Feedback and Oscillators

Lesson #15

Transient and Frequency Response

Section 9.6-10

Closed-Loop Gain in the Frequency Domain

- Assume that both the open-loop gain, $A(s)$ and the feedback, $\beta(s)$ are functions of frequency and we apply the Laplace transform (complex) variable $s=\sigma+j\omega$:

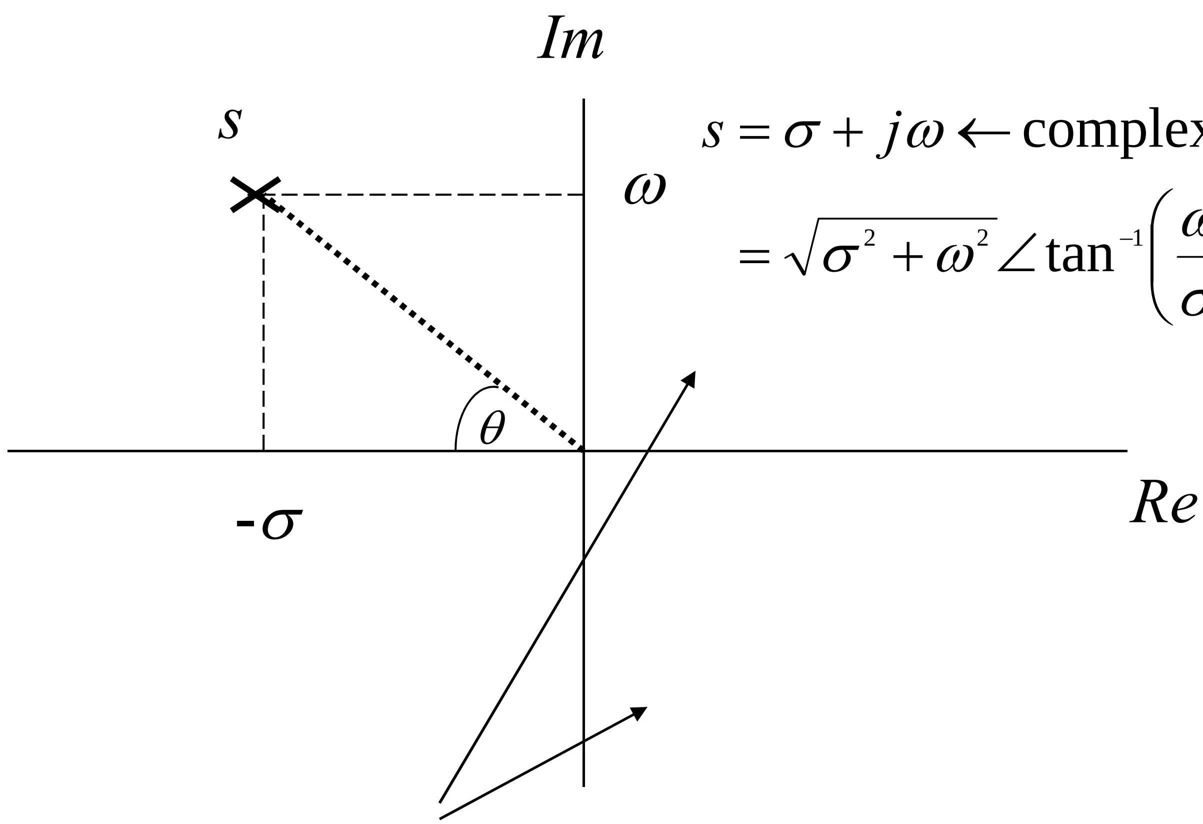
$$A_f(s) = \frac{A(s)}{1 + A(s)\beta(s)}$$

- Therefore,
  - the zeros of $A_f(s)$ are the values of s which satisfy $A(s)=0$
  - the poles of $A_f(s)$ are the values of s which satisfy $1+A(s)\beta(s)=0$

Poles

- Solutions to $1+A(s)\beta(s)=0$ define the transient response of the amplifier
- These solutions can be either real or complex values of s
- For real values $s=\pm\,\sigma$, the transient response will have the form exp ($\pm\,\sigma t$) and will dampen out ($s= -\,\sigma$ with time constant $1/\sigma$) or grow ($s= +\,\sigma$)
- Poles which are negative are desirable because the transient response dies out; while poles which are positive are undesirable because will cause the amplifier to function uncontrollably.

Complex Plane
Complex Number and Phasor Notation

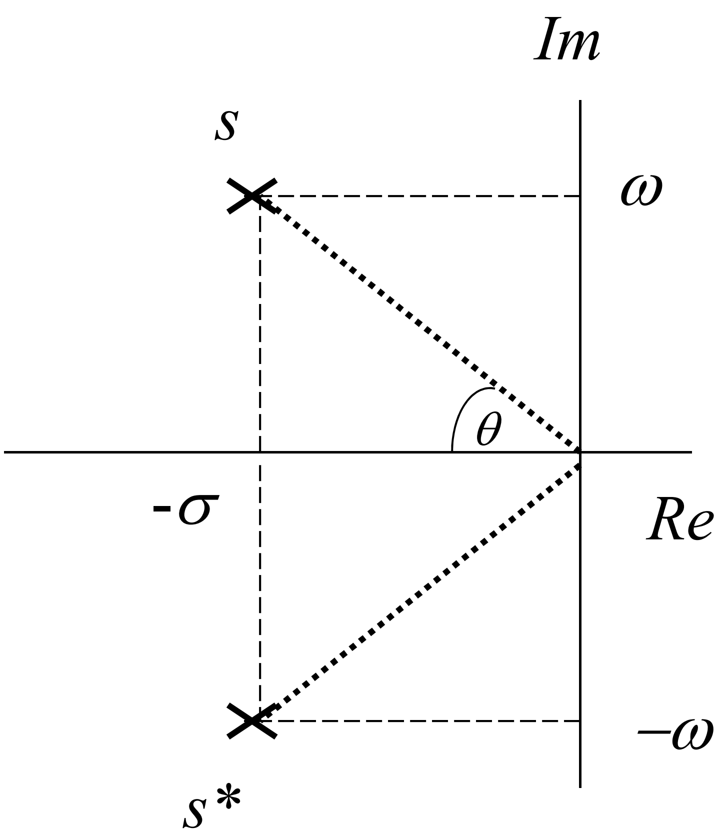


$$s = \sigma + j\omega \leftarrow \text{complex number}$$

$$= \sqrt{\sigma^2 + \omega^2} \angle \tan^{-1}\left(\frac{\omega}{\sigma}\right) \leftarrow \text{Phasor}$$

Note that poles in the right hand plane will have positive real parts and the transient response will grow exponentially

Complex Plane
Complex Conjugate Poles



$$s = \sigma + j\omega$$

$$s^* = \sigma - j\omega$$

$$s = \sqrt{\sigma^2 + \omega^2}\angle \tan^{-1}\left(\frac{\omega}{\sigma}\right)$$

$$s^* = \sqrt{\sigma^2 + \omega^2}\angle - \tan^{-1}\left(\frac{\omega}{\sigma}\right)$$

Complex conjugates yield

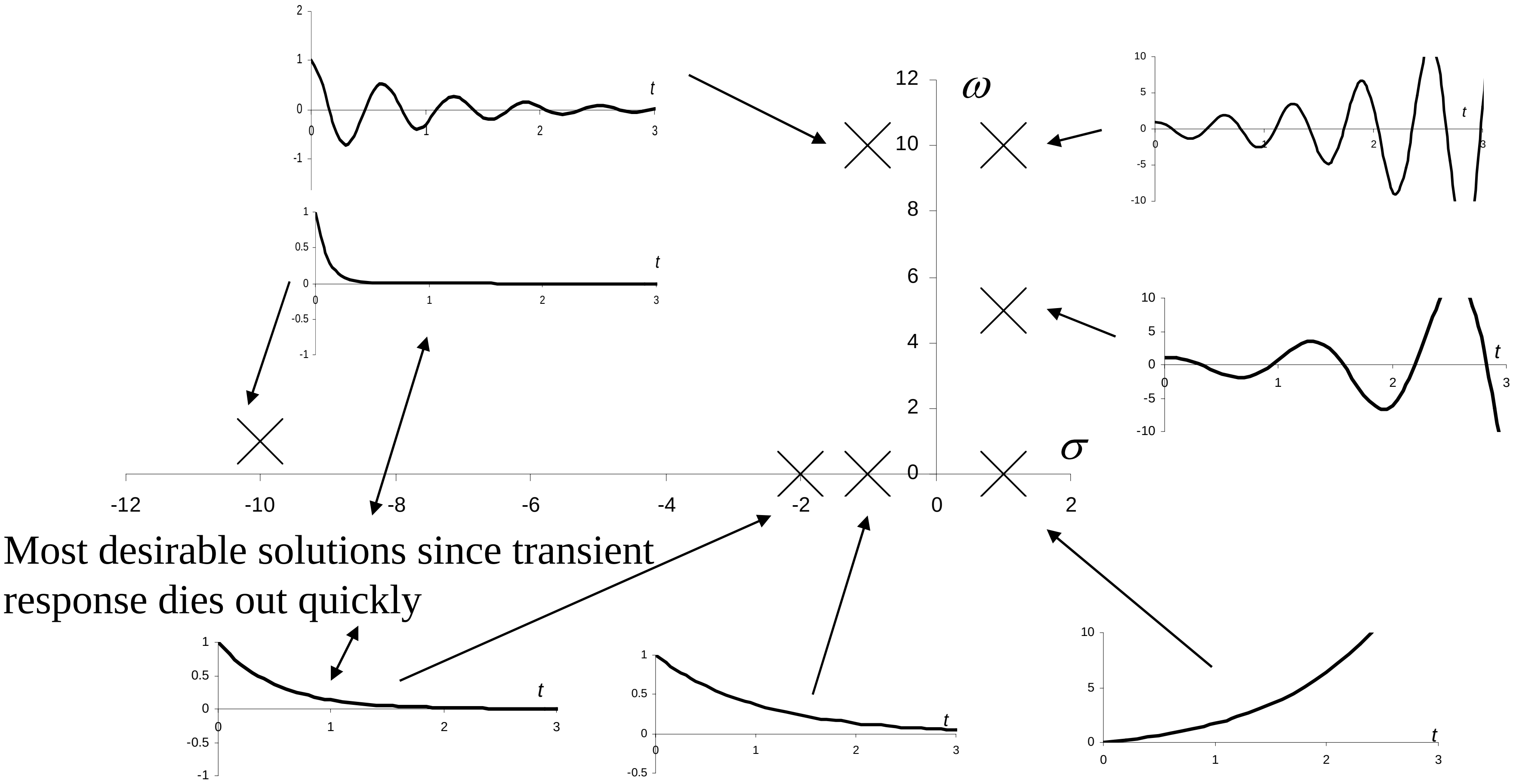
a transient response of the form:

$$e^{\pm\sigma t}(A\cos\omega t + B\sin\omega t)$$

Poles in the right hand plane will grow

while poles in the left hand plane dampen

Transient Solutions in the Complex Plane

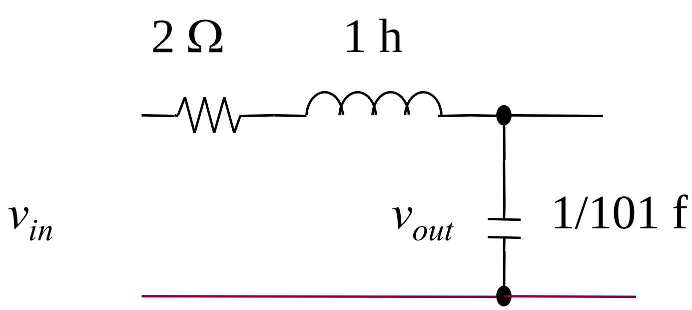


Note: Conjugates not shown

Frequency Response

- Plot the magnitude network function as a function of $s = j\omega$
- Sketch the magnitude by:
  - Calculate the magnitude for $\omega=0$ and $\omega \rightarrow \infty$
  - If there are real poles, estimate the breakpoint frequencies as $\omega = 1/s_{pk}$ and the value of the magnitude of the network function
  - If there are complex poles, estimate the maximum of the network function at the value of ω=Imaginary part for each pole
  - Set the function to zero at the zeros of the network function
- For network functions with only real poles:
  - Those with poles furthest from the origin have higher 3-dB cutoff frequencies
- For network functions with complex poles:
  - A gain peak will occur at the imaginary part ($j\omega$) of the pole
  - The gain peak will be smaller for those poles where the real part (σ) is greater than the imaginary part ($j\omega$) .

Example #1



$$V_0(s) = \frac{1/sC}{R + sL + 1/sC} V_{in}(s)$$

$$\frac{V_0(s)}{V_{in}(s)} = \frac{1}{sCR + s^2 LC + 1}$$

$$= \frac{1}{s\frac{1}{101}2 + s^2 1\frac{1}{101} + 1}$$

$$= \frac{101}{s^2 + 2s + 101} = \frac{101}{(s + 1 - j10)(s + 1 + j10)}$$

$s = -1 \pm j10$ transient response damped sinusoid

Frequency Response

$$\frac{V_0(s)}{V_{in}(s)} = \frac{101}{(s + 1 - j10)(s + 1 + j10)}$$

$$H(j\omega) = \frac{101}{(j\omega + 1 - j10)(j\omega + 1 + j10)}$$

$$|H(j\omega)| = \left|\frac{101}{(j\omega + 1 - j10)(j\omega + 1 + j10)}\right|$$

$$= \frac{101}{(\sqrt{1 + (\omega - 10)^2})(\sqrt{1 + (\omega + 10)^2})}$$

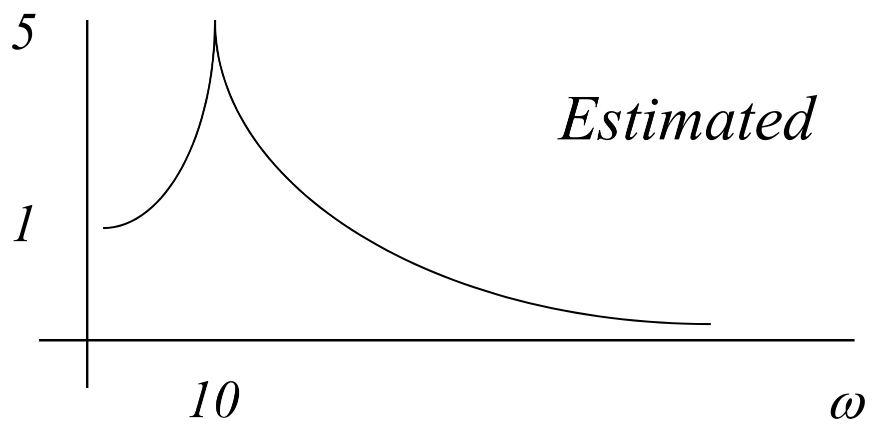
$$|H(0)| = \frac{101}{(\sqrt{1 + (10)^2})(\sqrt{1 + (10)^2})} = 1$$

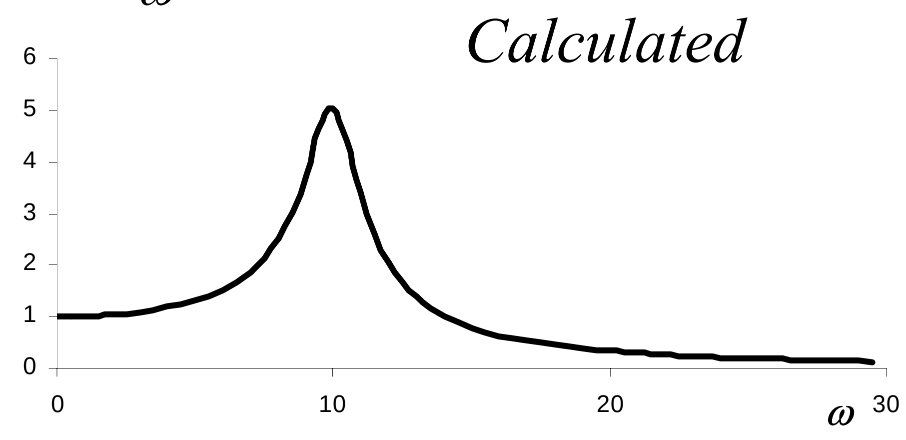
$$|H(j10)| = \frac{101}{(\sqrt{1 + (10 - 10)^2})(\sqrt{1 + (10 + 10)^2})}$$

$$= \frac{101}{\sqrt{1 + 20^2}} = \frac{101}{\sqrt{401}} = 5.04$$

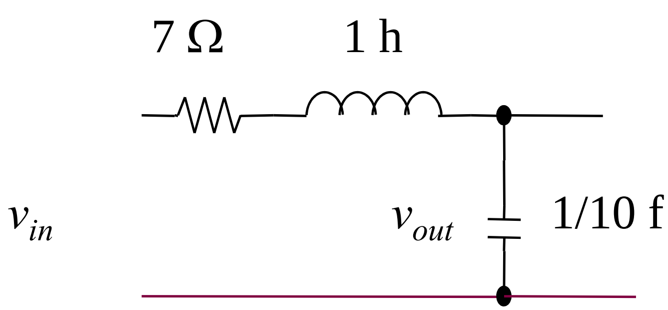
$$\lim_{\omega \to \infty} |H(j\omega)| \to 0$$

Example #1 (Continued)





Example #2



$$V_0(s) = \frac{1/sC}{R + sL + 1/sC} V_{in}(s)$$

$$\frac{V_0(s)}{V_{in}(s)} = \frac{1}{sCR + s^2LC + 1}$$

$$= \frac{1}{s\frac{1}{10}7 + s^2 1\frac{1}{10} + 1}$$

$$= \frac{10}{s^2 + 7s + 10} = \frac{10}{(s+2)(s+5)}$$

$s = -2, -5$ transient response

decaying exponentials

Frequency Response

$$\frac{V_0(s)}{V_{in}(s)} = \frac{10}{(s+2)(s+5)}$$

$$H(j\omega) = \frac{10}{(j\omega+2)(j\omega+5)}$$

$$|H(j\omega)| = \left|\frac{10}{(j\omega+2)(j\omega+5)}\right|$$

$$= \frac{10}{(\sqrt{4+\omega^2})(\sqrt{25+\omega^2})}$$

$$|H(0)| = \frac{10}{(\sqrt{4})(\sqrt{25})} = 1$$

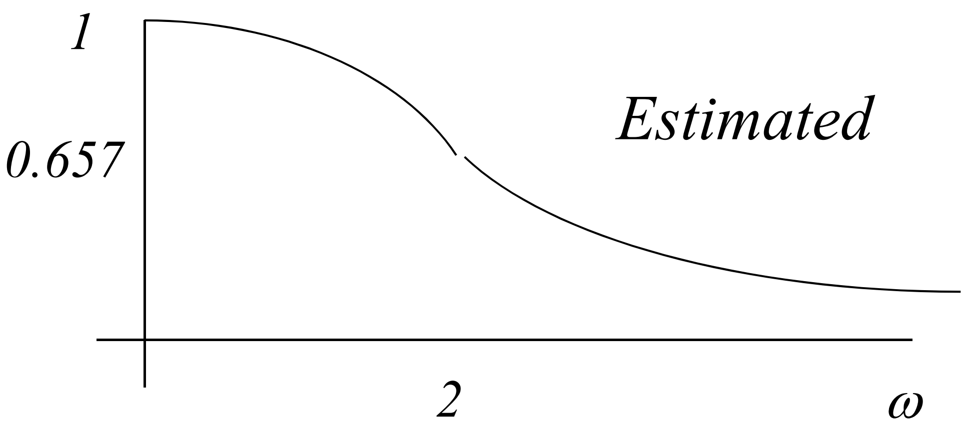
$$\lim_{\omega\to\infty} |H(j\omega)| \to 0$$

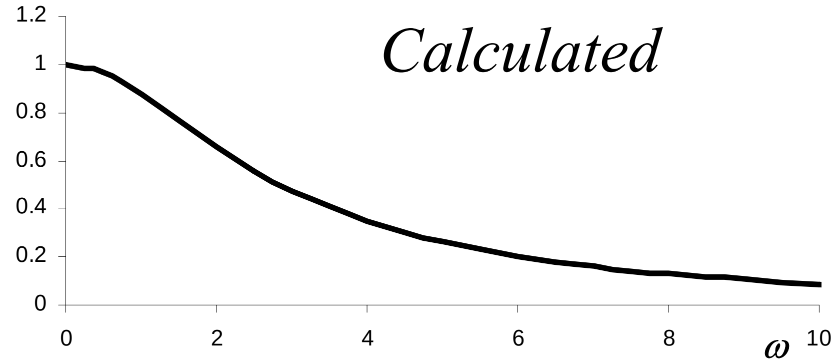
$$|H(2)| = \frac{10}{(\sqrt{4+4})(\sqrt{25+4})}$$

$$= \frac{10}{(2\sqrt{2})(\sqrt{29})} = \frac{5}{1.41 * 5.39}$$

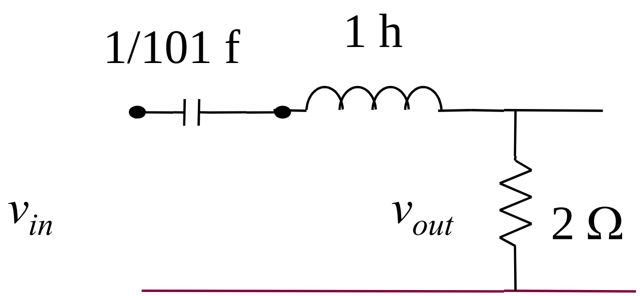
$$= \frac{5}{1.41 * 5.39} = .657$$

Example #2 (Continued)





Example #3



$$V_0(s) = \frac{R}{R + sL + 1/sC} V_{in}(s)$$

$$\frac{V_0(s)}{V_{in}(s)} = \frac{sCR}{sCR + s^2LC + 1}$$

$$= \frac{s\frac{1}{101}2}{s\frac{1}{101}2 + s^2 1\frac{1}{101} + 1}$$

$$= \frac{2s}{s^2 + 2s + 101} = \frac{2s}{(s+1-j10)(s+1+j10)}$$

$s = -1 \pm j10$ transient response damped sinusoid

Frequency Response

$$\frac{V_0(s)}{V_{in}(s)} = \frac{2s}{(s+1-j10)(s+1+j10)}$$

$$H(j\omega) = \frac{2s}{(j\omega+1-j10)(j\omega+1+j10)}$$

$$|H(j\omega)| = \left|\frac{2j\omega}{(j\omega+1-j10)(j\omega+1+j10)}\right|$$

$$= \frac{2\omega}{(\sqrt{1+(\omega-10)^2})(\sqrt{1+(\omega+10)^2})}$$

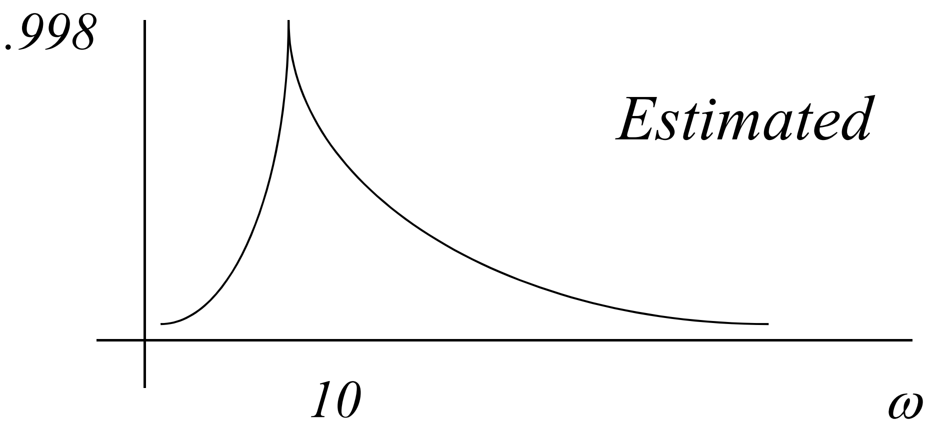
$$|H(0)| = \frac{2\times 0}{(\sqrt{1+(10)^2})(\sqrt{1+(10)^2})} = 0$$

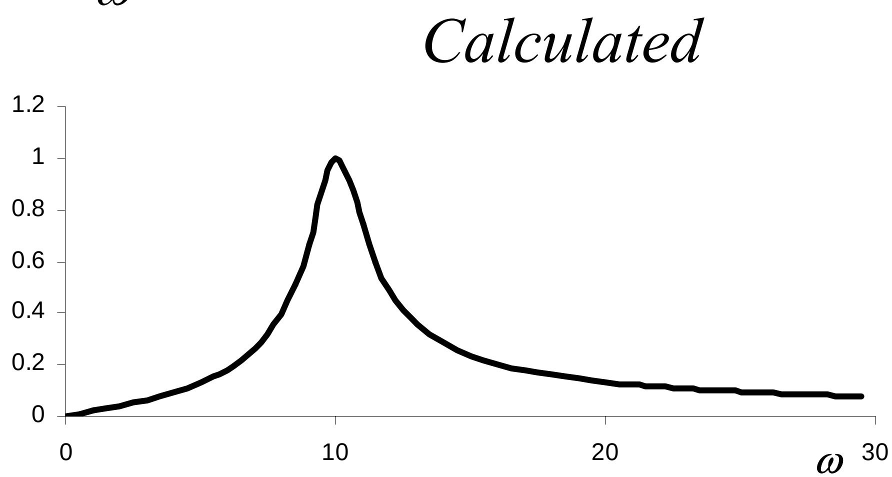
$$|H(j10)| = \frac{20j}{(\sqrt{1+(10-10)^2})(\sqrt{1+(10+10)^2})}$$

$$= \frac{20}{\sqrt{1+20^2}} = \frac{20}{\sqrt{401}} = .998$$

$$|H(j\omega)|_{\lim \omega\to\infty} \to 0$$

Example #3 (Continued)





Effects of Feedback on Pole Location Single Pole Amplifiers

Let's assume that the open-circuit gain of an amplifier is of the form:

$$A(s) = \frac{A_o}{(s/2\pi f_b) + 1}$$

Adding feedback to the amplifier:

$$A_f(s) = \frac{\dfrac{A_o}{(s/2\pi f_b) + 1}}{1 + \beta \dfrac{A_o}{(s/2\pi f_b) + 1}}$$

$$= \frac{A_o}{(s/2\pi f_b) + 1 + \beta A_o}$$

$$A_f(s) = \frac{A_o}{(s/2\pi f_b) + 1 + \beta A_o}$$

$$= \frac{A_o/(1 + \beta A_o)}{[s/2\pi f_b(1 + \beta A_o)] + 1}$$

$$= \frac{A_{of}}{(s/2\pi f_{bf}) + 1}$$

where:

$$A_{of} = \frac{A_o}{1 + \beta A_o},$$

$$f_{bf} = f_b(1 + \beta A_o)$$

Example #1

- Study the frequency response of an amplifier with open-loop mid-band gain =10^5 and break frequency f_b=10 Hz for feedback of β = 0.01, 0.1 and 1

Without feedback:

$$A_{o\,dB} = 20\log|A_o| = 20\log 10^5$$
$$= 20\times 5 = 100$$
$$f_b = 10\text{ Hz}$$

With $\beta = 0.01$:

$$A_{of} = \frac{A_o}{1+A_o\beta} = \frac{10^5}{1+.01\times 10^5}$$
$$= \frac{10^5}{1001} = 99.9$$
$$A_{of\,dB} = 20\log|A_{of}| = 20\log 99.9$$
$$= 40$$
$$f_{bf} = f_b(1+\beta A_o) = 10\times(1+0.01\times 10^5)$$
$$= 10\text{ k Hz}$$

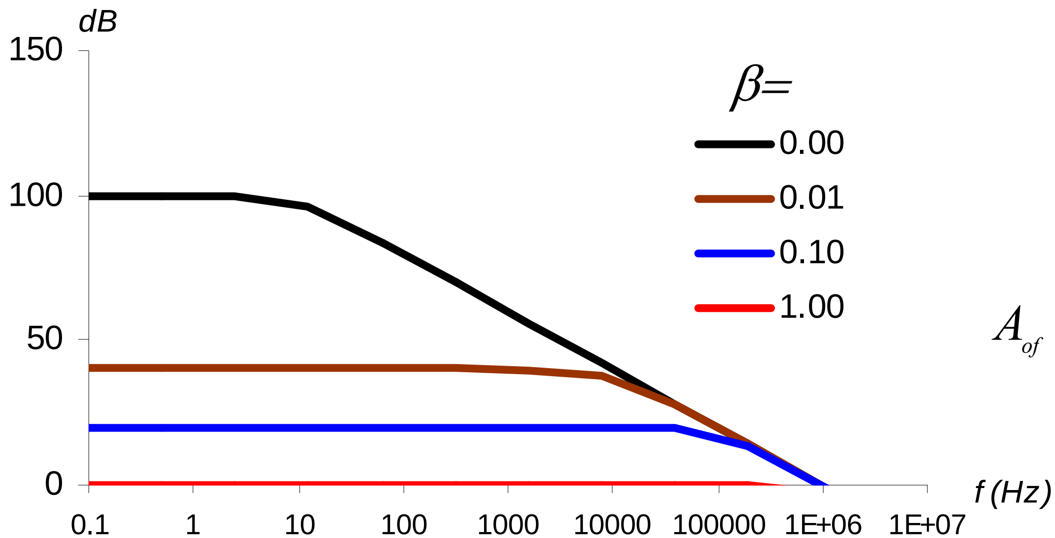
With $\beta = 0.1$:

$$A_{of} = \frac{A_o}{1+A_o\beta} = \frac{10^5}{1+.1\times 10^5} = 10$$
$$A_{of\,dB} = 20\log|A_{of}| = 20$$
$$f_{bf} = f_b(1+\beta A_o) = 100\text{ k Hz}$$

With $\beta = 1$:

$$A_{of} = \frac{A_o}{1+A_o\beta} = \frac{10^5}{1+1\times 10^5} = 1$$
$$A_{of\,dB} = 20\log|A_{of}| = 0$$
$$f_{bf} = f_b(1+\beta A_o) = 1\text{ M Hz}$$

Example #1 (Continued)
Gain-Bandwidth Product



$$A_{of} = \frac{A_o}{1+\beta A_o},$$

$$f_{bf} = f_b(1+\beta A_o)$$

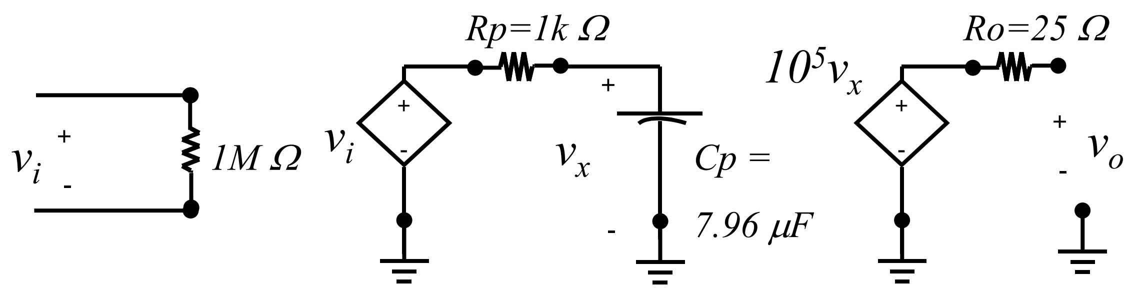
$$A_{of} \times f_{bf} = \frac{A_o}{1+\beta A_o} \times f_b(1+\beta A_o)$$

$$= A_o f_b$$

Note that as the feedback is increased (i.e., β increases) the pole moves further away from the origin. For single pole amplifiers, this also implies that the transient time constant decreases since $\tau = 1/2\pi f_{bf}$.

Example #2a

- For the Single Pole amplifier, find the pole for the open loop gain.
- Prepare a Bode Plot
- Find the Gain-Bandwidth Product



$$v_o = 10^5 v_x = 10^5 \frac{1/sC_p}{1/sC_p + R_p} v_i$$

$$A_{vo}(s) = \frac{v_o}{v_i} = \frac{10^5}{1 + sR_pC_p} = \frac{10^5}{1 + sR_pC_p}$$

$$= \frac{10^5}{1 + s(10^3)(7.96x10^{-6})} = \frac{10^5}{1 + 7.96x10^{-3} s}$$

$$A_{vo}(j\omega) = \frac{10^5}{1 + 7.96x10^{-3} j\omega}$$

$$A_{vo}(j0) = \frac{10^5}{1+0} = 10^5$$

$$A_{vo}(j0)_{dB} = 20\log 10^5 = 100dB$$

$$f_b = \frac{1}{2\pi(7.96x10^{-3})} = 20Hz$$

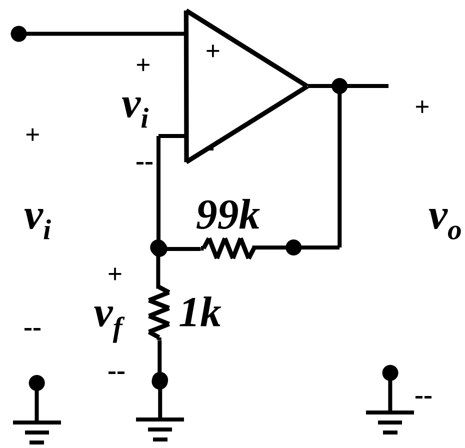
$$A_{vo}(j0) = A_o = \frac{10^5}{1+0} = 10^5$$

$$f_b = \frac{1}{2\pi(7.96x10^{-3})} = 20Hz$$

$$A_o \times f_b = 10^5 \times 20Hz = 2MHz$$

Example #2b

- Using the amplifier in 1b in the following feedback circuit, calculate:
  - β
  - Closed-loop gain at DC
  - Closed-loop bandwidth



$$\beta = \frac{1k}{1k + 99k} = .01$$

$$A_{of} = \frac{A_o}{1 + \beta A_o} = \frac{10^5}{1 + .01 \times 10^5}$$

$$= \frac{10^5}{1 + 10^3} = 99.9$$

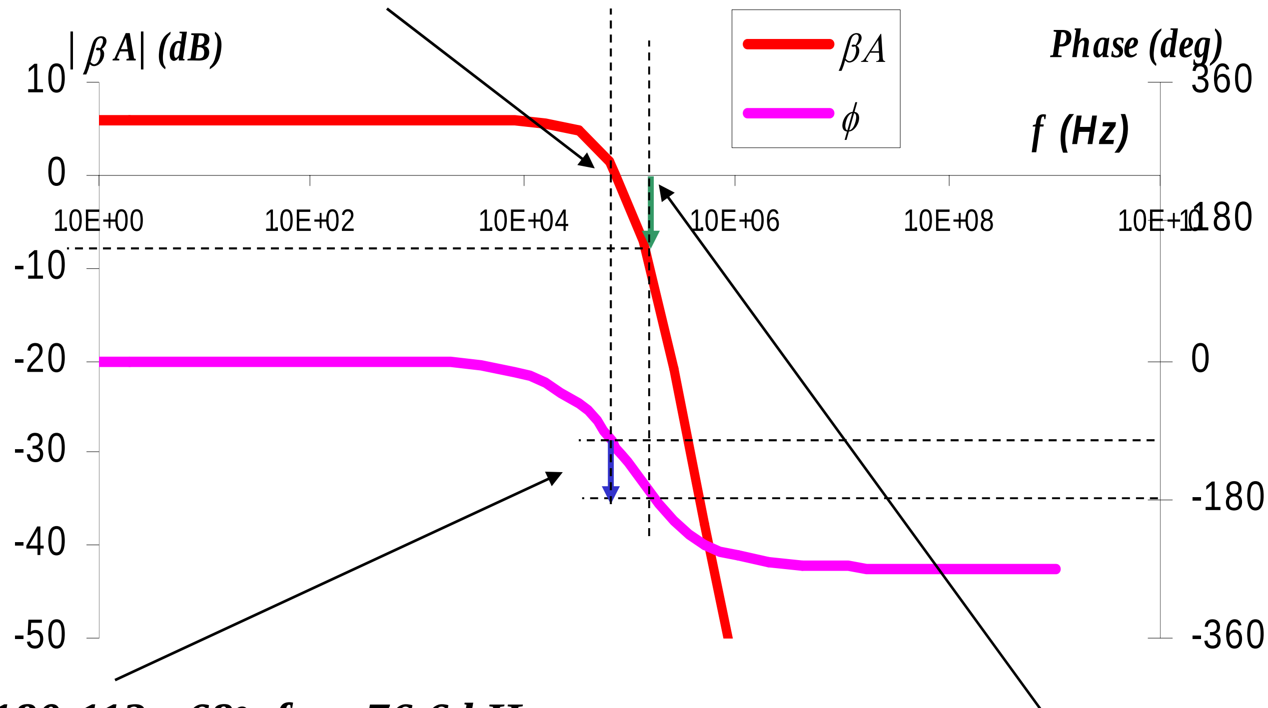
$$f_{bf} = f_b(1 + \beta A_o) = 20 \times (1 + 10^3)$$

$$= 20kHz$$

Gain and Phase Margin
Stability of Feedback Amplifiers

- Examine the Closed-loop gain as a function of frequency
- For a given frequency f_1, $\beta A(f_1) = -1$, then the closed-loop gain will be infinite (i.e., a pole at $s=j2\pi f_1$)
  - Note that without a source signal, then the input signal equals the feedback signal and $v_{in} = -\beta A(f_1)\ v_{out}$
  - When the phase of $\beta A(f_1) = 180^o$
    - if $|\beta A(f_1)| < 1$, the looped signal decays in amplitude –Stable
    - if $|\beta A(f_1)| > 1$, the looped signal grows in amplitude - Unstable
- We define Gain Margin, which is the amount (in dB) of gain below 0 dB when the phase of $\beta A(f_1) = 180^o$.
  - The larger the gain margin the more stable the amplifier (i.e., the poles are deeper into the left hand *s*-plane.
- We define the Phase Margin, which is defined at the frequency, f_{pm}, at which $\beta A(f_{pm})$ is unity and is equal to the difference between the phase of $\beta A(f_{pm})$ and 180^o.
  - The larger the phase margin the more stable the amplifier.

Stability of a Feedback Amplifier



PM = 180-112 =68º f_{PM}=76.6 kHz

GM = -9 dB f_{GM}=173 kHz

$$A(s) = \frac{1000}{(s/2\pi f_b + 1)^3}$$

$$f_b = 100kHz$$

$$\beta = 0.002$$

Homework

- Transient Response
  - Problems: 9.58-59
- Effects of Feedback on Pole Locations
  - Problems: 9.63-66
- Gain and Phase Margin
  - Problems: 9.72-73