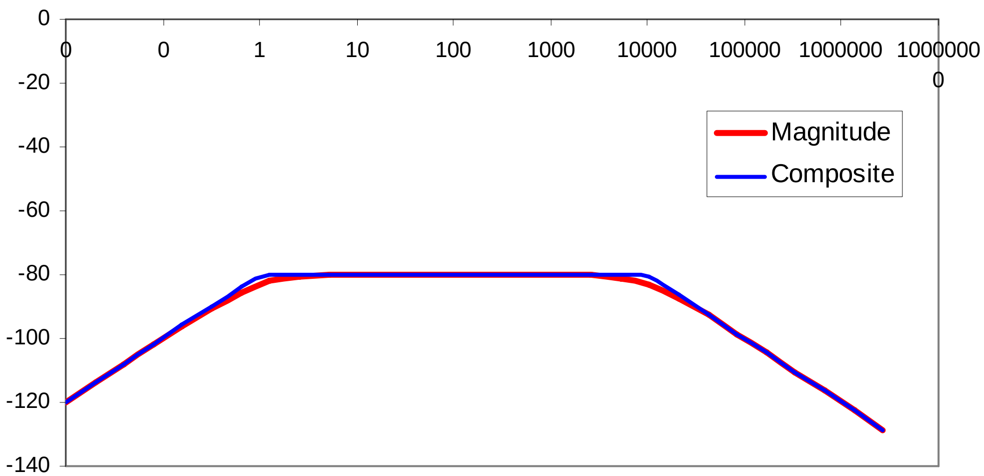
$$\text{Composite} = \text{Asymptote1} + \text{Asymptote2} + \text{Asymptote3} + \text{Aysmptote4} = 20\log(\frac{f_1}{f_2}) + 20\log(\frac{f}{f_2}) - 20\log(\frac{f}{f_1}) - 20\log(\frac{f}{f_2})$$

$\Rightarrow$ Constant + Zero + Pole at f_2 + Pole at f_1 $\Rightarrow$ slope=-20db/decade

Problem 1: $f_2 >> f_1$
Method 2: Coefficient of the Zero = $1/f_1$







Problem 2: $f_1 >> f_2$
Method 1: Coefficient of the Zero = $1/f_2$

$$A_{db} = 20\log\left[\frac{\frac{f}{f_2}}{\sqrt{1+(\frac{f}{f_1})^2}\sqrt{1+(\frac{f}{f_2})^2}}\right] = 20\log(\frac{f}{f_2}) - 20\log\sqrt{1+(\frac{f}{f_1})^2} - 20\log\sqrt{1+(\frac{f}{f_2})^2}$$

$$= 20\log(\frac{f}{f_2}) - 10\log(1+(\frac{f}{f_1})^2) - 10\log(1+(\frac{f}{f_2})^2)$$

where $f_1 >> f_2$

3 Asymptotes

Asymptote1$=20\log(\frac{f}{f_2}) \Rightarrow$ Zero at $f=0$

Asymptote2$=-20\log(\frac{f}{f_1}) \Rightarrow$ Pole at $f=f_1$

Asymptote3$=-20\log(\frac{f}{f_2}) \Rightarrow$ Pole at $f=f_2$

Problem 2: $f_1 >> f_2$
Method 1: Coefficient of the Zero = $1/f_2$

For $f > f_1$

$$\text{Composite} = \text{Asymptote1} = 20\log(\frac{f}{f_2}) \Rightarrow \text{Zero} \Rightarrow \text{slope=+20db/decade}$$

For $f_2 > f > f_1$

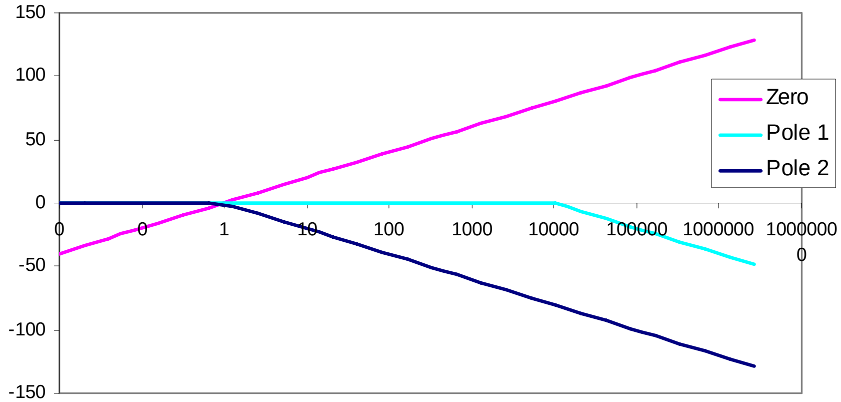
$$\text{Composite} = \text{Asymptote1} + \text{Asymptote2} = 20\log(\frac{f}{f_2}) - 20\log(\frac{f}{f_1}) \Rightarrow \text{Zero + Pole at } f_1 \Rightarrow \text{slope=0}$$

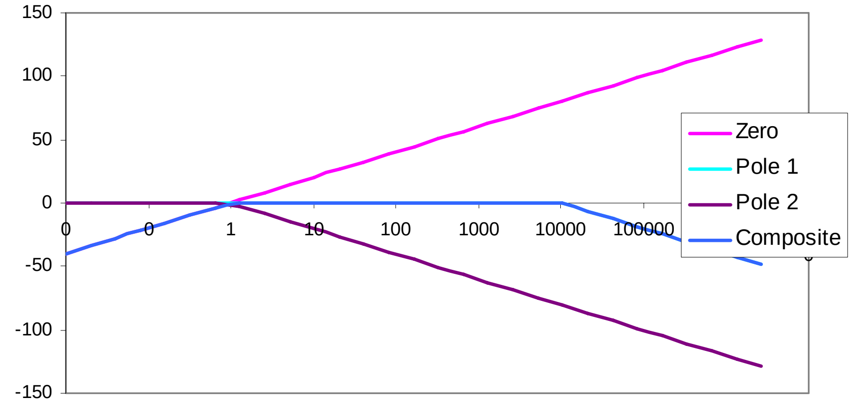
For $f > f_2$

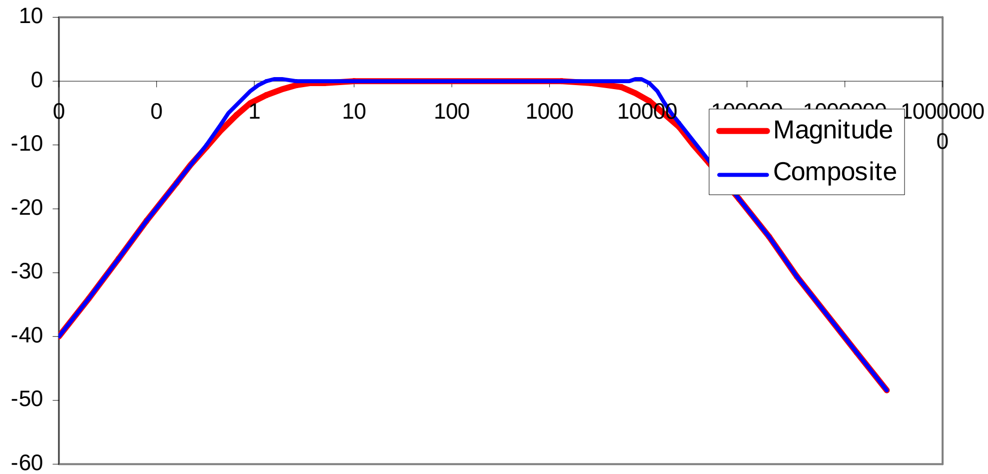
$$\text{Composite} = \text{Asymptote1} + \text{Asymptote2} + \text{Asymptote3}$$

$$= 20\log(\frac{f}{f_2}) - 20\log(\frac{f}{f_1}) - 20\log(\frac{f}{f_2})\text{Zero + Pole at } f_1 + \text{Pole at } f_1 \Rightarrow \text{slope=-20db/decade}$$

Problem 2: $f_1 >> f_2$
Method 1: Coefficient of the Zero = $1/f_2$







Problem 2: $f_1 >> f_2$
Method 2: Coefficient of the Zero = $1/f_1$

$$A = \frac{j\frac{f}{f_2}}{(1+j\frac{f}{f_1})(1+j\frac{f}{f_2})} = \frac{j\frac{f}{f_1}\frac{f_1}{f_2}}{(1+j\frac{f}{f_1})(1+j\frac{f}{f_2})} = \frac{f_1}{f_2}\frac{\frac{f}{f_1}}{\sqrt{1+(\frac{f}{f_1})^2}\sqrt{1+(\frac{f}{f_2})^2}}\angle\frac{\pi}{2}-\tan^{-1}(\frac{f}{f_1})-\tan^{-1}(\frac{f}{f_2})$$

$$A_{db} = 20\log[\frac{f_1}{f_2}\frac{\frac{f}{f_2}}{\sqrt{1+(\frac{f}{f_1})^2}\sqrt{1+(\frac{f}{f_2})^2}}] = 20\log(\frac{f_1}{f_2}) + 20\log(\frac{f}{f_2}) - 20\log\sqrt{1+(\frac{f}{f_1})^2} - 20\log\sqrt{1+(\frac{f}{f_2})^2}$$

$$= 20\log(\frac{f_1}{f_2}) + 20\log(\frac{f}{f_2}) - 10\log(1+(\frac{f}{f_1})^2) - 10\log(1+(\frac{f}{f_2})^2)$$

where $f_1 >> f_2$

4 Asymptotes

Asymptote1$=20\log(\frac{f_1}{f_2}) \Rightarrow$ constant

Asymptote2$=20\log(\frac{f}{f_2}) \Rightarrow$ Zero at $f=0$

Asymptote3$=-20\log(\frac{f}{f_1}) \Rightarrow$ Pole at $f=f_1$

Asymptote4$=-20\log(\frac{f}{f_2}) \Rightarrow$ Pole at $f=f_2$

Problem 2: $f_1 >> f_2$
Method 2: Coefficient of the Zero = $1/f_1$

For $f > f_1$

$$\text{Composite} = \text{Asymptote1} + \text{Asymptote2} = 20\log(\frac{f_1}{f_2}) + 20\log(\frac{f}{f_2})$$

$\Rightarrow$ Constant + Zero $\Rightarrow$ slope=-20db/decade

For $f_2 > f > f_1$

$$\text{Composite} = \text{Asymptote1} + \text{Asymptote2} + \text{Asymptote3} = 20\log(\frac{f_1}{f_2}) + 20\log(\frac{f}{f_2}) - 20\log(\frac{f}{f_1})$$

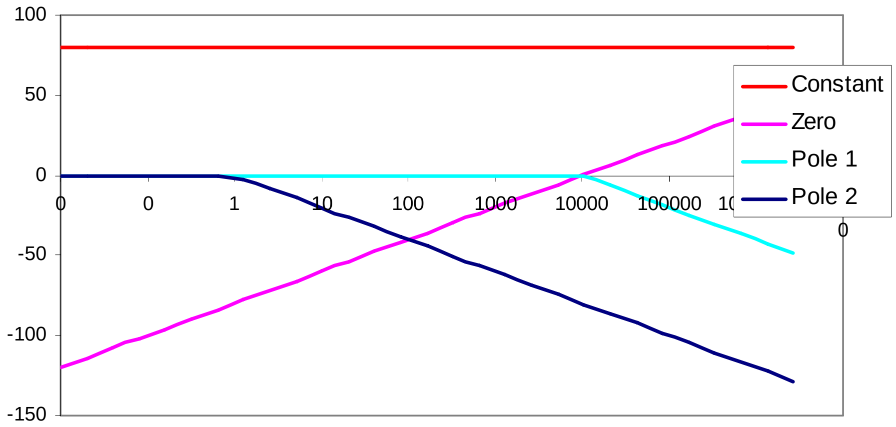
$\Rightarrow$ Constant + Zero + Pole at f_2 $\Rightarrow$ slope=0

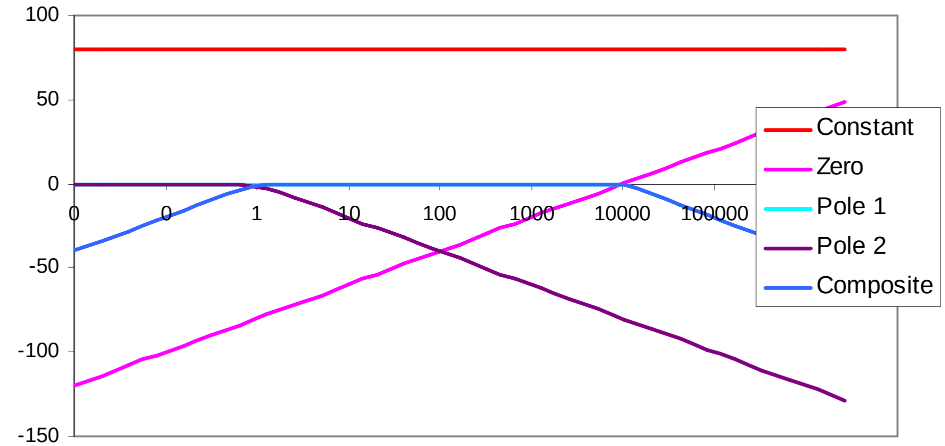
For $f > f_2$

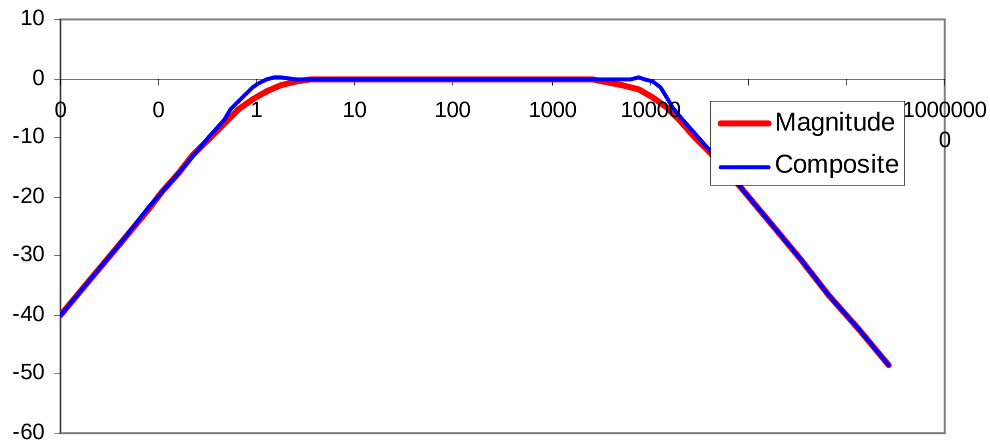
$$\text{Composite} = \text{Asymptote1} + \text{Asymptote2} + \text{Asymptote3} + \text{Aysmptote4} = 20\log(\frac{f_1}{f_2}) + 20\log(\frac{f}{f_2}) - 20\log(\frac{f}{f_1}) - 20\log(\frac{f}{f_2})$$

$\Rightarrow$ Constant + Zero + Pole at f_2 + Pole at f_1 $\Rightarrow$ slope=-20db/decade

Problem 2: $f_1 >> f_2$
Method 2: Coefficient of the Zero = $1/f_1$







Which one is easier?

- Whichever approach you chose you must be consistent and calculate the composite using all of the asymptotes in the region where they exist.