Frequency Response

Lesson #9

Circuit Analysis

Sections 8.1

Frequency Response

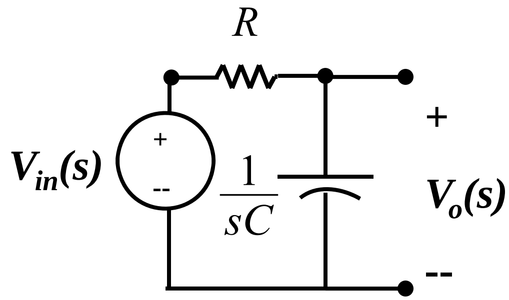
- To understand how electronic circuits are analyzed
- To understand electronic circuits responses to various frequencies

Bode Plots

- A method to understand how the frequency response of an electronic circuit can be viewed.
  - Signal processed by electronic circuits vary in frequency
    - Voice: 20 Hz to 20 kHz
    - Electrocardiograms: 0.05 Hz to 100 Hz
    - Video: DC to 4.5 MHz
  - Aids in designing how a circuit can be designed to avoid signal distortion.
- Some Mathematical Principles
  - Laplace Transform variable: s
    - Impedance of circuit parameters: Capacitance => $1/(sC)$
Inductance => sL
Resistance => R
  - To study frequency response: replace $s \Rightarrow j\omega$

Example:

- Network Function of a Low Pass Filter:



$$A_v(s) = \frac{V_o}{V_{in}}(s) = \frac{1/sC}{R + 1/sC} = \frac{1}{RCs + 1}$$

$$= \frac{1/RC}{s + 1/RC} = \frac{p}{s + p} = \frac{1}{\frac{s}{p} + 1}$$

$$A_v(j\omega) = \frac{1}{\frac{j\omega}{p} + 1} = \frac{1}{\frac{j2\pi f}{p} + 1} = \frac{1}{j(\frac{f}{f_p}) + 1}; \text{ where } f_p = \frac{p}{2\pi}$$

- Poles and Zeroes:
  - Poles: Values of s where the denominator of the network function are zero
  - Zeroes: Values of s where the numerator of the network function are zero
- Sanity Checks: at $\omega=0$, $\omega \rightarrow \infty$, at other ω' s (e.g., at poles or zero break frequencies or resonance frequencies)

Break Frequency

Let's subsitute $s \Rightarrow j\omega = j2\pi f$

$$A_v(f) = \frac{1}{j2\pi RCf + 1}$$

As an example: let's choose $f = 1/2\pi RC$

$$A_v(f) = \frac{1}{j1+1} = \frac{1}{\sqrt{1^2+1^2}\angle 45^\circ} = \frac{1}{\sqrt{2}}\angle -45^\circ = .707\angle -45^\circ$$

So if $Vin(t) = A\cos(2\pi ft)$, then at $f = 1/2\pi RC$, $V_o(t) = .707A\cos(t/RC - 45^\circ)$

Now let's generalize:

$$A_v(f) = \frac{\mathbf{V_o}}{\mathbf{V_{in}}} = \frac{1}{j(f/f_b)+1} = \frac{1}{\sqrt{1+(f/f_b)^2}\angle\tan^{-1}(f/f_b)} = \frac{1}{\sqrt{1+(f/f_b)^2}}\angle -\tan^{-1}(f/f_b);$$

where $f_b = \dfrac{1}{2\pi RC}$ and is called the break frequency (half power point or 3-dB point)

Bode Plot -
Logarithmic Plot of Magnitude vs. Frequency

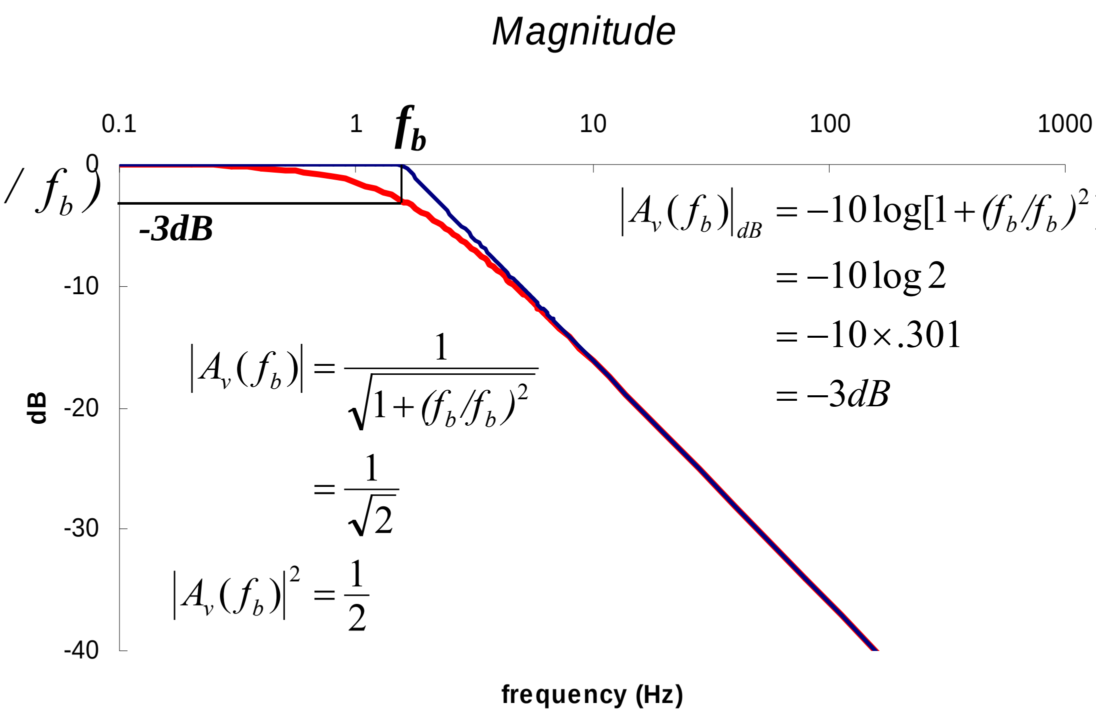
$$A_v(f) = \frac{1}{\sqrt{1+(f/f_b)^2}} \angle -\tan^{-1}(f/f_b)$$

$$|A_v(f)| = \frac{1}{\sqrt{1+(f/f_b)^2}}$$

$$|A_v(f)|_{dB} = -20\log(\sqrt{1+(f/f_b)^2})$$

$$= -10\log[1+(f/f_b)^2]$$

$$f_b = \pi/2 = 1.57$$



$$|A_v(f_b)|_{dB} = -10\log[1+(f_b/f_b)^2]$$
$$= -10\log 2$$
$$= -10\times.301$$
$$= -3dB$$

$$|A_v(f_b)| = \frac{1}{\sqrt{1+(f_b/f_b)^2}}$$
$$= \frac{1}{\sqrt{2}}$$
$$|A_v(f_b)|^2 = \frac{1}{2}$$

We see that for $f \approx 0, |A_v(f)|_{dB} = 0$

But for $f >> f_b$, $|A_v(f)|_{dB} = -10\log[(f/f_b)^2] = -20\log(f/f_b)$

which is a straight line of slope -20dB for each multiple of 10 (decade) in frequency and with intercept at f_b

Bode Plot - Logarithmic Plot of Angle vs. Frequency

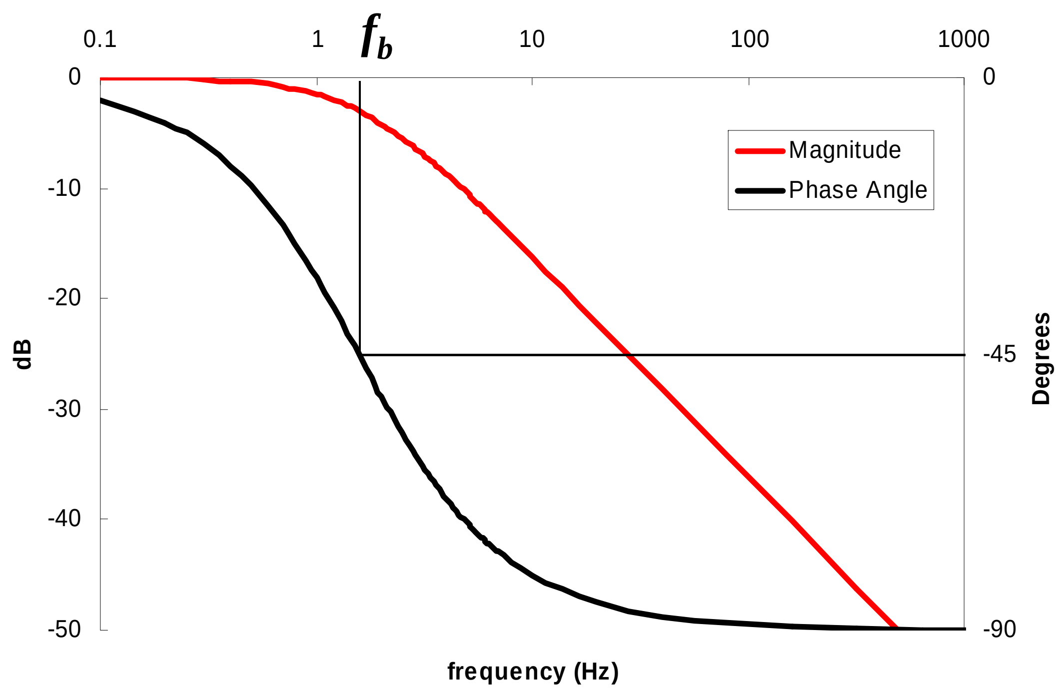
$\phi(f) = -\tan^{-1}(f/f_b)$

$f_b = \pi/2 = 1.57$

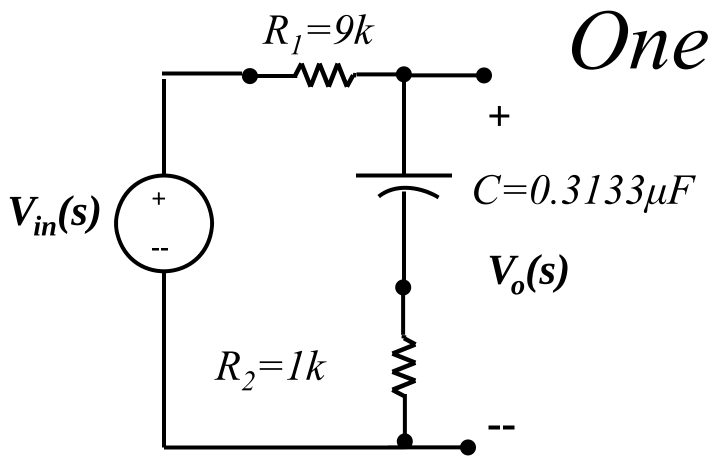
We see that for $f \approx 0$, $\phi(f) = 0^{\circ}$

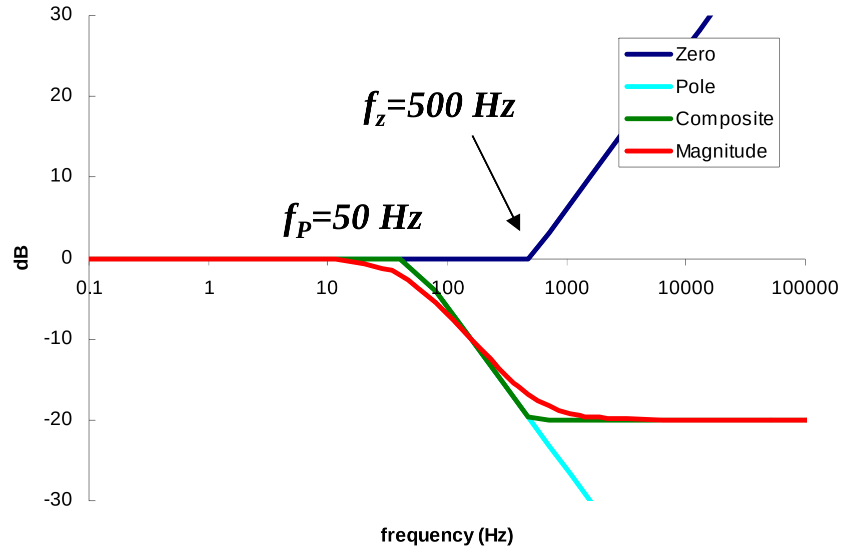
for $f = f_b$, $\phi(f_b) = -45^{\circ}$

But for $f >> f_b$, $\phi(f) \Rightarrow -90^{\circ}$



An Example: One Pole and One Zero





$$A_v(s) = \frac{V_o}{V_{in}}(s) = \frac{R_2 + 1/sC}{R_1 + R_2 + 1/sC}$$

$$= \frac{sCR_2 + 1}{sC(R_1 + R_2) + 1}$$

$$A_v(f) = \frac{j2\pi fCR_2 + 1}{j2\pi fC(R_1 + R_2) + 1}$$

$$A_v(f) = \frac{j(f/f_z) + 1}{j(f/f_p) + 1} = \frac{\sqrt{1 + (f/f_z)^2}}{\sqrt{1 + (f/f_p)^2}} \angle \tan^{-1}(f/f_z) - \tan^{-1}(f/f_p),$$

$$\text{where } f_z = \frac{1}{2\pi CR_2} \;=\; 500 \text{ Hz}$$

$$\text{and } f_p = \frac{1}{2\pi C(R_1 + R_2)} = 50 \text{ Hz}$$

$$\left|A_v(f)\right|_{dB} = 20\log\sqrt{1 + (f/f_z)^2} - 20\log\sqrt{1 + (f/f_p)^2}$$

$$\varphi(f) = \tan^{-1}(f/f_z) - \tan^{-1}(f/f_p)$$

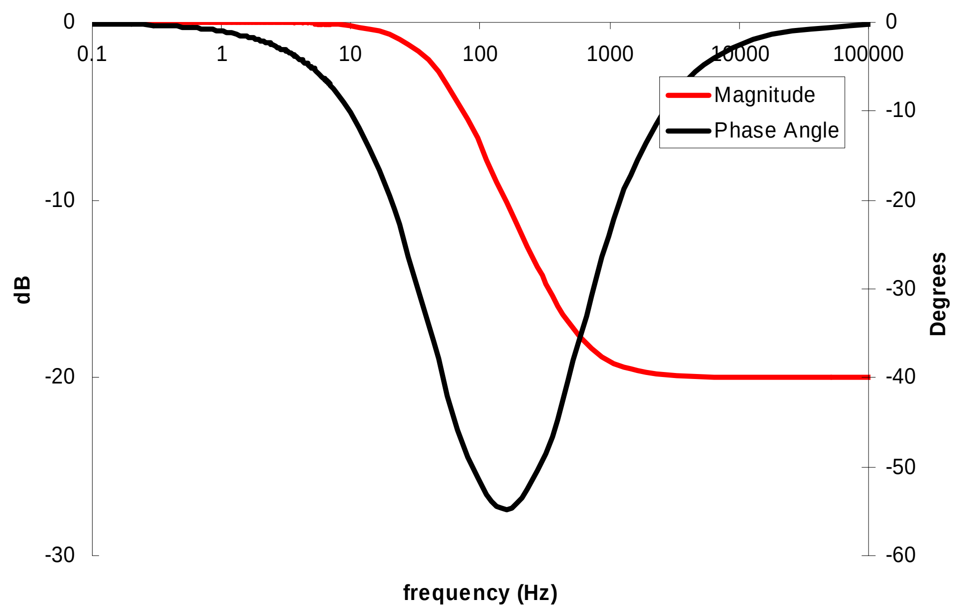
$$A_v(f) = \frac{j2\pi fCR_2 + 1}{j2\pi fC(R_1 + R_2) + 1}$$

$$A_v(f) = \frac{j2\pi fCR_2 + 1}{j2\pi fC(R_1 + R_2) + 1} \Rightarrow \frac{R_2}{(R_1 + R_2)} \quad as\ f \to \infty$$

$$A_v(f)_{f \to \infty} = \frac{1k}{(1k + 9k)} \;=.1$$

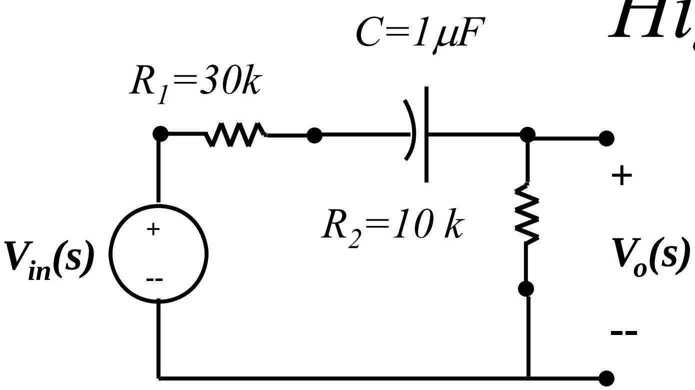
$$\left|A_v(f)\right|_{dB\ f \to \infty} = 20\log .1 = -20\ dB$$

Example (Continued)



$$\varphi(f) = \tan^{-1}(f / f_z) - \tan^{-1}(f / f_p)$$

Another Example: High Pass Filter



$$A_v(s) = \frac{V_o}{V_{in}}(s) = \frac{R_2}{R_1 + R_2 + 1/sC}$$

$$= \frac{sCR_2}{sC(R_1 + R_2) + 1}, \text{ pole at } s = -1/[(R_1 + R_2)C] \text{ and zero at } s = 0$$

$$A_v(f) = \frac{j2\pi fCR_2}{j2\pi fC(R_1 + R_2) + 1} = \left(\frac{R_2}{R_1 + R_2}\right)\left(\frac{j(f/f_p)}{j(f/f_p) + 1}\right)$$

$$\text{where } f_p = \frac{1}{2\pi(R_1 + R_2)C}$$

$$A_v(f) = \left(\frac{R_2}{R_1 + R_2}\right)\frac{f/f_p}{\sqrt{1 + (f/f_p)^2}}\angle 90^\circ - \tan^{-1}(f/f_p),$$

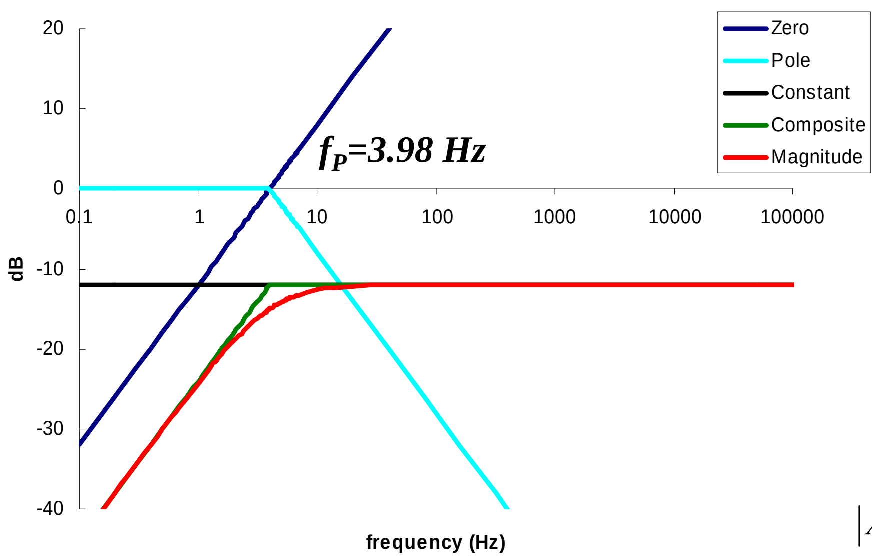
$$\text{where } f_p = \frac{1}{2\pi C(R_1 + R_2)} = 3.98 \text{ Hz}$$

$$A_v(f) = .25\frac{f/3.98}{\sqrt{1 + (f/3.98)^2}}\angle 90^\circ - \tan^{-1}(f/3.98_p)$$

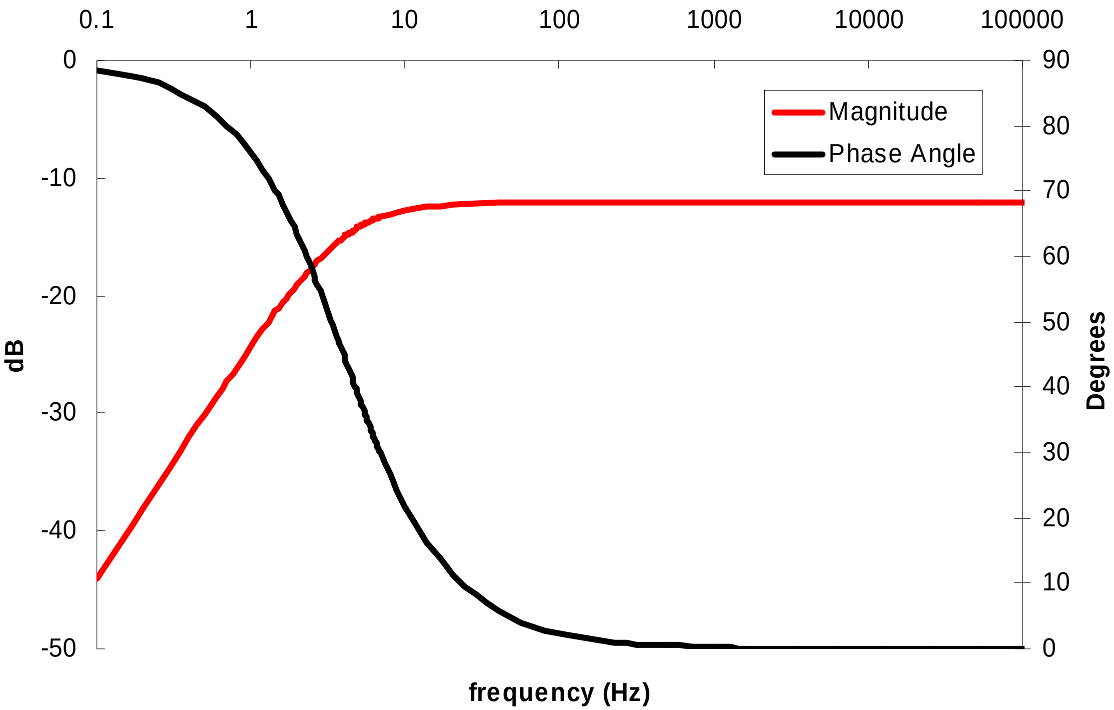
$$|A_v(f)|_{dB} = 20\log .25 + 20\log f/3.98 - 20\log\sqrt{1 + (f/3.98)^2}$$

$$= -12 + 20\log f/3.98 - 20\log\sqrt{1 + (f/3.98)^2}$$

$$|A_v(f)|_{dB\, f\to\infty} = -12$$

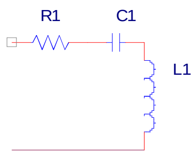


Another Example: High Pass Filter (Continued)



$$\phi(f) = 90^\circ - \tan^{-1}(f/3.98)$$

RLC Circuits



$$H(j\omega)=\frac{j\omega L}{R+j\omega L+\frac{1}{j\omega C}}$$

$$=\frac{j\omega L}{R+j(\omega L-\frac{1}{\omega C})}$$

$$=\frac{\omega L}{\sqrt{R^2+(\omega L-\frac{1}{\omega C})^2}}\angle 90^\circ-\tan^{-1}[\frac{(\omega L-\frac{1}{\omega C})}{R}]$$

OR

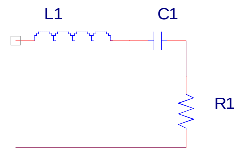
$$=\frac{-\omega^2 LC}{(1-\omega^2 LC)+j\omega RC}$$

$$=\frac{\omega^2 LC}{\sqrt{(1-\omega^2 LC)^2+(\omega RC)^2}}\angle 180^\circ-\tan^{-1}[\frac{\omega RC}{(1-\omega^2 LC)}]$$

$$H(j0)=0\angle 180^\circ$$

$$H(j\frac{1}{\sqrt{LC}})=\frac{1}{R}\sqrt{\frac{L}{C}}\angle 90^\circ$$

$$H(j\omega\to\infty)=1\angle 0^\circ$$



$$H(j\omega)=\frac{R}{R+j\omega L+\frac{1}{j\omega C}}$$

$$=\frac{R}{R+j(\omega L-\frac{1}{\omega C})}$$

$$=\frac{R}{\sqrt{R^2+(\omega L-\frac{1}{\omega C})^2}}\angle -\tan^{-1}[\frac{(\omega L-\frac{1}{\omega C})}{R}]$$

OR

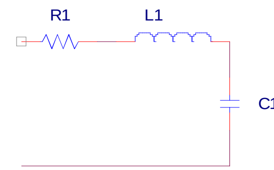
$$=\frac{j\omega RC}{(1-\omega^2 LC)+j\omega RC}$$

$$=\frac{\omega RC}{\sqrt{(1-\omega^2 LC)^2+(\omega RC)^2}}\angle 90^\circ-\tan^{-1}[\frac{\omega RC}{(1-\omega^2 LC)}]$$

$$H(j0)=0\angle 90^\circ$$

$$H(j\frac{1}{\sqrt{LC}})=1\angle 0^\circ$$

$$H(j\omega\to\infty)=0\angle -90^\circ$$



$$H(j\omega)=\frac{\frac{1}{j\omega C}}{R+j\omega L+\frac{1}{j\omega C}}$$

$$=\frac{\frac{1}{j\omega C}}{R+j(\omega L-\frac{1}{\omega C})}$$

$$=\frac{\frac{1}{\omega C}}{\sqrt{R^2+(\omega L-\frac{1}{\omega C})^2}}\angle -90^\circ-\tan^{-1}[\frac{(\omega L-\frac{1}{\omega C})}{R}]$$

OR

$$=\frac{1}{(1-\omega^2 LC)+j\omega RC}$$

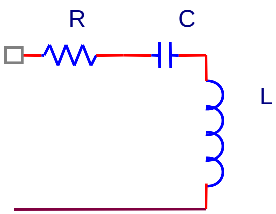
$$=\frac{1}{\sqrt{(1-\omega^2 LC)^2+(\omega RC)^2}}\angle -\tan^{-1}[\frac{\omega RC}{(1-\omega^2 LC)}]$$

$$H(j0)=1\angle 0^\circ$$

$$H(j\frac{1}{\sqrt{LC}})=\frac{1}{R}\sqrt{\frac{L}{C}}\angle -90^\circ$$

$$H(j\omega\to\infty)=1\angle 0^\circ$$

RLC Circuits



$$H(s)=\frac{sL}{R+sL+\frac{1}{sC}}=\frac{(sC)(sL)}{(sC)(sL)+sRC+1}=\frac{s^2}{s^2+s\frac{R}{L}+\frac{1}{LC}}=\frac{s^2}{(s-s_1)(s-s_2)}$$

$$\text{where } s_1,s_2=\frac{-RC\pm\sqrt{(RC)^2-4LC}}{2LC}=\frac{-\frac{R}{L}\pm\sqrt{(\frac{R}{L})^2-\frac{4}{LC}}}{2};\text{Let } p_1=-s_1,\,p_2=-s_2$$

$$p_1+p_2=\frac{\frac{R}{L}+\sqrt{(\frac{R}{L})^2-\frac{4}{LC}}}{2}+\frac{\frac{R}{L}-\sqrt{(\frac{R}{L})^2-\frac{4}{LC}}}{2}=\frac{R}{L};\,p_1\times p_2=\frac{\frac{R}{L}+\sqrt{(\frac{R}{L})^2-\frac{4}{LC}}}{2}\times\frac{\frac{R}{L}-\sqrt{(\frac{R}{L})^2-\frac{4}{LC}}}{2}=\frac{1}{4}[(\frac{R}{L})^2+\frac{R}{L}(\sqrt{(\frac{R}{L})^2-\frac{4}{LC}}-\sqrt{(\frac{R}{L})^2-\frac{4}{LC}})-\{(\frac{R}{L})^2-\frac{4}{LC}\}]=\frac{1}{LC}$$

$$H(s)=\frac{s^2}{(s+p_1)(s+p_2)}\Rightarrow H(j\omega)=\frac{(j\omega)^2}{(j\omega+p_1)(j\omega+p_2)}=\frac{-\frac{\omega^2}{p_1p_2}}{(j\frac{\omega}{p_1}+1)(j\frac{\omega}{p_2}+1)}=\frac{-\frac{(2\pi f)^2}{p_1p_2}}{(j\frac{2\pi f}{p_1}+1)(j\frac{2\pi f}{p_2}+1)}=\frac{-\frac{f^2}{f_1f_2}}{(j\frac{f}{f_1}+1)(j\frac{f}{f_2}+1)};\text{where } f_1=\frac{p_1}{2\pi};\,f_2=\frac{p_2}{2\pi}$$

$$=\frac{\frac{f^2}{f_1f_2}}{\sqrt{(\frac{f}{f_1})^2+1}\sqrt{(\frac{f}{f_2})^2+1}}\angle\pi-\tan^{-1}(\frac{f}{f_1})-\tan^{-1}(\frac{f}{f_2})$$

OR

$$=\frac{-\frac{f^2}{f_1f_2}}{(1-\frac{f^2}{f_1f_2})+j(\frac{f}{f_2}+\frac{f}{f_1})}=\frac{\frac{f^2}{f_1f_2}}{\sqrt{(1-\frac{f^2}{f_1f_2})^2+(\frac{f}{f_2}+\frac{f}{f_1})^2}}\angle\pi-\tan^{-1}(\frac{\frac{f}{f_2}+\frac{f}{f_1}}{1-\frac{f^2}{f_1f_2}})$$

RLC Circuits

$$H(j0)=0\angle 180^\circ$$

$$H(jf_1)=\frac{\frac{f_1}{f_2}}{\sqrt{2}\sqrt{(\frac{f_1}{f_2})^2+1}}\angle\pi-\tan^{-1}(1)-\tan^{-1}(\frac{f_1}{f_2})=\frac{f_1}{\sqrt{2}\sqrt{(f_1)^2+(f_2)^2}}\angle\frac{3\pi}{4}-\tan^{-1}(\frac{f_1}{f_2})$$

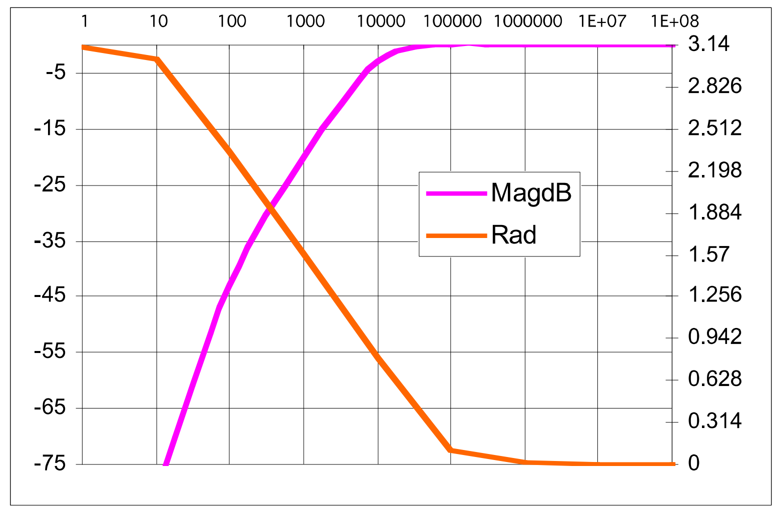
$$H(j\sqrt{f_1f_2})=\frac{1}{\frac{\sqrt{f_1f_2}}{f_2}+\frac{\sqrt{f_1f_2}}{f_1}}\angle\pi-\frac{\pi}{2}=\frac{\frac{1}{\sqrt{f_1f_2}}}{\frac{f_1+f_2}{f_1f_2}}\angle\frac{\pi}{2}=\frac{\sqrt{f_1f_2}}{f_1+f_2}\angle\frac{\pi}{2}=\frac{\sqrt{\frac{1}{LC}}}{\frac{R}{L}}\angle\frac{\pi}{2}$$

$$H(jf_2)=\frac{\frac{f_2}{f_1}}{\sqrt{2}\sqrt{(\frac{f_2}{f_1})^2+1}}\angle\pi-\tan^{-1}(\frac{f_2}{f_1})-\tan^{-1}(1)=\frac{f_2}{\sqrt{2}\sqrt{(f_1)^2+(f_2)^2}}\angle\frac{3\pi}{4}-\tan^{-1}(\frac{f_2}{f_1})$$

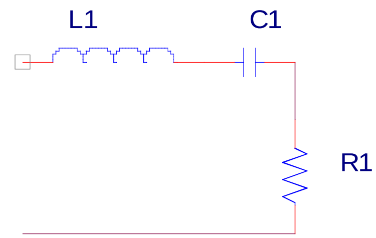
$$\text{If } f_2>>f_1; H(jf_2)=\frac{f_2}{\sqrt{2}\sqrt{(f_1)^2+(f_2)^2}}\angle\frac{3\pi}{4}-\tan^{-1}(\frac{f_2}{f_1})\approx\frac{1}{\sqrt{2}}\angle\frac{3\pi}{4}-\frac{\pi}{2}=\frac{1}{\sqrt{2}}\angle\frac{\pi}{4}$$

$$H(j\omega\to\infty)=1\angle 0^\circ$$

f_1=100 Hz
f_2=1000 Hz



RLC Circuits



$$H(s)=\frac{R}{R+sL+\frac{1}{sC}}=\frac{(sC)(R)}{(sC)(sL)+sRC+1}=\frac{s\frac{R}{L}}{s^2+s\frac{R}{L}+\frac{1}{LC}}=\frac{s\frac{R}{L}}{(s-s_1)(s-s_2)}$$

$$\text{where } s_1,s_2=\frac{-RC\pm\sqrt{(RC)^2-4LC}}{2LC}=\frac{-\frac{R}{L}\pm\sqrt{(\frac{R}{L})^2-\frac{4}{LC}}}{2};\text{Let } p_1=-s_1,p_2=-s_2$$

$$p_1+p_2=\frac{\frac{R}{L}+\sqrt{(\frac{R}{L})^2-\frac{4}{LC}}}{2}+\frac{\frac{R}{L}-\sqrt{(\frac{R}{L})^2-\frac{4}{LC}}}{2}=\frac{R}{L};\,p_1\times p_2=\frac{\frac{R}{L}+\sqrt{(\frac{R}{L})^2-\frac{4}{LC}}}{2}\times\frac{\frac{R}{L}-\sqrt{(\frac{R}{L})^2-\frac{4}{LC}}}{2}=\frac{1}{4}[(\frac{R}{L})^2+\frac{R}{L}(\sqrt{(\frac{R}{L})^2-\frac{4}{LC}}-\sqrt{(\frac{R}{L})^2-\frac{4}{LC}})-\{(\frac{R}{L})^2-\frac{4}{LC}\}]=\frac{1}{LC}$$

$$H(s)=\frac{s(p_1+p_2)}{(s+p_1)(s+p_2)}\Rightarrow H(j\omega)=\frac{j\omega(p_1+p_2)}{(j\omega+p_1)(j\omega+p_2)}=\frac{j\omega\frac{(p_1+p_2)}{p_1p_2}}{(j\frac{\omega}{p_1}+1)(j\frac{\omega}{p_2}+1)}=\frac{jf\frac{(f_1+f_2)}{f_1f_2}}{(j\frac{f}{f_1}+1)(j\frac{f}{f_2}+1)};\text{where } f_1=\frac{p_1}{2\pi};\,f_2=\frac{p_2}{2\pi}$$

$$=\frac{f\frac{(f_1+f_2)}{f_1f_2}}{\sqrt{(\frac{f}{f_1})^2+1}\sqrt{(\frac{f}{f_2})^2+1}}\angle\frac{\pi}{2}-\tan^{-1}(\frac{f}{f_1})-\tan^{-1}(\frac{f}{f_2})$$

OR

$$=\frac{jf\frac{(f_1+f_2)}{f_1f_2}}{(1-\frac{f^2}{f_1f_2})+j(\frac{f}{f_2}+\frac{f}{f_1})}=\frac{f\frac{(f_1+f_2)}{f_1f_2}}{\sqrt{(1-\frac{f^2}{f_1f_2})^2+(\frac{f}{f_2}+\frac{f}{f_1})^2}}\angle\frac{\pi}{2}-\tan^{-1}(\frac{\frac{f}{f_2}+\frac{f}{f_1}}{1-\frac{f^2}{f_1f_2}})$$

$$\frac{f_1f_2}{f_1+f_2}=\frac{f_1f_2}{f_1(1+\frac{f_2}{f_1})}=\frac{f_2}{(1+\frac{f_2}{f_1})}<f_2;\frac{f_1f_2}{f_1+f_2}=\frac{f_1}{(1+\frac{f_1}{f_2})}<f_1;$$

RLC Circuits

$$H(j0)=0\angle 90^\circ$$

$$H(jf_1)=\frac{\frac{(f_1+f_2)}{f_2}}{\sqrt{2}\sqrt{(\frac{f_1}{f_2})^2+1}}\angle\frac{\pi}{2}-\tan^{-1}(1)-\tan^{-1}(\frac{f_1}{f_2})=\frac{f_1+f_2}{\sqrt{2}\sqrt{(f_1)^2+(f_2)^2}}\angle\frac{\pi}{4}-\tan^{-1}(\frac{f_1}{f_2})$$

$$H(j\sqrt{f_1f_2})=\frac{\frac{(f_1+f_2)}{f_1f_2}}{(\frac{1}{f_2}+\frac{1}{f_1})}\angle\frac{\pi}{2}-\frac{\pi}{2}=1\angle 0$$

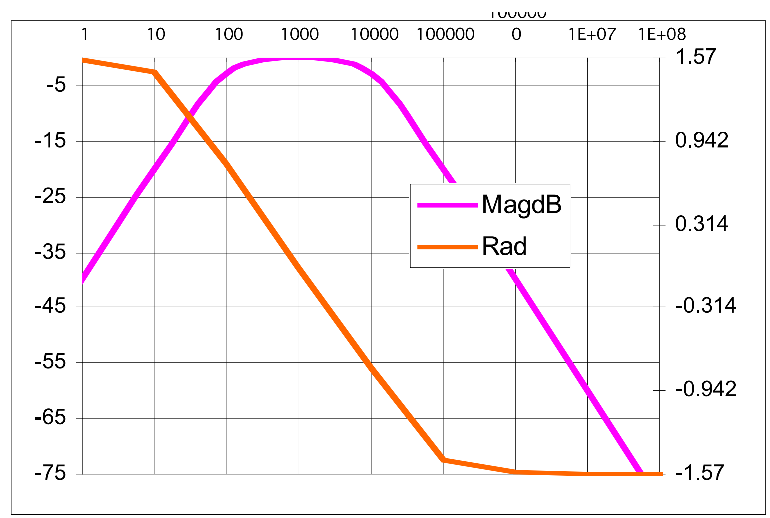
$$H(jf_2)=H(jf_1)=\frac{\frac{(f_1+f_2)}{f_1}}{\sqrt{2}\sqrt{(\frac{f_2}{f_1})^2+1}}\angle\frac{\pi}{2}-\tan^{-1}(\frac{f_2}{f_1})-\tan^{-1}(1)=\frac{f_1+f_2}{\sqrt{2}\sqrt{(f_2)^2+(f_1)^2}}\angle\frac{\pi}{4}-\tan^{-1}(\frac{f_2}{f_1})$$

$$\text{If } f_2>>f_1;\ H(jf_2)=\frac{f_1+f_2}{\sqrt{2}\sqrt{(f_2)^2+(f_1)^2}}\angle\frac{\pi}{4}-\tan^{-1}(\frac{f_2}{f_1})\approx\frac{1}{\sqrt{2}}\angle-\frac{\pi}{4}$$

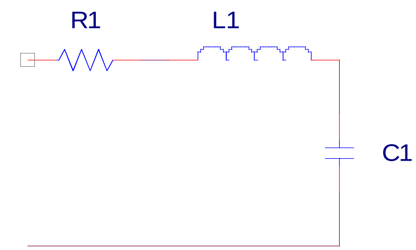
$$H(j\omega\to\infty)=0\angle-90^\circ$$

f_1=100 Hz

f_2=1000 Hz



RLC Circuits



$$H(s)=\frac{\frac{1}{sC}}{R+sL+\frac{1}{sC}}=\frac{1}{(sC)(sL)+sRC+1}=\frac{\frac{1}{LC}}{s^2+s\frac{R}{L}+\frac{1}{LC}}=\frac{\frac{1}{LC}}{(s-s_1)(s-s_2)}$$

$$\text{where } s_1,s_2=\frac{-RC\pm\sqrt{(RC)^2-4LC}}{2LC}=\frac{-\frac{R}{L}\pm\sqrt{(\frac{R}{L})^2-\frac{4}{LC}}}{2};\text{Let } p_1=-s_1,\, p_2=-s_2$$

$$p_1+p_2=\frac{\frac{R}{L}+\sqrt{(\frac{R}{L})^2-\frac{4}{LC}}}{2}+\frac{\frac{R}{L}-\sqrt{(\frac{R}{L})^2-\frac{4}{LC}}}{2}=\frac{R}{L};\, p_1\times p_2=\frac{\frac{R}{L}+\sqrt{(\frac{R}{L})^2-\frac{4}{LC}}}{2}\times\frac{\frac{R}{L}-\sqrt{(\frac{R}{L})^2-\frac{4}{LC}}}{2}=\frac{1}{4}[(\frac{R}{L})^2+\frac{R}{L}(\sqrt{(\frac{R}{L})^2-\frac{4}{LC}}-\sqrt{(\frac{R}{L})^2-\frac{4}{LC}})-\{(\frac{R}{L})^2-\frac{4}{LC}\}]=\frac{1}{LC}$$

$$H(s)=\frac{p_1p_2}{(s+p_1)(s+p_2)}\Rightarrow H(j\omega)=\frac{p_1p_2}{(j\omega+p_1)(j\omega+p_2)}=\frac{1}{(j\frac{\omega}{p_1}+1)(j\frac{\omega}{p_2}+1)}=\frac{1}{(j\frac{f}{f_1}+1)(j\frac{f}{f_2}+1)};\text{where } f_1=\frac{p_1}{2\pi};\, f_2=\frac{p_2}{2\pi}$$

$$=\frac{1}{\sqrt{(\frac{f}{f_1})^2+1}\sqrt{(\frac{f}{f_2})^2+1}}\angle-\tan^{-1}(\frac{f}{f_1})-\tan^{-1}(\frac{f}{f_2})$$

OR

$$=\frac{1}{(1-\frac{f^2}{f_1f_2})+j(\frac{f}{f_2}+\frac{f}{f_1})}=\frac{1}{\sqrt{(1-\frac{f^2}{f_1f_2})^2+(\frac{f}{f_2}+\frac{f}{f_1})^2}}\angle-\tan^{-1}(\frac{\frac{f}{f_2}+\frac{f}{f_1}}{1-\frac{f^2}{f_1f_2}})$$

RLC Circuits

$$H(j0)=1\angle 0^\circ$$

$$H(jf_1)=\frac{1}{\sqrt{2}\sqrt{(\frac{f_1}{f_2})^2+1}}\angle -\tan^{-1}(1)-\tan^{-1}(\frac{f_1}{f_2})=\frac{f_2}{\sqrt{2}\sqrt{(f_1)^2+(f_2)^2}}\angle -\frac{\pi}{4}-\tan^{-1}(\frac{f_1}{f_2})$$

$$\text{If } f_2>>f_1; H(jf_2)=\frac{f_2}{\sqrt{2}\sqrt{(f_2)^2+(f_1)^2}}\angle -\frac{\pi}{4}-\tan^{-1}(\frac{f_1}{f_2})\approx\frac{1}{\sqrt{2}}\angle -\frac{\pi}{4}$$

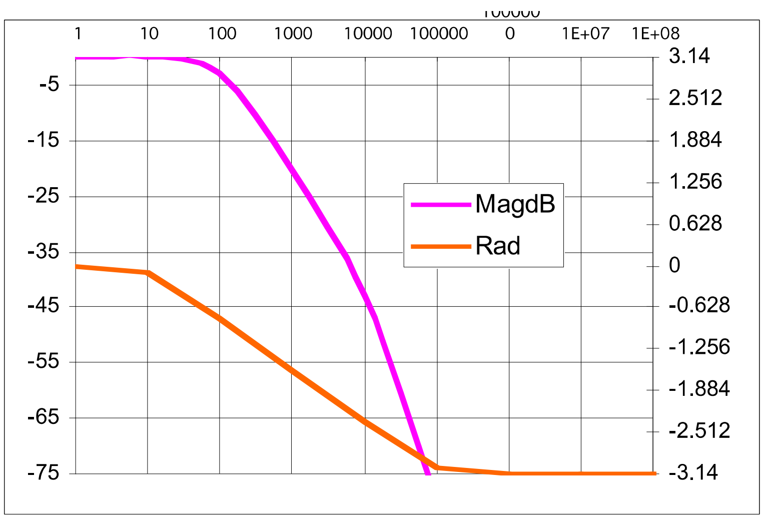
$$H(j\sqrt{f_1f_2})=\frac{1}{(\frac{1}{f_2}+\frac{1}{f_1})}\angle -\frac{\pi}{2}=\frac{f_1f_2}{f_1+f_2}\angle -\frac{\pi}{2}$$

$$H(jf_2)=H(jf_1)=\frac{1}{\sqrt{2}\sqrt{(\frac{f_2}{f_1})^2+1}}\angle -\tan^{-1}(\frac{f_2}{f_1})-\tan^{-1}(1)=\frac{f_1}{\sqrt{2}\sqrt{(f_2)^2+(f_1)^2}}\angle -\frac{\pi}{4}-\tan^{-1}(\frac{f_2}{f_1})$$

$$H(j\omega\to\infty)=0\angle -\pi$$

f_1=100 Hz

f_2=1000 Hz



Homework

- Bode Plots
  - Problems: 8.7-9