MOSFET EQUIVALENT CIRCUITS

Lesson #4

Section 5.4-6

Small-Signal Equivalent Circuits

- As done for BJTs, we will investigate an equivalent circuit when the signal variations are small compared to the bias points
- Some nomenclature:
  - The values of the FET parameters at the Q point (i.e., the DC value) will be denoted by the capital letters with the subscript Q: I_{DQ} for the Q point drain current.
  - The portion of the signal which varies with time will be denoted by lower case letters and lower case subscripts: $i_d(t)$ for variable portion of the drain current
  - The total signal will be denoted by lower case letters with upper case subscripts: $i_D(t)$ is the total drain current and

$$i_D(t) = I_{DQ} + i_d(t)$$

$$v_{GS}(t) = V_{GSQ} + v_{gs}(t)$$

Small-Signal Equivalent Circuits (Continued)

$$i_D(t) = I_{DQ} + i_d(t)$$

$$v_{GS}(t) = V_{GSQ} + v_{gs}(t)$$

Recall that

$$i_D(t) = K(v_{GS}(t) - V_{to})^2$$

$$I_{DQ} + i_d(t) = K(V_{GSQ} + v_{gs}(t) - V_{to})^2$$

$$= K(V_{GSQ} - V_{to})^2 + 2K(V_{GSQ} - V_{to})v_{gs}(t) + Kv_{gs}(t)^2$$

But the Q point is also given by

$$I_{DQ} = K(V_{GSQ} - V_{to})^2$$

Therefore,

$$i_d(t) = 2K(V_{GSQ} - V_{to})v_{gs}(t) + Kv_{gs}(t)^2$$

Small-Signal Equivalent Circuits (Continued)

Assuming the we are dealing with small signals about the Q point and

$$\left|v_{gs}(t)\right| << \left|V_{GSQ} - V_{to}\right|$$

then

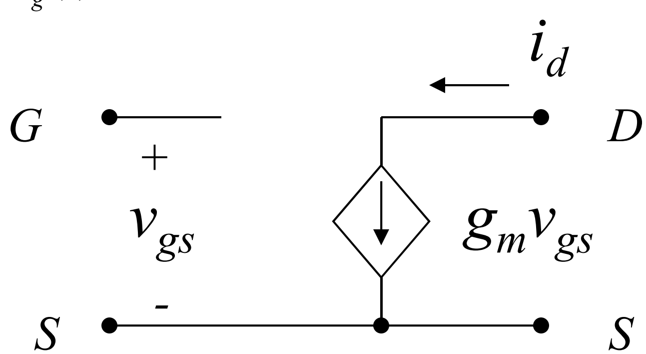
$$i_d(t) = 2K(V_{GSQ} - V_{to})v_{gs}(t)$$

Let's define, g_m, the transconductance as $g_m = 2K(V_{GSQ} - V_{to})$

$$i_d(t) = g_m v_{gs}(t)$$

also

$$i_g(t) = 0$$



Other forms of g_m

if $g_m = 2K(V_{GSQ} - V_{to})$

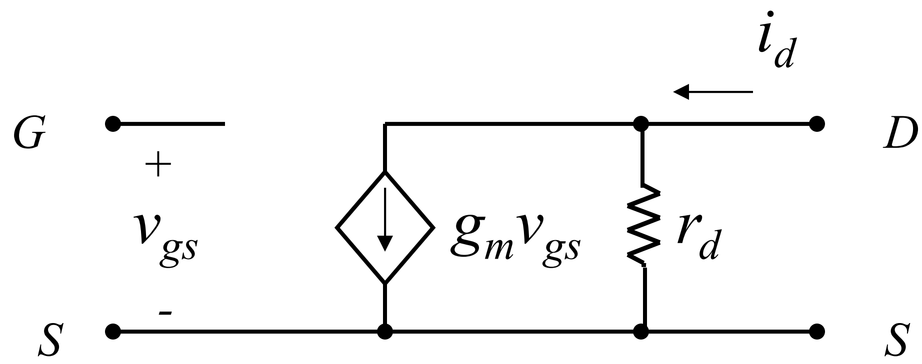
And

$$I_{DQ} = K(V_{GSQ} - V_{to})^2 \Rightarrow (V_{GSQ} - V_{to}) = \sqrt{\frac{I_{DQ}}{K}}$$

Then

$$g_m = 2K\sqrt{\frac{I_{DQ}}{K}} = 2\sqrt{KI_{DQ}} = 2\sqrt{K}\sqrt{I_{DQ}} = 2\sqrt{(\frac{W}{L})(\frac{KP}{2})}\sqrt{I_{DQ}}$$
$$= \sqrt{(W/L)}\sqrt{2KP}\sqrt{I_{DQ}}$$

Another More Complex Equivalent Circuit



Up till now we assumed the the FET curves in the saturation region were flat; however, if there is a slight slope, a small resistance in the drain should be added to reflect this. So:

$$i_d(t) = g_m v_{gs}(t) + \frac{v_{ds}(t)}{r_d}$$

Therefore,

$$g_m = \left.\frac{i_d}{v_{gs}}\right|_{v_{ds}=0} \cong \left.\frac{\Delta i_D}{\Delta v_{GS}}\right|_{v_{DS}=V_{DSQ}}$$

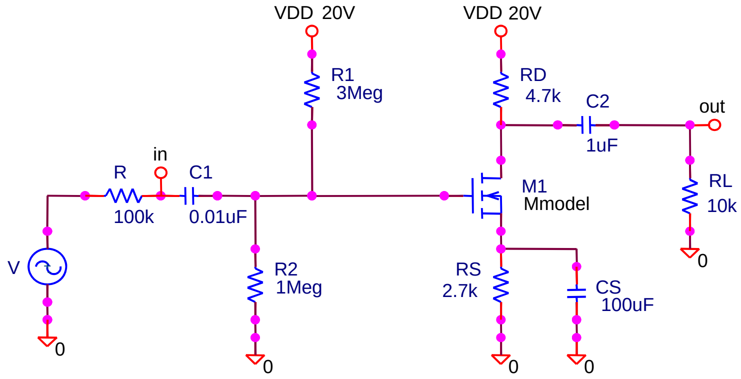
$$g_m = \left.\frac{\partial i_D}{\partial v_{GS}}\right|_{\text{Q-point}}$$

And

$$\frac{1}{r_d} = \left.\frac{i_d}{v_{ds}}\right|_{v_{gs}=0} \cong \left.\frac{\Delta i_D}{\Delta v_{DS}}\right|_{v_{GS}=V_{GSQ}}$$

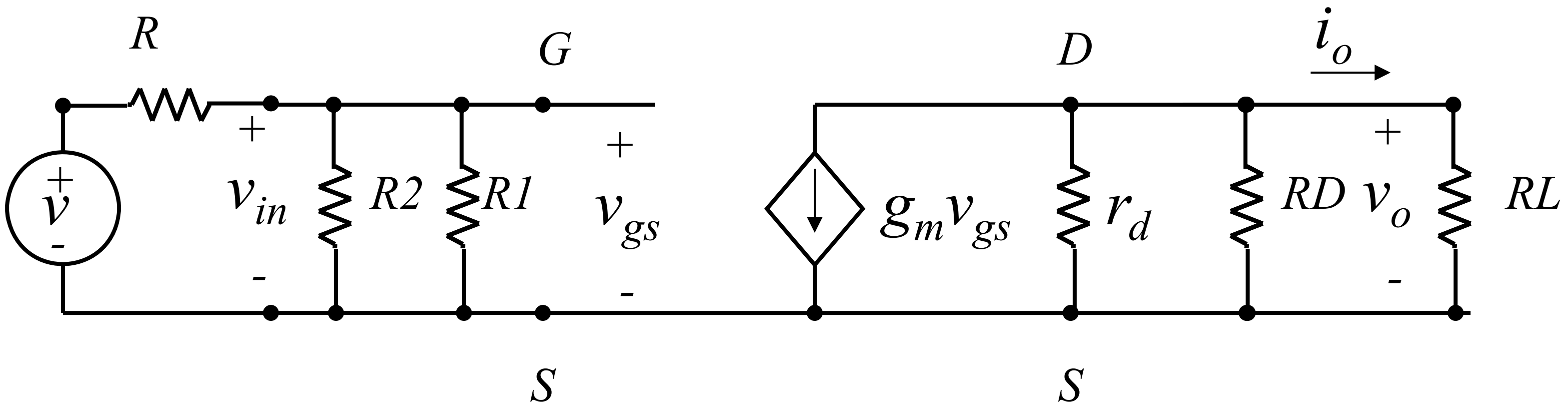
$$\frac{1}{r_d} = \left.\frac{\partial i_D}{\partial v_{DS}}\right|_{\text{Q-point}}$$

Application of the Small Signal Equivalent Circuit
Common Source Amplifier



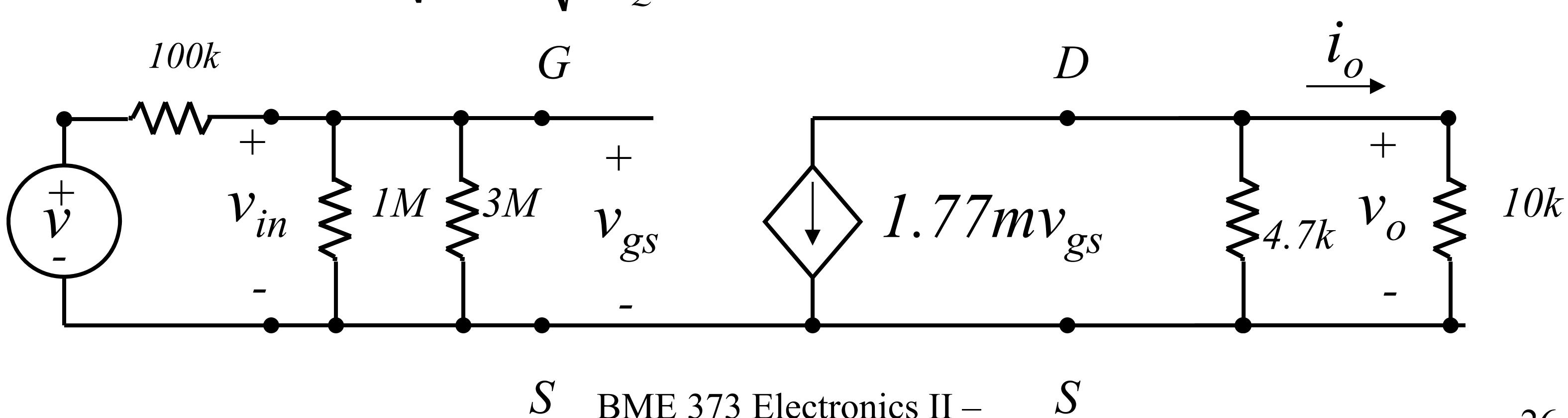
- Before we can apply the FET small signal equivalent circuit, we must review and reduce the circuit elements of this amplifier to those which are affected by a varying AC signal. In particular,
  - The coupling capacitors C1 and C2 are shorted
  - The bias resistor RS is shorted by the bypass capacitor CS
  - The DC voltage sources are shorted
- Note that the DC equivalent circuit is identical to the Self Bias circuit we previous investigated. Recall $I_{DQ} = 0.7833$ and $V_{DSQ} = 14.203$

Common Source Amplifier Equivalent Circuit

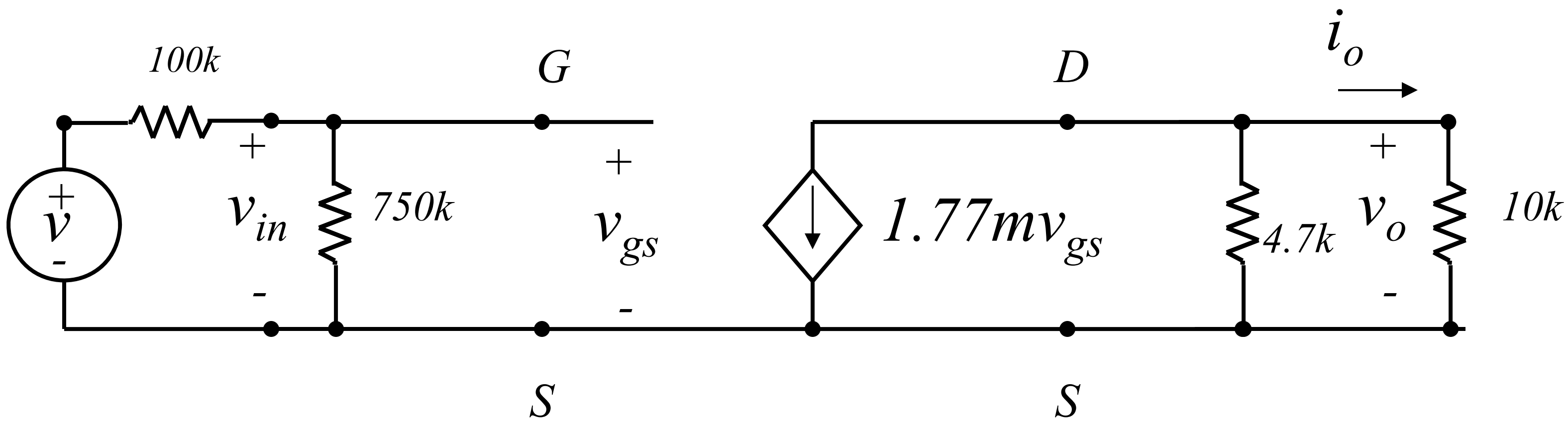


Assume that saturation region is flat and so $r_d = \infty$ and $KP=50mA/V^2$, $V_{to}=2$, $L=10\mu m$, $W=400\mu m$, $v=100sin(2000\pi t)\ mV$

$$g_m = \sqrt{2KP}\sqrt{W/L}\sqrt{I_{DQ}} = \sqrt{100e^{-6}}\sqrt{40}\sqrt{0.7833} = 1.77mS$$



Common Source Amplifier Equivalent Circuit

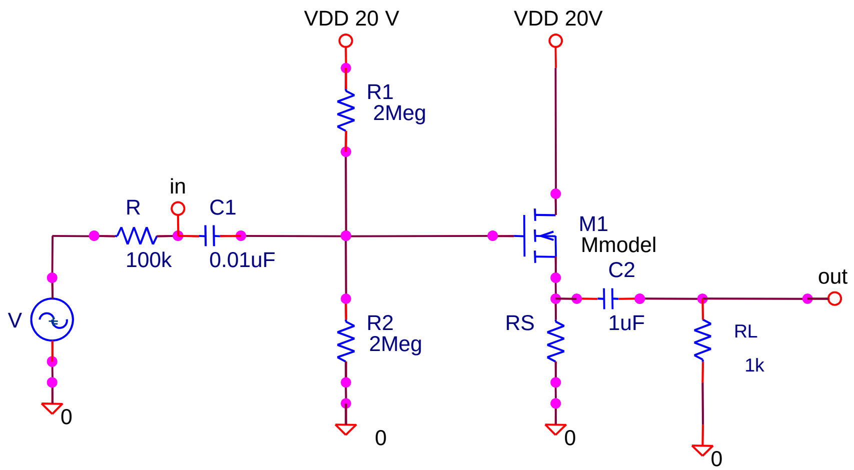


$$
\begin{aligned}
v_o &= i_o 10k \\
&= (-1.77\times10^{-3} v_{gs} \frac{4.7}{4.7+10})\times 10k \\
&= -5.66 v_{gs} \\
&= -5.66\times(\frac{750}{850}\times 100\sin(2000\pi t)mV) \\
&= -500\sin(2000\pi t)mV
\end{aligned}
$$

$$A_v = \frac{v_o}{v_{in}} = -5.66$$

$$A_{vi} = \frac{v_o}{v} = -5$$

Source Follower



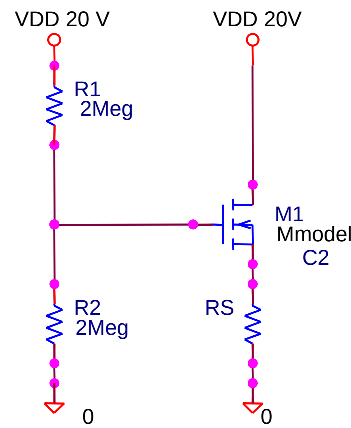
- In this circuit, we would like to find *RS* to support a Q-point value of the drain current, I_{DQ}, equal to 10 *mA*.
- Then solve for the gain of the amplifier using the small signal equivalent circuit.
- Assume $KP = 50\mu A/V^2$, $L = 2\ \mu m$, $W = 160\ \mu m$, $V_{to} = 1\ V$.

- To calculate *RS* let's first draw the equivalent DC circuit for the Q-point.
- This becomes the self bias circuit and we have

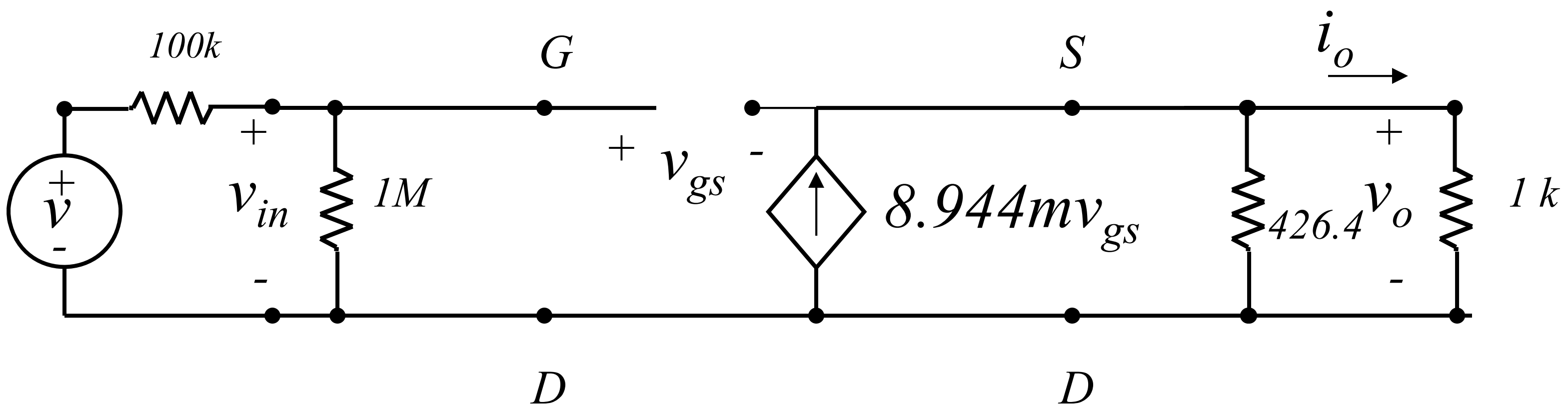
$$V_{GSQ} = [R2\ /\ (R1+R2)]\ x\ VDD - I_{DQ} * RS$$

$$I_{DQ} = K(V_{GSQ} - V_{t0})^2$$

- Solving the 2nd equation for $V_{GSQ} = 3.236\ V$ and substituting it into the 1st equation to yield $RS = 426.4\ \Omega$



Source Follower (Continued)



- Now we apply the small signal equivalent circuit realizing that the drain is grounded.

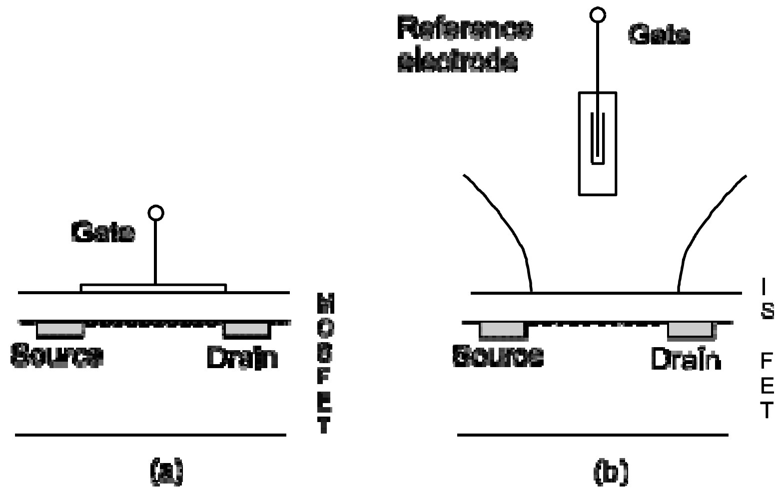
$$g_m = \sqrt{2KP}\sqrt{W/L}\sqrt{I_{DQ}} = \sqrt{100\times10^{-6}}\sqrt{80}\sqrt{10\times10^{-3}} = 8.944mS$$

$$v_0 = g_m v_{GS} \times RL' = g_m v_{GS} \times \left(\frac{RS \times RL}{RS + RL}\right) = 8.944\times10^{-3} \times \left[\frac{(426.4)(1000)}{(1000+426.4)}\right] v_{GS}$$

$$v_{in} = v_{GS} + v_0 = \frac{v_0}{g_m RL'} + v_0$$

$$\frac{v_0}{v_{in}} = \frac{g_m RL'}{1+g_m RL'} = .7278$$

Ion Sensing Field Effect Transistor (ISFET)



- $\Delta\phi = RT/F \ln (c_1/c_2)$
- R is the gas constant, T the absolute temperature (K) and F the Faraday constant and c_i, are ion concentrations in the solution and oxide.
- Using hydrogen ions can be used to measure pH[1] and DNA[2]

[1] Bergveld, P. **ISFET, Theory and Practice,** IEEE SENSOR CONFERENCE TORONTO, OCTOBER 2003

[2] DNA Electronics, http://dnae.co.uk/technology/overview/

Homework

- Small Signal Equivalent Circuits
  - Problems: 5.28, 5.29, 5.31, 5.32, 5.33
- Common-Source Amplifier
  - Problems: 5.36, 5.37, 5.38, 5.40
- Source Follower
  - Problems: 5.45, 5.47